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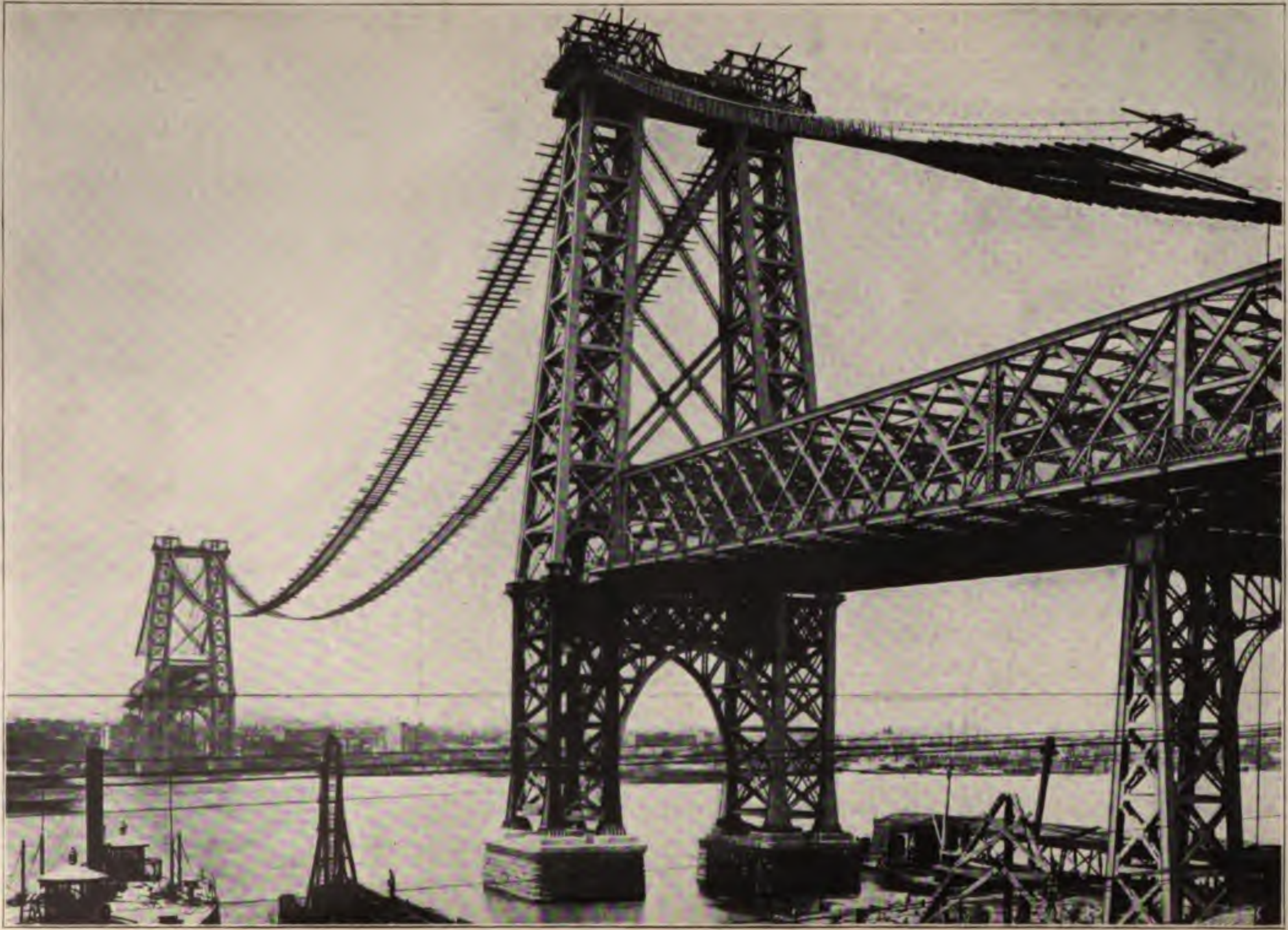

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CONSTRUCTION, HYDRAULIC ENGINEERING, IRRIGATION,
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Assisted by a Corps of
CIVIL AND CONSULTING ENGINEERS AND TECHNICAL EXPERTS OF THE
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THE editors have freely consulted the standard technical literature of America and Europe in the preparation of these volumes. They desire to express their indebtedness, particularly, to the following eminent authorities, whose well-known treatises should be in the library of everyone interested in Civil Engineering.

Grateful acknowledgment is here made also for the invaluable co-operation of the foremost Civil, Structural, Railroad, Hydraulic, and Sanitary Engineers in making these volumes thoroughly representative of the very best and latest practice in every branch of the broad field of Civil Engineering; also for the valuable drawings and data, illustrations, suggestions, criticisms, and other courtesies.

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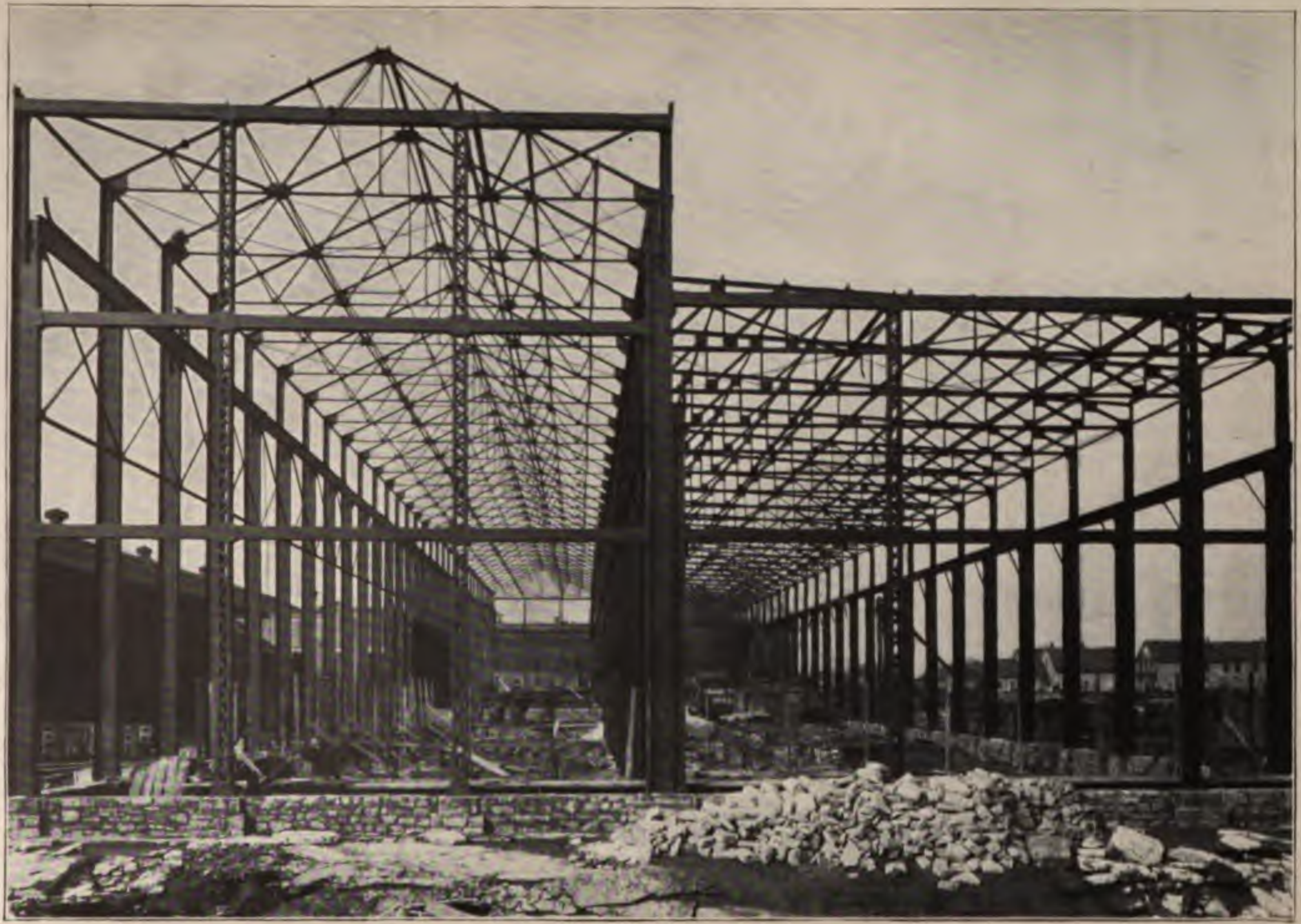

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Foreword

THE marvelous developments of the present day in the field of Civil Engineering, as seen in the extension of railroad lines, the improvement of highways and waterways, the increasing application of steel and reinforced concrete to construction work, the development of water power and irrigation projects, etc., have created a distinct necessity for an authoritative work of general reference embodying the results and methods of the latest engineering achievement. The Cyclopedia of Civil Engineering is designed to fill this acknowledged need.

¶ The aim of the publishers has been to create a work which, while adequate to meet all demands of the technically trained expert, will appeal equally to the self-taught practical man, who, as a result of the unavoidable conditions of his environment, may be denied the advantages of training at a resident technical school. The Cyclopedia covers not only the fundamentals that underlie all civil engineering, but their application to all types of engineering problems; and, by placing the reader in direct contact with the experience of teachers fresh from practical work, furnishes him that adjustment to advanced modern needs and conditions which is a necessity even to the technical graduate.

¶ The Cyclopedia of Civil Engineering is a compilation of representative Instruction Books of the American School of Correspondence, and is based upon the method which this school has developed and effectively used for many years in teaching the principles and practice of engineering in its different branches. The success attained by this institution as a factor in the machinery of modern technical education is in itself the best possible guarantee for the present work.

¶ Therefore, while these volumes are a marked innovation in technical literature — representing, as they do, the best ideas and methods of a large number of *different* authors, each an acknowledged authority in his work — they are by no means an experiment, but are in fact based on what long experience has demonstrated to be the best method yet devised for the education of the busy workingman. They have been prepared only after the most careful study of modern needs as developed under conditions of actual practice at engineering headquarters and in the field.

¶ Grateful acknowledgment is due the corps of authors and collaborators — engineers of wide practical experience, and teachers of well-recognized ability — without whose co-operation this work would have been impossible.

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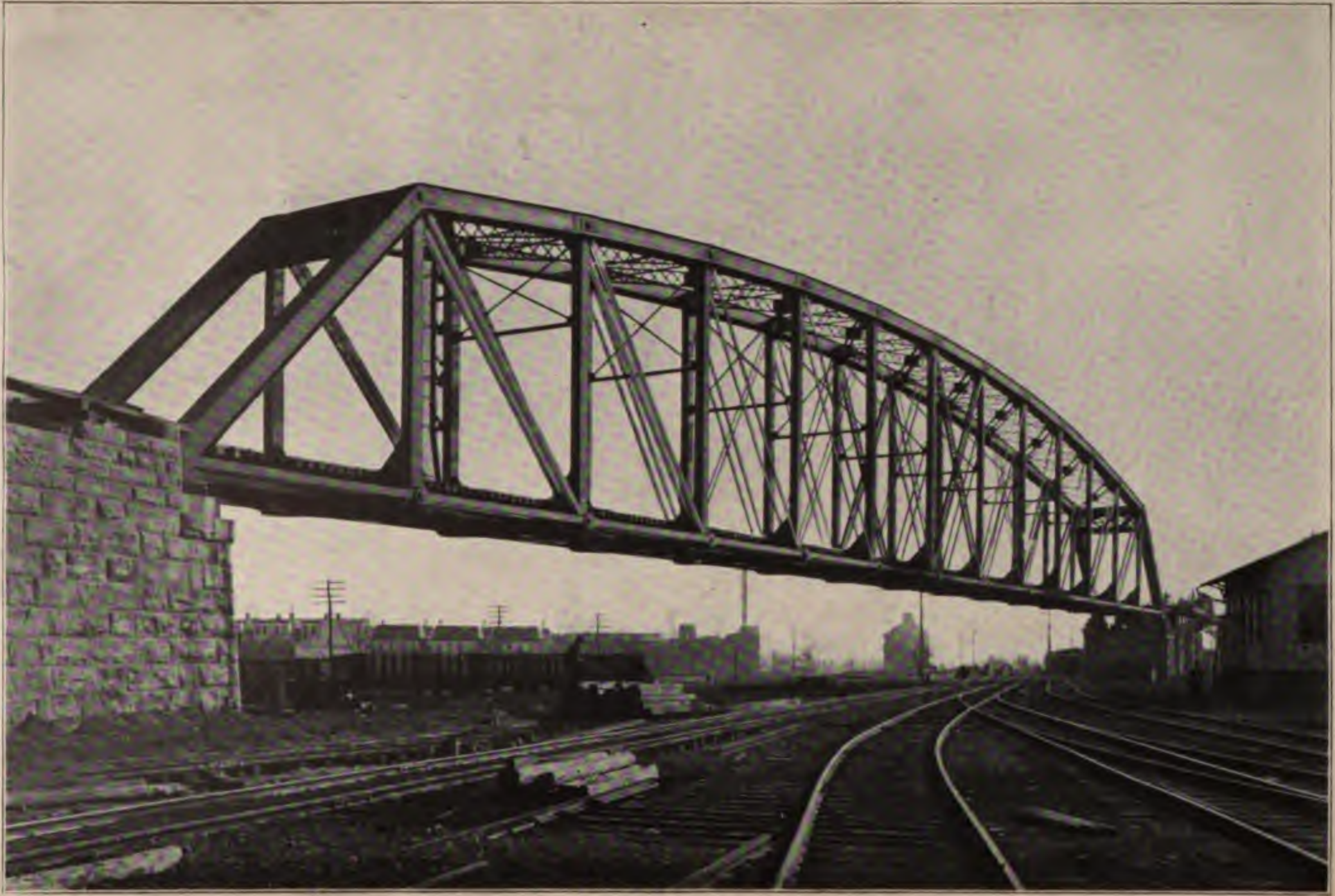

BRIDGE AT 52D STREET OVER FREIGHT YARD OF THE PENNSYLVANIA RAILROAD, PITTSBURG, PA.

One single-track, through, pin-connected span of 387 feet 10 inches. Built in 1902.

STRENGTH OF MATERIALS.

PART I.

SIMPLE STRESS.

1. Stress. When forces are applied to a body they tend in a greater or less degree to break it. Preventing or tending to prevent the rupture, there arise, generally, forces between every two adjacent parts of the body. Thus, when a load is suspended by means of an iron rod, the rod is subjected to a downward pull at its lower end and to an upward pull at its upper end, and these two forces tend to pull it apart. At any cross-section of the rod the iron on either side "holds fast" to that on the other, and these forces which the parts of the rod exert upon each other prevent the tearing of the rod. For example, in Fig. 1, let *a* represent the rod and its suspended load, 1,000 pounds; then the pull on the lower end equals 1,000 pounds. If we neglect the weight of the rod, the pull on the upper end is also 1,000 pounds, as shown in Fig. 1 (*b*); and the upper part A exerts on the lower part B an upward pull Q equal to 1,000 pounds, while the lower part exerts on the upper a force P also equal to 1,000 pounds. These two forces, P and Q, prevent rupture of the rod at the "section" C; at any other section there are two forces like P and Q preventing rupture at that section.

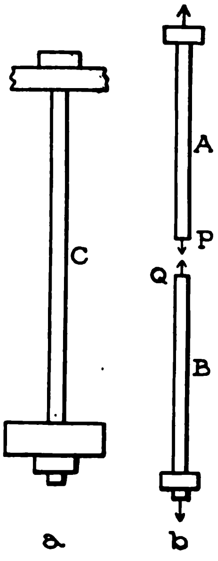

Fig. 1.

By *stress at a section* of a body is meant the force which the part of the body on either side of the section exerts on the other. Thus, the stress at the section C (Fig. 1) is P (or Q), and it equals 1,000 pounds.

2. Stresses are usually expressed (in America) in pounds, sometimes in tons. Thus the stress P in the preceding article is

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1,000 pounds, or ½ ton. Notice that this value has nothing to do with the size of the cross-section on which the stress acts.

3. Kinds of Stress. (a) When the forces acting on a body (as a rope or rod) are such that they tend to tear it, the stress at any cross-section is called a *tension* or a *tensile stress*. The stresses P and Q, of Fig. 1, are tensile stresses. Stretched ropes, loaded "tie rods" of roofs and bridges, etc., are under tensile stress.

(b.) When the forces acting on a body (as a short post, brick, etc.) are such that they tend to crush it, the stress at any section at right angles to the direction of the crushing forces is called a *pressure* or a *compressive stress*. Fig. 2 (*a*) represents a loaded post, and Fig. 2 (*b*) the upper and lower parts. The upper part presses down on B, and the lower part presses up on A, as shown. P or Q is the compressive stress in the post at section C. Loaded posts, or struts, piers, etc., are under compressive stress.

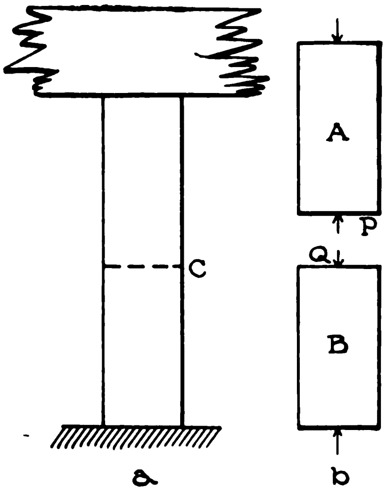

Fig. 2.

(c.) When the forces acting on a body (as a rivet in a bridge joint) are such that they tend to cut or "shear" it across, the stress at a section along which there is a tendency to cut is called a *shear* or a *shearing stress*. This kind of stress takes its name from the act of cutting with a pair of shears. In a material which is being cut in this way, the stresses that are being "overcome" are shearing stresses. Fig. 3 (*a*) represents a riveted joint, and Fig. 3 (*b*) two parts of the rivet. The forces applied to the joint are such that A tends to slide to the left, and B to the right; then B exerts on A a force P toward the right, and A on B a force Q toward the left as shown. P or Q is the shearing stress in the rivet.

Tensions, Compressions and Shears are called *simple stresses*. Forces may act upon a body so as to produce a combination of simple stresses on some section; such a combination is called a *complex* *stress.* The stresses in beams are usually complex. There are other terms used to describe stress; they will be defined farther on.

4. **Unit-Stress.** It is often necessary to specify not merely the amount of the entire stress which acts on an area, but also the amount which acts on each unit of area (square inch for example). By unit-stress is meant stress per unit area.

To find the value of a unit-stress: *Divide the whole stress by the whole area of the section on which it acts, or over which it is distributed.* Thus, let

P denote the value of the whole stress,
A the area on which it acts, and
S the value of the unit-stress; then

$$S=\frac{P}{A}, \text{ also } P=AS. \qquad (1)$$

Strictly these formulas apply only when the stress P is uniform, that is, when it is uniformly distributed over the area, each square inch for example sustaining the same amount of stress. When the stress is not uniform, that is, when the stresses on different square inches are not equal, then $P \div A$ equals the *average value* of the unit-stress.

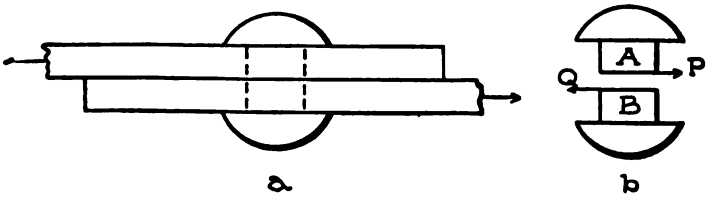

Fig. 3.

5. Unit-stresses are usually expressed (in America) in pounds per square inch, sometimes in tons per square inch. If P and A in equation 1 are expressed in pounds and square inches respectively, then S will be in pounds per square inch; and if P and A are expressed in tons and square inches, S will be in tons per square inch.

Examples. 1. Suppose that the rod sustaining the load in Fig. 1 is 2 square inches in cross-section, and that the load weighs 1,000 pounds. What is the value of the unit-stress?

Here $P = 1{,}000$ pounds, $A = 2$ square inches; hence.

$$S = \frac{1{,}000}{2} = 500 \text{ pounds per square inch.}$$

2. Suppose that the rod is one-half square inch in cross-section. What is the value of the unit-stress?

$A = \frac{1}{2}$-square inch, and, as before, $P = 1{,}000$ pounds; hence

$$S = 1{,}000 \div \frac{1}{2} = 2{,}000 \text{ pounds per square inch.}$$

Notice that one must always divide the whole stress by the area to get the unit-stress, whether the area is greater or less than one.

6. Deformation. Whenever forces are applied to a body it changes in size, and usually in shape also. This change of size and shape is called deformation. Deformations are usually measured in inches; thus, if a rod is stretched 2 inches, the "elongation" = 2 inches.

7. Unit-Deformation. It is sometimes necessary to specify not merely the value of a total deformation but its amount per unit length of the deformed body. Deformation per unit length of the deformed body is called unit-deformation.

To find the value of a unit-deformation: *Divide the whole deformation by the length over which it is distributed.* Thus, if

D denotes the value of a deformation,
l the length,
s the unit-deformation, then

$$s = \frac{D}{l}, \text{ also } D = ls. \qquad (2)$$

Both D and l should always be expressed in the same unit.

Example. Suppose that a 4-foot rod is elongated $\frac{1}{2}$ inch. What is the value of the unit-deformation?

Here $D = \frac{1}{2}$ inch, and $l = 4$ feet $= 48$ inches;
hence $s = \frac{1}{2} \div 48 = \frac{1}{96}$ inch per inch.

That is, each inch is elongated $\frac{1}{96}$ inch.

Unit-elongations are sometimes expressed in per cent. To express an elongation in per cent: *Divide the elongation in inches by the original length in inches, and multiply by 100.*

8. Elasticity. Most solid bodies when deformed will regain more or less completely their natural size and shape when the de-forming forces cease to act. This property of regaining size and shape is called elasticity.

We may classify bodies into kinds depending on the degree of elasticity which they have, thus:

1. *Perfectly elastic* bodies; these will regain their original form and size no matter how large the applied forces are if less than breaking values. Strictly there are no such materials, but rubber, practically, is perfectly elastic.

2. *Imperfectly elastic* bodies; these will fully regain their original form and size if the applied forces are not too large, and practically even if the loads are large but less than the breaking value. Most of the constructive materials belong to this class.

3. *Inelastic* or *plastic* bodies; these will not regain in the least their original form when the applied forces cease to act. Clay and putty are good examples of this class.

9. Hooke's Law, and Elastic Limit. If a gradually increasing force is applied to a perfectly elastic material, the deformation increases proportionally to the force; that is, if P and P′ denote two values of the force (or stress), and D and D′ the values of the deformation produced by the force,

$$\text{then } P:P'::D:D'.$$

This relation is also true for imperfectly elastic materials, provided that the loads P and P′ do not exceed a certain limit depending on the material. Beyond this limit, the deformation increases much faster than the load; that is, if within the limit an addition of 1,000 pounds to the load produces a stretch of 0.01 inch, beyond the limit an equal addition produces a stretch larger and usually much larger than 0.01 inch.

Beyond this limit of proportionality a part of the deformation is permanent; that is, if the load is removed the body only partially recovers its form and size. The permanent part of a deformation is called *set*.

The fact that for most materials the deformation is proportional to the load within certain limits, is known as Hooke's Law. The unit-stress within which Hooke's law holds, or above which the deformation is not proportional to the load or stress, is called *elastic limit*.

10. Ultimate Strength. By ultimate tensile, compressive, or shearing strength of a material is meant the greatest tensile, compressive, or shearing unit-stress which it can withstand.

As before mentioned, when a material is subjected to an increasing load the deformation increases faster than the load beyond the elastic limit, and much faster near the stage of rupture. Not only do tension bars and compression blocks elongate and shorten respectively, but their cross-sectional areas change also; tension bars thin down and compression blocks "swell out" more or less. The value of the ultimate strength for any material is ascertained by subjecting a specimen to a gradually increasing tensile, compressive, or shearing stress, as the case may be, until rupture occurs, and measuring the greatest load. *The breaking load divided by the area of the original cross-section sustaining the stress, is the value of the ultimate strength.*

Example. Suppose that in a tension test of a wrought-iron rod $\frac{1}{2}$ inch in diameter the greatest load was 12,540 pounds. What is the value of the ultimate strength of that grade of wrought iron?

The original area of the cross-section of the rod was

$0.7854\ (\text{diameter})^2 = 0.7854 \times \frac{1}{4} = 0.1964$ square inches; hence the ultimate strength equals

$$12{,}540 \div 0.1964 = 63{,}850 \text{ pounds per square inch.}$$

11. Stress-Deformation Diagram. A "test" to determine the elastic limit, ultimate strength, and other information in regard to a material is conducted by applying a gradually increasing load until the specimen is broken, and noting the deformation corresponding to many values of the load. The first and second columns of the following table are a record of a tension test on a steel rod one inch in diameter. The numbers in the first column are the values of the pull, or the loads, at which the elongation of the specimen was measured. The elongations are given in the second column. The numbers in the third and fourth columns are the values of the unit-stress and unit-elongation corresponding to the values of the load opposite to them. The numbers in the third column were obtained from those in the first by dividing the latter by the area of the cross-section of the rod, 0.7854 square inches. Thus,

$$3{,}930 \div 0.7854 = 5{,}000$$

$$7{,}850 \div 0.7854 = 10{,}000, \text{ etc.}$$

Total Pull in pounds, P	Deformation in inches, D	Unit-Stress in pounds per square inch, S	Unit-Deformation, s
3930	0.00136	5000	0.00017
7850	.00280	10000	.00035
11780	.00404	15000	.00050
15710	.00538	20000	.00067
19635	.00672	25000	.00084
23560	.00805	30000	.00101
27490	.00942	35000	.00118
31415	.01080	40000	.00135
35345	.01221	45000	.00153
39270	.0144	50000	.00180
43200	.0800	55000	.0100
47125	.1622	60000	.0202
51050	.201	65000	.0251
54980	.281	70000	.0351
58910	.384	75000	.048
62832	.560	80000	.070
65200	1.600	83000	.200

The numbers in the fourth column were obtained by dividing those in the second by the length of the specimen (or rather the length of that part whose elongation was measured), 8 inches. Thus,

$$0.00136 \div 8 = 0.00017,$$
$$.00280 \div 8 = .00035, \text{ etc.}$$

Looking at the first two columns it will be seen that the elongations are practically proportional to the loads up to the ninth load, the increase of stretch for each increase in load being about 0.00135 inch; but beyond the ninth load the increases of stretch are much greater. Hence the elastic limit was reached at about the ninth load, and its value is about 45,000 pounds per square inch. The greatest load was 65,200 pounds, and the corresponding unit-stress, 83,000 pounds per square inch, is the ultimate strength.

Nearly all the information revealed by such a test can be well represented in a diagram called a *stress-deformation diagram*. It is made as follows: Lay off the values of the unit-deformation (fourth column) along a horizontal line, according to some convenient scale, from some fixed point in the line. At the points on the horizontal line representing the various unit-elongations, lay off perpendicular distances equal to the corresponding unit-stresses. Then connect by a smooth curve the upper ends of all those distances, last distances laid off. Thus, for instance, the highest unit-elongation (0.20) laid off from *o* (Fig. 4) fixes the point *a*, and a perpendicular distance to represent the highest unit-stress (83,000) fixes the point *b*. All the points so laid off give the curve *ocb*. The part *oc*, within the elastic limit, is straight and nearly vertical while the remainder is curved and more or less horizontal, especially toward the point of rupture *b*. Fig. 5 is a typical stress-deformation diagram for timber, cast iron, wrought iron, soft and hard steel, in tension and compression.

12. Working Stress and Strength, and Factor of Safety. The greatest unit-stress in any part of a structure when it is sustaining its loads is called the *working stress* of that part. If it is under tension, compression and shearing stresses, then the corresponding highest unit-stresses in it are called its working stress in tension, in compression, and in shear respectively; that is, we speak of as many working stresses as it has kinds of stress.

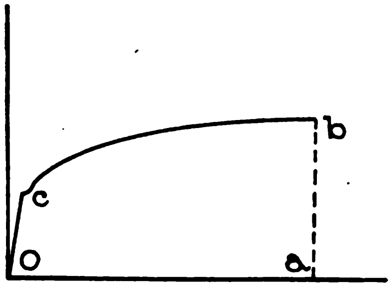

Fig. 4.

By *working strength* of a material to be used for a certain purpose is meant the highest unit-stress to which the material ought to be subjected when so used. Each material has a working strength for tension, for compression, and for shear, and they are in general different.

By *factor of safety* is meant the ratio of the ultimate strength of a material to its working stress or strength. Thus, if

S_u denotes ultimate strength,
S_w denotes working stress or strength, and
f' denotes factor of safety, then

$$f' = \frac{S_u}{S_w}; \text{ also } S_w = \frac{S_u}{f}. \qquad (3)$$

When a structure which has to stand certain loads is about to be designed, it is necessary to select working strengths or factors of safety for the materials to be used. Often the selection is a matter of great importance, and can be wisely performed only by an experienced engineer, for this is a matter where hard-and-fast rules should not govern but rather the judgment of the expert. But there are certain principles to be used as guides in making a selection, chief among which are:

1. The working strength should be considerably below the elastic limit. (Then the deformations will be small and not permanent.)

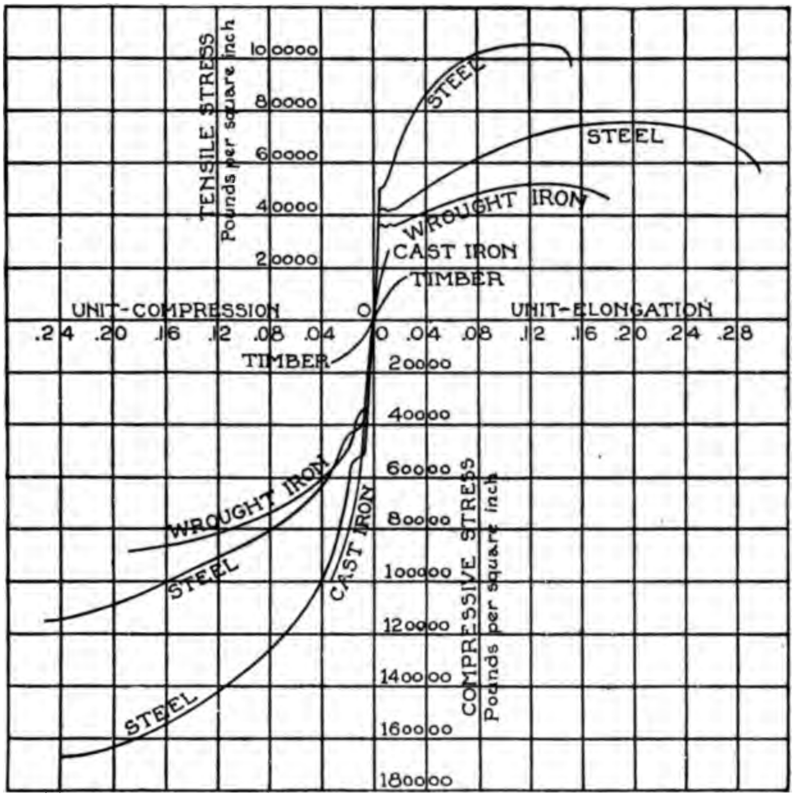

Fig. 5. (After Johnson.)

2. The working strength should be smaller for parts of a structure sustaining varying loads than for those whose loads are steady. (Actual experiments have disclosed the fact that the strength of a specimen depends on the kind of load put upon it, and that in a general way it is less the less steady the load is.)

3. The working strength must be taken low for non-uniform material, where poor workmanship may be expected, when the

loads are uncertain, etc. Principles 1 and 2 have been reduced to figures or formulas for many particular cases, but the third must remain a subject for display of judgment, and even good guessing in many cases.

The following is a table of factors of safety* which will be used in the problems:

Factors of Safety.

Materials.	For steady stress. (Buildings.)	For varying stress. (Bridges.)	For shocks. (Machines.)
Timber	8	10	15
Brick and stone	15	25	30
Cast iron	6	15	20
Wrought iron	4	6	10
Steel	5	7	15

They must be regarded as average values and are not to be adopted in every case in practice.

Examples. 1. A wrought-iron rod 1 inch in diameter sustains a load of 30,000 pounds. What is its working stress? If its ultimate strength is 50,000 pounds per square inch, what is its factor of safety?

The area of the cross-section of the rod equals $0.7854 \times (\text{diameter})^2 = 0.7854 \times 1^2 = 0.7854$ square inches. Since the whole stress on the cross-section is 30,000 pounds, equation 1 gives for the unit working stress

$$S = \frac{30{,}000}{0.7854} = 38{,}197 \text{ pounds per square inch.}$$

Equation 3 gives for factor of safety

$$f = \frac{50{,}000}{38{,}197} = 1.3$$

2. How large a steel bar or rod is needed to sustain a steady pull of 100,000 pounds if the ultimate strength of the material is 65,000 pounds?

The load being steady, we use a factor of safety of 5 (see table above); hence the working strength to be used (see equation 3) is

$$S = \frac{65{,}000}{5} = 13{,}000 \text{ pounds per square inch.}$$

The proper area of the cross-section of the rod can now be computed from equation 1 thus:

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*Taken from Merriman's "Mechanics of Materials."

$$A=\frac{P}{S}=\frac{100,000}{13,000}=7.692 \text{ square inches.}$$

A bar 2×4 inches in cross-section would be a little stronger than necessary. To find the diameter (d) of a round rod of sufficient strength, we write $0.7854\ d^2=7.692$, and solve the equation for d; thus:

$$d^2=\frac{7.692}{0.7854}=9.794, \text{ or } d=3.129 \text{ inches.}$$

3. How large a steady load can a short timber post safely sustain if it is 10×10 inches in cross-section and its ultimate compressive strength is 10,000 pounds per square inch ?

According to the table (page 12) the proper factor of safety is 8, and hence the working strength according to equation 3 is

$$S=\frac{10,000}{8}=1,250 \text{ pounds per square inch.}$$

The area of the cross-section is 100 square inches; hence the safe load (see equation 1) is

$$P=100\times1,250=125,000 \text{ pounds.}$$

4. When a hole is punched through a plate the shearing strength of the material has to be overcome. If the ultimate shearing strength is 50,000 pounds per square inch, the thickness of the plate $\frac{1}{2}$ inch, and the diameter of the hole $\frac{3}{4}$ inch, what is the value of the force to be overcome ?

The area shorn is that of the cylindrical surface of the hole or the metal punched out; that is

$$3.1416\times\text{diameter}\times\text{thickness}=3.1416\times\tfrac{3}{4}\times\tfrac{1}{2}=1.178 \text{ sq. in.}$$

Hence, by equation 1, the total shearing strength or resistance to punching is

$$P=1.178\times50,000=58,900 \text{ pounds.}$$

STRENGTH OF MATERIALS UNDER SIMPLE STRESS.

13. Materials in Tension. Practically the only materials used extensively under tension are timber, wrought iron and steel, and to some extent cast iron.

14. Timber. A successful tension test of wood is difficult, as the specimen usually crushes at the ends when held in the testing machine, splits, or fails otherwise than as desired. Hence the tensile strengths of woods are not well known, but the following may be taken as approximate average values of the ultimate strengths of the woods named, when "dry out of doors."

Hemlock,	7,000	pounds per square inch.
White pine,	8,000	" "
Yellow pine, long leaf,	12,000	" "
" " , short leaf,	10,000	" "
Douglas spruce,	10,000	" "
White oak,	12,000	" "
Red oak,	9,000	" "

15. Wrought Iron. The process of the manufacture of wrought iron gives it a "grain," and its tensile strengths along and across the grain are unequal, the latter being about three-fourths of the former. The ultimate tensile strength of wrought iron along the grain varies from 45,000 to 55,000 pounds per square inch. Strength along the grain is meant when not otherwise stated.

The strength depends on the size of the piece, it being greater for small than for large rods or bars, and also for thin than for thick plates. The elastic limit varies from 25,000 to 40,000 pounds per square inch, depending on the size of the bar or plate even more than the ultimate strength. Wrought iron is very ductile, a specimen tested in tension to destruction elongating from 5 to 25 per cent of its length.

16. Steel. Steel has more or less of a grain but is practically of the same strength in all directions. To suit different purposes, steel is made of various grades, chief among which may be mentioned rivet steel, sheet steel (for boilers), medium steel (for bridges and buildings), rail steel, tool and spring steel. In general, these grades of steel are hard and strong in the order named, the ultimate tensile strength ranging from about 50,000 to 160,000 pounds per square inch.

There are several grades of structural steel, which may be described as follows:*

1. Rivet steel:

Ultimate tensile strength, 48,000 to 58,000 pounds per square inch.
Elastic limit, not less than one-half the ultimate strength.
Elongation, 26 per cent.
Bends 180 degrees flat on itself without fracture.

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*Taken from "Manufacturer's Standard Specifications."

2. Soft steel:
   - Ultimate tensile strength, 52,000 to 62,000 pounds per square inch.
   - Elastic limit, not less than one-half the ultimate strength.
   - Elongation, 25 per cent.
   - Bends 180 degrees flat on itself.
3. Medium steel:
   - Ultimate tensile strength, 60,000 to 70,000 pounds per square inch.
   - Elastic limit, not less than one-half the ultimate strength.
   - Elongation, 22 per cent.
   - Bends 180 degrees to a diameter equal to the thickness of the specimen without fracture.

17. Cast Iron. As in the case of steel, there are many grades of cast iron. The grades are not the same for all localities or districts, but they are based on the appearance of the fractures, which vary from coarse dark grey to fine silvery white.

The ultimate tensile strength does not vary uniformly with the grades but depends for the most part on the percentage of "combined carbon" present in the iron. This strength varies from 15,000 to 35,000 pounds per square inch, 20,000 being a fair average.

Cast iron has no well-defined elastic limit (see curve for cast iron, Fig. 5). Its ultimate elongation is about one per cent.

EXAMPLES FOR PRACTICE.

1. A steel wire is one-eighth inch in diameter, and the ultimate tensile strength of the material is 150,000 pounds per square inch. How large is its breaking load? Ans. 1,840 pounds.

2. A wrought-iron rod (ultimate tensile strength 50,000 pounds per square inch) is 2 inches in diameter. How large a steady pull can it safely bear? Ans. 39,270 pounds.

18. Materials in Compression. Unlike the tensile, the compressive strength of a specimen or structural part depends on its dimension in the direction in which the load is applied, for, in compression, a long bar or rod is weaker than a short one. At present we refer only to the strength of short pieces such as do not bend under the load, the longer ones (columns) being discussed farther on.

Different materials break or fail under compression, in two very different ways:

1. Ductile materials (structural steel, wrought iron, etc.),

and wood compressed across the grain, do not fail by breaking into two distinct parts as in tension, but the former bulge out and flatten under great loads, while wood splits and mashes down. There is no particular point or instant of failure under increasing loads, and such materials have no definite ultimate strength in compression.

2. Brittle materials (brick, stone, hard steel, cast iron, etc.), and wood compressed along the grain, do not mash gradually, but fail suddenly and have a definite ultimate strength in compression. Although the surfaces of fracture are always much inclined to the direction in which the load is applied (about 45 degrees), the ultimate strength is computed by dividing the total breaking load by the cross-sectional area of the specimen.

The principal materials used under compression in structural work are timber, wrought iron, steel, cast iron, brick and stone.

19. Timber. As before noted, timber has no definite ultimate compressive strength across the grain. The U. S. Forestry Division has adopted certain amounts of compressive *deformation* as marking stages of failure. Three per cent compression is regarded as "a working limit allowable," and fifteen per cent as "an extreme limit, or as failure." The following (except the first) are values for compressive strength from the Forestry Division Reports, all in pounds per square inch:

	Ultimate strength along the grain.	3% Compression across the grain
Hemlock	6,000	
White pine	5,400	700
Long-leaf yellow pine	8,000	1,260
Short-leaf yellow pine	6,500	1,050
Douglas spruce	5,700	800
White oak	8,500	2,200
Red oak	7,200	2,300

20. Wrought Iron. The elastic limit of wrought iron, as before noted, depends very much upon the size of the bars or plate, it being greater for small bars and thin plates. Its value for compression is practically the same as for tension, 25,000 to 40,000 pounds per square inch.

21. Steel. The hard steels have the highest compressive strength; there is a recorded value of nearly 400,000 pounds per square inch, but 150,000 is probably a fair average.

The elastic limit in compression is practically the same as in tension, which is about 60 per cent of the ultimate tensile strength, or, for structural steel, about 25,000 to 42,000 pounds per square inch.

22. Cast Iron. This is a very strong material in compression, in which way, principally, it is used structurally. Its ultimate strength depends much on the proportion of "combined carbon" and silicon present, and varies from 50,000 to 200,000 pounds per square inch, 90,000 being a fair average. As in tension, there is no well-defined elastic limit in compression (see curve for cast iron, Fig. 5).

23. Brick. The ultimate strengths are as various as the kinds and makes of brick. For soft brick, the ultimate strength is as low as 500 pounds per square inch, and for pressed brick it varies from 4,000 to 20,000 pounds per square inch, 8,000 to 10,000 being a fair average. The ultimate strength of good paving brick is still higher, its average value being from 12,000 to 15,000 pounds per square inch.

24. Stone. Sandstone, limestone and granite are the principal building stones. Their ultimate strengths in pounds per square inch are about as follows:

Sandstone,* 5,000 to 16,000, average 8,000.
Limestone,* 8,000 " 16,000, " 10,000.
Granite, 14,000 " 24,000, " 16,000.

*Compression at right angles to the "bed" of the stone.

EXAMPLES FOR PRACTICE.

1. A limestone 12×12 inches on its bed is used as a pier cap, and bears a load of 120,000 pounds. What is its factor of safety? Ans. 12.

2. How large a post (short) is needed to sustain a steady load of 100,000 pounds if the ultimate compressive strength of the wood is 10,000 pounds per square inch? Ans. 10×10 inches.

25. Materials in Shear. The principal materials used under shearing stress are timber, wrought iron, steel and cast iron. Partly on account of the difficulty of determining shearing strengths, these are not well known.

26. Timber. The ultimate shearing strengths of the more important woods *along the grain* are about as follows:

Hemlock,	300	pounds per square inch.
White pine,	400	“ “
Long-leaf yellow pine,	850	“ “
Short-leaf “ “	775	“ “
Douglas spruce,	500	“ “
White oak,	1,000	“ “
Red oak,	1,100	“ “

Wood rarely fails by shearing across the grain. Its ultimate shearing strength in that direction is probably four or five times the values above given.

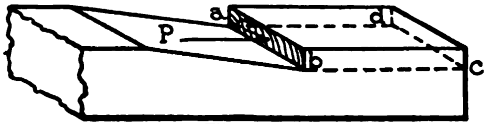


Fig. 6 *a*.

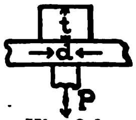


Fig. 6 *b*.

27. Metals. The ultimate shearing strength of wrought iron, steel, and cast iron is about 80 per cent of their respective ultimate tensile strengths.

EXAMPLES FOR PRACTICE.

1. How large a pressure P (Fig. 6 *a*) exerted on the shaded area can the timber stand before it will shear off on the surface *abcd*, if *ab* = 6 inches and *bc* = 10 inches, and the ultimate shearing strength of the timber is 400 pounds per square inch?

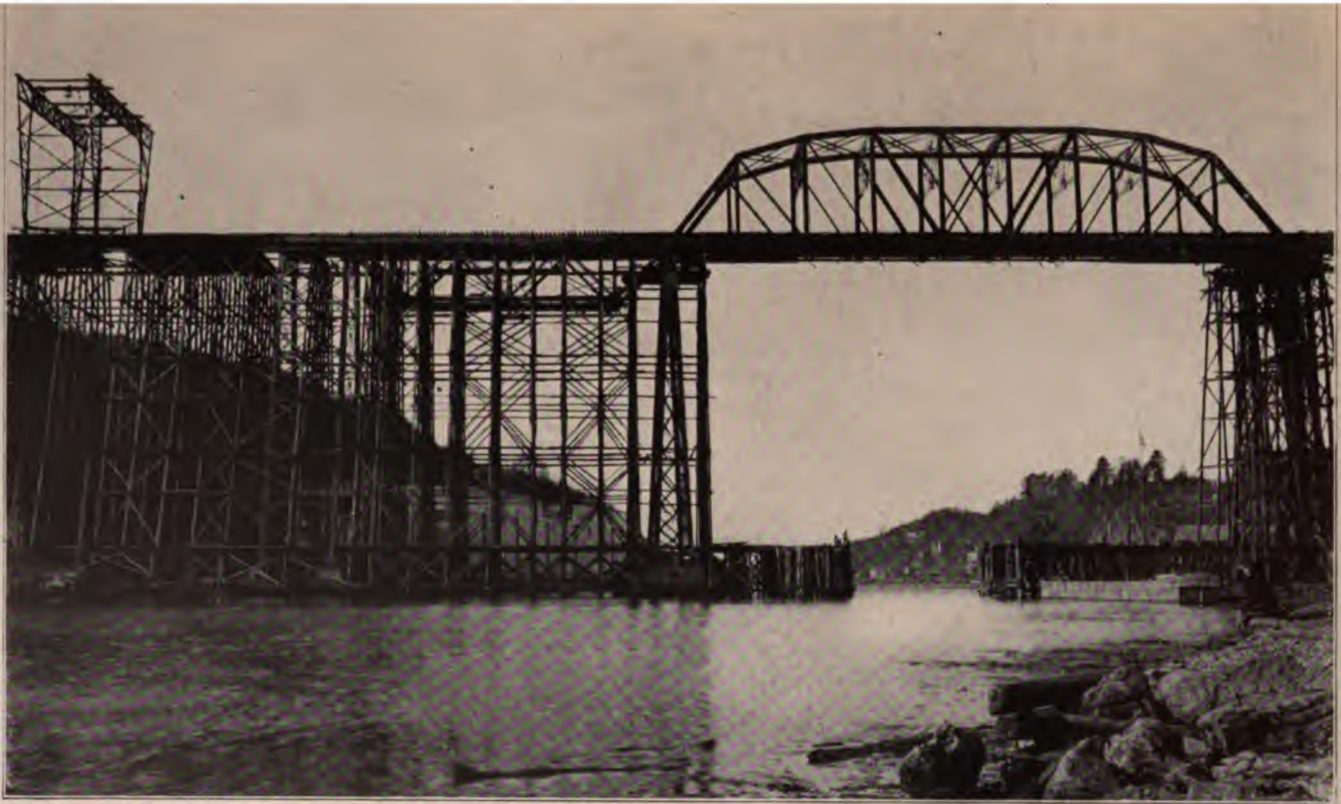
Ans. 24,000 pounds.

2. When a bolt is under tension, there is a tendency to tear the bolt and to "strip" or shear off the head. The shorn area would be the surface of the cylindrical hole left in the head. Compute the tensile and shearing unit-stresses when P (Fig. 6 *b*) equals 30,000 pounds, d = 2 inches, and t = 3 inches.

Ans. { Tensile unit-stress, 9,550 pounds per square inch.
Shearing unit-stress, 1,595 pounds per square inch.

REACTIONS OF SUPPORTS.

28. Moment of a Force. By moment of a force with respect to a point is meant its tendency to produce rotation about that point. Evidently the tendency depends on the magnitude of the force and on the perpendicular distance of the line of action of the force from the point: the greater the force and the perpendicular distance, the greater the tendency; hence *the moment*



RONDOUT CREEK BRIDGE KINGSTON, NEW YORK

On line of West Shore Railroad. Built in 1902. Consists of the following double-track spans: One through pin-connected, 270 ft.; one deck lattice, 143 ft.; eighteen deck plate-girder spans ranging from 10 ft. to 77 ft. 6 in. Height from water to rail, 160 ft.

Courtesy of American Bridge Company

of a force with respect to a point equals the product of the force and the perpendicular distance from the force to the point.

The point with respect to which the moment of one or more forces is taken is called an *origin* or *center of moments*, and the perpendicular distance from an origin of moments to the line of action of a force is called the *arm* of the force with respect to that origin. Thus, if F_1 and F_2 (Fig. 7) are forces, their arms with respect to O′ are a_1' and a_2' respectively, and their moments are $F_1a'_1$ and $F_2a'_2$. With respect to O″ their arms are a_1'' and a_2'' respectively, and their moments are F_1a_1'' and F_2a_2''.

If the force is expressed in pounds and its arm in feet, the moment is in foot-pounds; if the force is in pounds and the arm in inches, the moment is in inch-pounds.

29. A *sign* is given to the moment of a force for convenience; the rule used herein is as follows: *The moment of a force about a point is positive or negative according as it tends to turn the body about that point in the clockwise or counter-clockwise direction**.

Thus the moment (Fig. 7)
of F_1 about O′ is negative, about O″ positive;
" F_2 " O′ " " , about O″ negative.

30. Principle of Moments. In general, a single force of proper magnitude and line of action can balance any number of forces. That single force is called the *equilibrant* of the forces, and the single force that would balance the equilibrant is called the *resultant* of the forces. Or, otherwise stated, the resultant of any number of forces is a force which produces the same effect. It can be proved that—*The algebraic sum of the moments of any number of forces with respect to a point, equals the moment of their resultant about that point.*

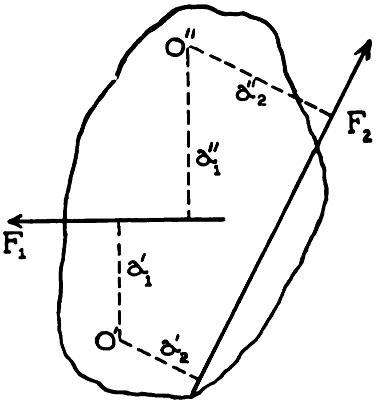


Fig. 7.

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*By clockwise direction is meant that in which the hands of a clock rotate; and by counter-clockwise, the opposite direction.

This is a useful principle and is called "principle of moments."

31. All the forces acting upon a body which is at rest are said to be *balanced* or *in equilibrium*. No force is required to balance such forces and hence their equilibrant and resultant are zero.

Since their resultant is zero, *the algebraic sum of the moments of any number of forces which are balanced or in equilibrium equals zero.*

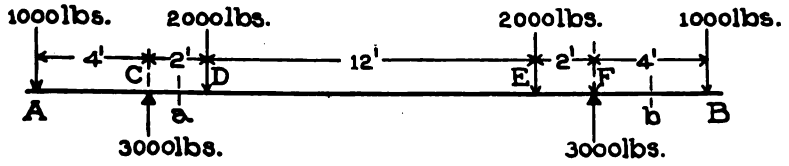


Fig. 8.

This is known as the principle of moments for forces in equilibrium; for brevity we shall call it also "the principle of moments."

The principle is easily verified in a simple case. Thus, let AB (Fig. 8) be a beam resting on supports at C and F. It is evident from the symmetry of the loading that each reaction equals one-half of the whole load, that is, $\frac{1}{2}$ of 6,000=3,000 pounds. (We neglect the weight of the beam for simplicity.)

With respect to C, for example, the moments of the forces are, taking them in order from the left:

—1,000 × 4 = — 4,000 foot-pounds
3,000 × 0 = 0 "
2,000 × 2 = 4,000 "
2,000 × 14 = 28,000 "
—3,000 × 16 = —48,000 "
1,000 × 20 = 20,000 "

The algebraic sum of these moments is seen to equal zero.

Again, with respect to B the moments are:

—1,000 × 24 = —24,000 foot-pounds
3,000 × 20 = 60,000 "
—2,000 × 18 = —36,000 "
—2,000 × 6 = —12,000 "
3,000 × 4 = 12,000 "
1,000 × 0 = 0 "

The sum of these moments also equals zero. In fact, no matter where the center of moments is taken, it will be found in this and any other balanced system of forces that the algebraic sum of their moments equals zero. The chief use that we shall make of this principle is in finding the supporting forces of loaded beams.

32. Kinds of Beams. A *cantilever beam* is one resting on one support or fixed at one end, as in a wall, the other end being free.

A *simple beam* is one resting on two supports.

A *restrained beam* is one fixed at both ends; a beam fixed at one end and resting on a support at the other is said to be restrained at the fixed end and simply supported at the other.

A *continuous beam* is one resting on more than two supports.

33. Determination of Reactions on Beams. The forces which the supports exert on a beam, that is, the "supporting forces," are called *reactions*. We shall deal chiefly with simple beams. The reaction on a cantilever beam supported at one point evidently equals the total load on the beam.

When the loads on a horizontal beam are all vertical (and this is the usual case), the supporting forces are also vertical and *the sum of the reactions equals the sum of the loads*. This principle is sometimes useful in determining reactions, but in the case of simple beams the principle of moments is sufficient. The general method of determining reactions is as follows:

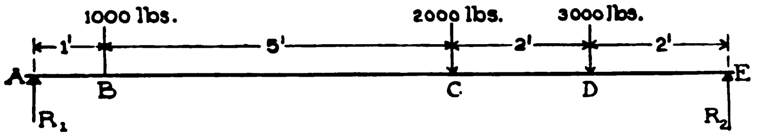


Fig. 9.

1. Write out two equations of moments for all the forces (loads and reactions) acting on the beam with origins of moments at the supports.
2. Solve the equations for the reactions.
3. As a check, try if the sum of the reactions equals the sum of the loads.

Examples. 1. Fig. 9 represents a beam supported at its ends and sustaining three loads. We wish to find the reactions due to these loads.

Let the reactions be denoted by R_1 and R_2 as shown; then the moment equations are:
For origin at A,

$$1{,}000 \times 1 + 2{,}000 \times 6 + 3{,}000 \times 8 - R_2 \times 10 = 0.$$

For origin at E,

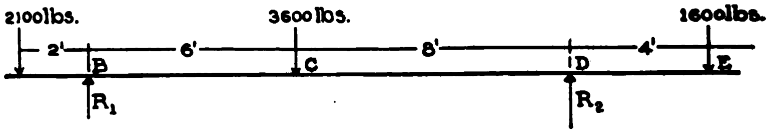


Fig. 10.

$$R_1 \times 10 - 1{,}000 \times 9 - 2{,}000 \times 4 - 3{,}000 \times 2 = 0.$$

The first equation reduces to

$$10\ R_2 = 1{,}000 + 12{,}000 + 24{,}000 = 37{,}000;\ \text{or}$$

$R_2 = 3{,}700$ pounds.

The second equation reduces to

$$10\ R_1 = 9{,}000 + 8{,}000 + 6{,}000 = 23{,}000;\ \text{or}$$

$R_1 = 2{,}300$ pounds.

The sum of the loads is 6,000 pounds and the sum of the reactions is the same; hence the computation is correct.

2. Fig. 10 represents a beam supported at B and D (that is, it has overhanging ends) and sustaining three loads as shown. We wish to determine the reactions due to the loads.

Let R_1 and R_2 denote the reactions as shown; then the moment equations are:
For origin at B,

$$-2{,}100 \times 2 + 0 + 3{,}600 \times 6 - R_2 \times 14 + 1{,}600 \times 18 = 0.$$

For origin at D,

$$-2{,}100 \times 16 + R_1 \times 14 - 3{,}600 \times 8 + 0 + 1{,}600 \times 4 = 0.$$

The first equation reduces to

$$14\ R_2 = -4{,}200 + 21{,}600 + 28{,}800 = 46{,}200;\ \text{or}$$

$R_2 = 3{,}300$ pounds.

The second equation reduces to

$$14\ R_1 = 33{,}600 + 28{,}800 - 6{,}400 = 56{,}000;\ \text{or}$$

$R_1 = 4{,}000$ pounds.

The sum of the loads equals 7,300 pounds and the sum of the reactions is the same; hence the computation checks.

3. What are the total reactions in example 1 if the beam weighs 400 pounds?

(1.) Since we already know the reactions due to the loads (2,300 and 3,700 pounds at the left and right ends respectively (see illustration 1 above), we need only to compute the reactions due to the weight of the beam and add. Evidently the reactions due to the weight equal 200 pounds each; hence the

left reaction $= 2{,}300+200=2{,}500$ pounds, and the
right " $= 3{,}700+200=3{,}900$ " .

(2.) Or, we might compute the reactions due to the loads and weight of the beam together and directly. In figuring the moment due to the weight of the beam, we imagine the weight as concentrated at the middle of the beam; then its moments with respect to the left and right supports are (400×5) and $-(400\times5)$ respectively. The moment equations for origins at A and E are like those of illustration 1 except that they contain one more term, the moment due to the weight; thus they are respectively:

$$1{,}000\times1+2{,}000\times6+3{,}000\times8-R_2\times10+400\times5=0,$$

$$R_1\times10-1{,}000\times9-2{,}000\times4-3{,}000\times2-400\times5=0.$$

The first one reduces to

$$10\,R_2=39{,}000,\text{ or }R_2=3{,}900\text{ pounds};$$

and the second to

$$10\,R_1=25{,}000,\text{ or }R_1=2{,}500\text{ pounds}.$$

4. What are the total reactions in example 2 if the beam weighs 42 pounds per foot?

As in example 3, we might compute the reactions due to the weight and then add them to the corresponding reactions due to the loads (already found in example 2), but we shall determine the total reactions due to load and weight directly.

The beam being 20 feet long, its weight is 42×20, or 840 pounds. Since the middle of the beam is 8 feet from the left and 6 feet from the right support, the moments of the weight with respect to the left and right supports are respectively:

$$840\times8=6{,}720,\text{ and }-840\times6=-5{,}040\text{ foot-pounds}.$$

The moment equations for all the forces applied to the beam for origins at B and D are like those in example 2, with an additional term, the moment of the weight; they are respectively:

$$-2{,}100\times2+0+3{,}600\times6-R_2\times14+1{,}600\times18+6{,}720=0,$$

$$-2{,}100\times16+R_1\times14-3{,}600\times8+0+1{,}600\times4-5{,}040=0.$$

The first equation reduces to

$$14\ R_2 = 52{,}920, \text{ or } R_2 = 3{,}780 \text{ pounds},$$

and the second to

$$14\ R_1 = 61{,}040, \text{ or } R_1 = 4{,}360 \text{ pounds}.$$

The sum of the loads and weight of beam is 8,140 pounds; and since the sum of the reactions is the same, the computation checks.

EXAMPLES FOR PRACTICE.

1. AB (Fig. 11) represents a simple beam supported at its ends. Compute the reactions, neglecting the weight of the beam.

Ans. { Right reaction = 1,443.75 pounds.
Left reaction = 1,556.25 pounds.

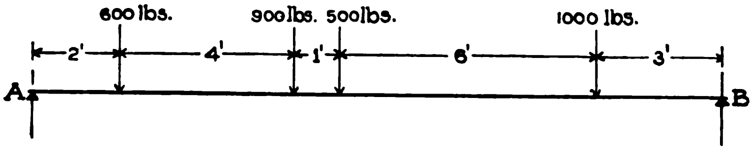


Fig. 11.

2. Solve example 1 taking into account the weight of the beam, which suppose to be 400 pounds.

Ans. { Right reaction = 1,643.75 pounds.
Left reaction = 1,756.25 pounds.

3. Fig. 12 represents a simple beam weighing 800 pounds supported at A and B, and sustaining three loads as shown. What are the reactions?

Ans. { Right reaction = 2,014.28 pounds.
Left reaction = 4,785.72 pounds.

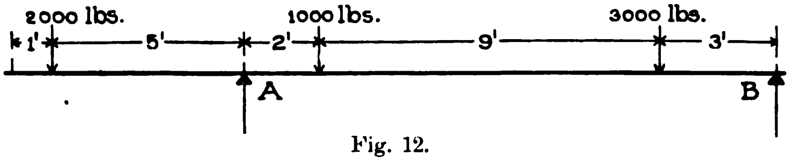


Fig. 12.

4. Suppose that in example 3 the beam also sustains a uniformly distributed load (as a floor) over its entire length, of 500 pounds per foot. Compute the reactions due to all the loads and the weight of the beam.

Ans. { Right reaction = 4,871.43 pounds.
Left reaction = 11,928.57 pounds.

EXTERNAL SHEAR AND BENDING MOMENT.

On almost every cross-section of a loaded beam there are three kinds of stress, namely tension, compression and shear. The first two are often called *fibre stresses* because they act along the real fibres of a wooden beam or the imaginary ones of which we may suppose iron and steel beams composed. Before taking up the subject of these stresses in beams it is desirable to study certain quantities relating to the loads, and on which the stresses in a beam depend. These quantities are called *external shear* and *bending moment*, and will now be discussed.

34. External Shear. By external shear at (or for) any section of a loaded beam is meant the algebraic sum of all the loads (including weight of beam) and reactions on *either side* of the section. This sum is called external shear because, as is shown later, it equals the shearing stress (internal) at the section. For brevity, we shall often say simply "shear" when external shear is meant.

35. Rule of Signs. In computing external shears, it is customary to give the plus sign to the reactions and the minus sign to the loads. But in order to get the same sign for the external shear whether computed from the right or left, we *change the sign* of the sum when computed from the loads and reactions *to the right.* Thus for section *a* of the beam in Fig. 8 the algebraic sum is, when computed from the left,

$$-1{,}000+3{,}000=+2{,}000 \text{ pounds;}$$

and when computed from the right,

$$-1{,}000+3{,}000-2{,}000-2{,}000=-2{,}000 \text{ pounds.}$$

The external shear at section *a* is +2,000 pounds.

Again, for section *b* the algebraic sum is, when computed from the left,

$$-1{,}000+3{,}000-2{,}000-2{,}000+3{,}000=+1{,}000 \text{ pounds;}$$

and when computed from the right, $-1{,}000$ pounds.

The external shear at the section is +1,000 pounds.

It is usually convenient to compute the shear at a section from the forces to the right or left according as there are fewer forces (loads and reactions) on the right or left sides of the section.

36. Units for Shears. It is customary to express external shears in pounds, but any other unit for expressing force and weight (as the ton) may be used.

37. Notation. We shall use V to stand for external shear at any section, and the shear at a particular section will be denoted by that letter subscripted; thus V_1, V_2, etc., stand for the shears at sections one, two, etc., feet from the left end of a beam.

The shear has different values just to the left and right of a support or concentrated load. We shall denote such values by V' and V''; thus V_5' and V_5'' denote the values of the shear at sections a little less and a little more than 5 feet from the left end respectively.

Examples. 1. Compute the shears for sections one foot apart in the beam represented in Fig. 9, neglecting the weight of the beam. (The right and left reactions are 3,700 and 2,300 pounds respectively; see example 1, Art. 33.)

All the following values of the shear are computed from the left. The shear just to the right of the left support is denoted by V_0'', and $V_0'' = 2{,}300$ pounds. The shear just to the left of B is denoted by V_1', and since the only force to the left of the section is the left reaction, $V_1' = 2{,}300$ pounds. The shear just to the right of B is denoted by V_1'', and since the only forces to the left of this section are the left reaction and the 1,000-pound load, $V_1'' = 2{,}300 - 1{,}000 = 1{,}300$ pounds. To the left of all sections between B and C, there are but two forces, the left reaction and the 1,000-pound load; hence the shear at any of those sections equals $2{,}300 - 1{,}000 = 1{,}300$ pounds, or

$$V_2 = V_3 = V_4 = V_5 = V_6' = 1{,}300 \text{ pounds.}$$

The shear just to the right of C is denoted by V_6''; and since the forces to the left of that section are the left reaction and the 1,000- and 2,000-pound loads,

$$V_6'' = 2{,}300 - 1{,}000 - 2{,}000 = -700 \text{ pounds.}$$

Without further explanation, the student should understand that

$$V_7 = +2{,}300 - 1{,}000 - 2{,}000 = -700 \text{ pounds,}$$

$$V_8' = -700,$$

$$V_8'' = +2{,}300 - 1{,}000 - 2{,}000 - 3{,}000 = -3{,}700,$$

$$V_9 = V_{10}' = -3{,}700,$$

$$V_{10}'' = +2.300 - 1{,}000 - 2{,}000 - 3{,}000 + 3{,}700 = 0$$

2. A simple beam 10 feet long, and supported at each end, weighs 400 pounds, and bears a uniformly distributed load of 1,600 pounds. Compute the shears for sections two feet apart.

Evidently each reaction equals one-half the sum of the load and weight of the beam, that is, $\frac{1}{2}$ (1,600+400) = 1,000 pounds. To the left of a section 2 feet from the left end, the forces acting on the beam consist of the left reaction, the load on that part of the beam, and the weight of that part; then since the load and weight of the beam *per foot* equal 200 pounds,

$$V_2 = 1{,}000 - 200 \times 2 = 600 \text{ pounds.}$$

To the left of a section four feet from the left end, the forces are the left reaction, the load on that part of the beam, and the weight; hence

$$V_4 = 1{,}000 - 200 \times 4 = 200 \text{ pounds.}$$

Without further explanation, the student should see that

$$\begin{aligned} V_6 &= 1{,}000 - 200 \times 6 = -200 \text{ pounds,} \\ V_8 &= 1{,}000 - 200 \times 8 = -600 \text{ pounds,} \\ V_{10}' &= 1{,}000 - 200 \times 10 = -1{,}000 \text{ pounds,} \\ V_{10}'' &= 1{,}000 - 200 \times 10 + 1{,}000 = 0. \end{aligned}$$

3. Compute the values of the shear in example 1, taking into account the weight of the beam (400 pounds). (The right and left reactions are then 3,900 and 2,500 pounds respectively; see example 3, Art. 33.)

We proceed just as in example 1, except that in each computation we include the weight of the beam to the left of the section (or to the right when computing from forces to the right). The weight of the beam being 40 pounds per foot, then (computing from the left)

$$\begin{aligned} V_0'' &= +2{,}500 \text{ pounds,} \\ V_1' &= +2{,}500 - 40 = +2{,}460, \\ V_1'' &= +2{,}500 - 40 - 1{,}000 = +1{,}460, \\ V_2 &= +2{,}500 - 1{,}000 - 40 \times 2 = +1{,}420, \\ V_3 &= +2{,}500 - 1{,}000 - 40 \times 3 = +1{,}380, \\ V_4 &= +2{,}500 - 1{,}000 - 40 \times 4 = +1{,}340, \\ V_5 &= +2{,}500 - 1{,}000 - 40 \times 5 = +1{,}300, \\ V_6' &= +2{,}500 - 1{,}000 - 40 \times 6 = +1{,}260, \\ V_6'' &= +2{,}500 - 1{,}000 - 40 \times 6 - 2{,}000 = -740, \\ V_7 &= +2{,}500 - 1{,}000 - 2{,}000 - 40 \times 7 = -780, \end{aligned}$$

$$\begin{aligned}
V_8' &= +2{,}500-1{,}000-2{,}000-40\times 8 = -820,\\
V_8'' &= +2{,}500-1{,}000-2{,}000-40\times 8-3{,}000 = -3{,}820,\\
V_9 &= +2{,}500-1{,}000-2{,}000-3{,}000-40\times 9 = -3{,}860,\\
V'_{10} &= +2{,}500-1{,}000-2{,}000-3{,}000-40\times 10 = -3{,}900,\\
V''_{10} &= +2{,}500-1{,}000-2{,}000-3{,}000-40\times 10 + 3{,}900 = 0.
\end{aligned}$$

Computing from the right, we find, as before, that

$$\begin{aligned}
V_7 &= -(3{,}900-3{,}000-40\times 3) = -780 \text{ pounds},\\
V_8' &= -(3{,}900-3{,}000-40\times 2) = -820,\\
V_8'' &= -(3{,}900-40\times 2) = -3{,}820,
\end{aligned}$$

etc., etc.

EXAMPLES FOR PRACTICE.

1. Compute the values of the shear for sections of the beam represented in Fig. 10, neglecting the weight of the beam. (The right and left reactions are 3,300 and 4,000 pounds respectively; see example 2, Art. 33.)

$$\text{Ans.}\begin{cases}
V_1 = V_2' = -2{,}100 \text{ pounds},\\
V_2'' = V_3 = V_4 = V_5 = V_6 = V_7 = V'_8 = +1{,}900,\\
V_8'' = V_9 = V_{10} = V_{11} = V_{12} = V_{13} = V_{14} = V_{15} = V'_{16} = -1{,}700,\\
V''_{16} = V_{17} = V_{18} = V_{19} = V'_{20} = +1{,}600.
\end{cases}$$

2. Solve the preceding example, taking into account the weight of the beam, 42 pounds per foot. (The right and left reactions are 3,780 and 4,360 pounds respectively; see example 4, Art. 33.)

$$\text{Ans.}\begin{cases}
V_0'' = -2{,}100 \text{ lbs.} & V_7 = +1{,}966 \text{ lbs.} & V_{14} = -1{,}928 \text{ lbs.}\\
V_1 = -2{,}142 & V_8' = +1{,}924 & V_{15} = -1{,}970\\
V_2' = -2{,}184 & V_8'' = -1{,}676 & V_{16}' = -2{,}012\\
V_2'' = +2{,}176 & V_9 = -1{,}718 & V_{16}'' = +1{,}768\\
V_3 = +2{,}134 & V_{10} = -1{,}760 & V_{17} = +1{,}726\\
V_4 = +2{,}092 & V_{11} = -1{,}802 & V_{18} = +1{,}684\\
V_5 = +2{,}050 & V_{12} = -1{,}844 & V_{19} = +1{,}642\\
V_6 = +2{,}008 & V_{13} = -1{,}886 & V_{20}' = +1{,}600
\end{cases}$$

3. Compute the values of the shear at sections one foot apart in the beam of Fig. 11, neglecting the weight. (The right and left reactions are 1,444 and 1,556 pounds respectively; see example 1, Art. 33.)

$$\text{Ans.}\begin{cases} V_0'' = V_1 = V_2' = +1{,}556 \text{ pounds}, \\ V_2'' = V_3 = V_4 = V_5 = V_6' = +956, \\ V_6'' = V_7' = +56, \\ V_7'' = V_8 = V_9 = V_{10} = V_{11} = V_{12} = V_{13}' = -444, \\ V_{13}'' = V_{14} = V_{15} = V_{16}' = -1{,}444. \end{cases}$$

4. Compute the vertical shear at sections one foot apart in the beam of Fig. 12, taking into account the weight of the beam, 800 pounds, and a distributed load of 500 pounds per foot. (The right and left reactions are 4,870 and 11,930 pounds respectively; see examples 3 and 4, Art. 33.)

$$\text{Ans.}\left\{\begin{array}{lll} V_0 = 0 & V_7 = +6{,}150 \text{ lbs.} & V_{15} = +830 \text{ lbs.} \\ V_1' = -540 \text{ lbs.} & V_8' = +5{,}610 & V_{16} = +290 \\ V_1'' = -2{,}540 & V_8'' = +4{,}610 & V_{17}' = -250 \\ V_2 = -3{,}080 & V_9 = +4{,}070 & V_{17}'' = -3{,}250 \\ V_3 = -3{,}620 & V_{10} = +3{,}530 & V_{18} = -3{,}790 \\ V_4 = -4{,}160 & V_{11} = +2{,}990 & V_{19} = -4{,}330 \\ V_5 = -4{,}700 & V_{12} = +2{,}450 & V_{20}' = -4{,}870 \\ V_6' = -5{,}240 & V_{13} = +1{,}910 & V_{20}'' = 0 \\ V_6'' = +6{,}690 & V_{14} = +1{,}370 & \end{array}\right.$$

38. Shear Diagrams. The way in which the external shear varies from section to section in a beam can be well represented by means of a diagram called a *shear diagram*. To construct such a diagram for any loaded beam,

1. Lay off a line equal (by some scale) to the length of the beam, and mark the positions of the supports and the loads. (This is called a "base-line.")

2. Draw a line such that the distance of any point of it from the base equals (by some scale) the shear at the corresponding section of the beam, and so that the line is above the base where the shear is positive, and below it where negative. (This is called a *shear line*, and the distance from a point of it to the base is called the "ordinate" from the base to the shear line at that point.)

We shall explain these diagrams further by means of illustrative examples.

Examples. 1. It is required to construct the shear diagram for the beam represented in Fig. 13, *a* (a copy of Fig. 9).

Lay off A'E' (Fig. 13, *b*) to represent the beam, and mark the positions of the loads B', C' and D'. In example 1, Art. 37, we computed the values of the shear at sections one foot apart; hence we lay off ordinates at points on A'E' one foot apart, to represent those shears.

Use a scale of 4,000 pounds to one inch. Since the shear for any section in AB is 2,300 pounds, we draw a line *ab* parallel to the base 0.575 inch (2,300÷4,000) therefrom; this is the shear line for the portion AB. Since the shear for any section in BC equals 1,300 pounds, we draw a line *b'c* parallel to the base and 0.325 inch (1,300÷4,000) therefrom; this is the shear line for the portion BC. Since the shear for any section in CD is −700 pounds, we draw a line *c'd* below the base and 0.175 inch (700÷4,000) therefrom; this is the shear line for the portion CD. Since the shear for any section in DE equals −3,700 lbs., we draw a line *d'e* below the base and 0.925 inch (3,700÷4,000) therefrom; this is the shear line for the portion DE. Fig. 13, *b*, is the required shear diagram.

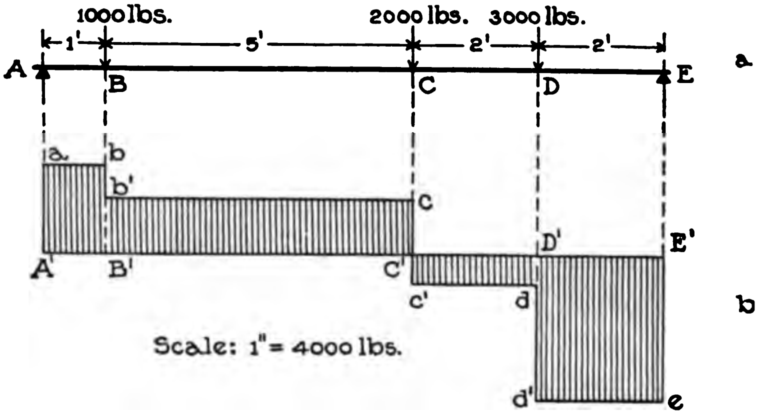
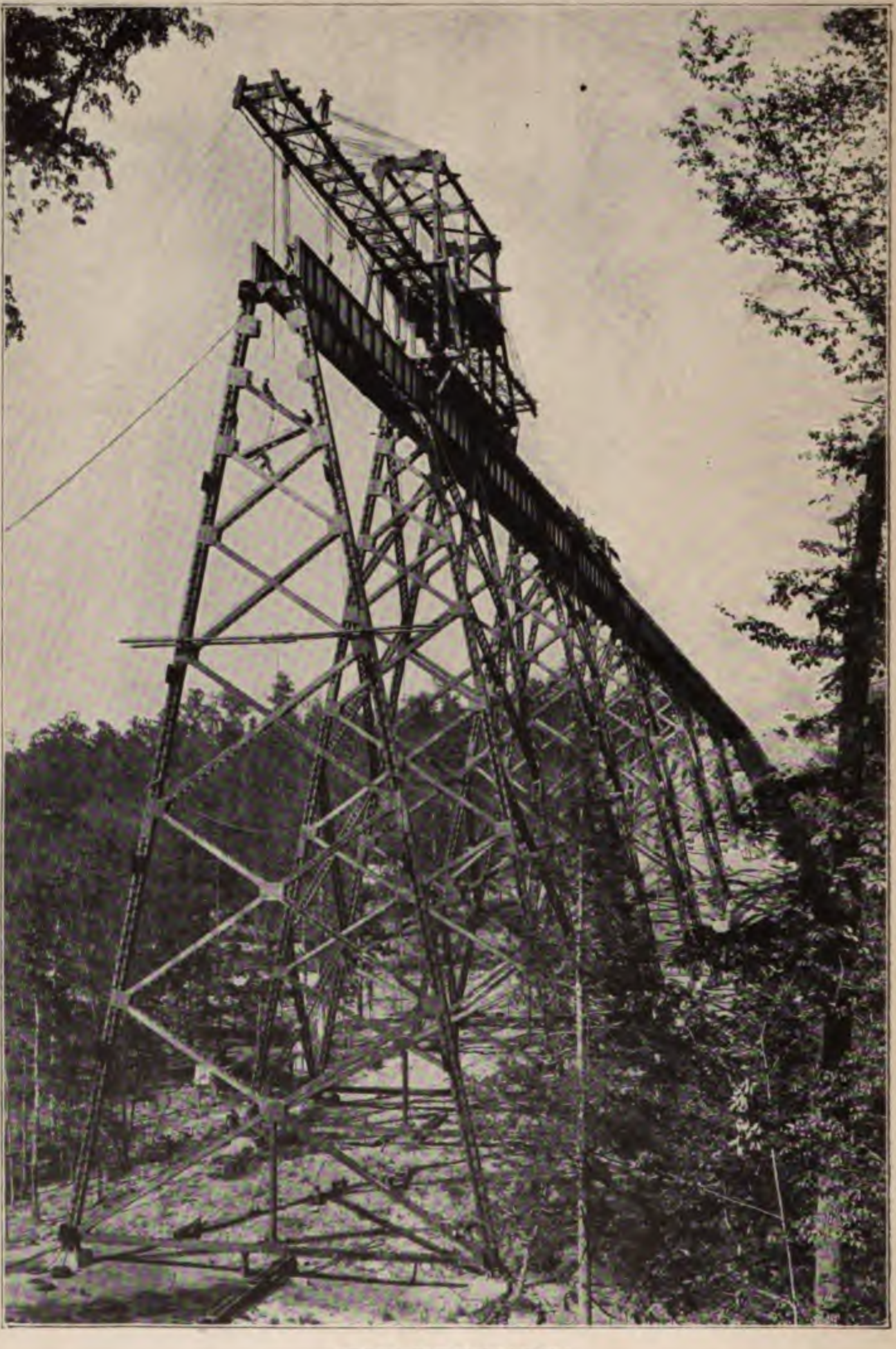


Fig. 13.

2. It is required to construct the shear diagram for the beam of Fig. 14, *a* (a copy of Fig. 9), taking into account the weight of the beam, 400 pounds.

The values of the shear for sections one foot apart were computed in example 3, Art. 37, so we have only to erect ordinates at the various points on a base line A'E' (Fig. 14, *b*), equal to those



BRUSH CREEK VIADUCT

In Alabama, on Birmingham extension of Illinois Central Railroad. Height, 185 feet. View showing trolley traveler used in construction work.

R. E. Gaut, Engineer of Bridges and Buildings.

values. We shall use the same scale as in the preceding illustration, 4,000 pounds to an inch. Then the lengths of the ordinates corresponding to the values of the shear (see example 3, Art. 37) are respectively:

$2{,}500 \div 4{,}000 = 0.625$ inch
$2{,}460 \div 4{,}000 = 0.615$ "
$1{,}460 \div 4{,}000 = 0.365$ "
etc. etc.

Laying these ordinates off from the base (upwards or downwards according as they correspond to positive or negative shears), we get *ab*, *b'c*, *c'd*, and *d'e* as the shear lines.

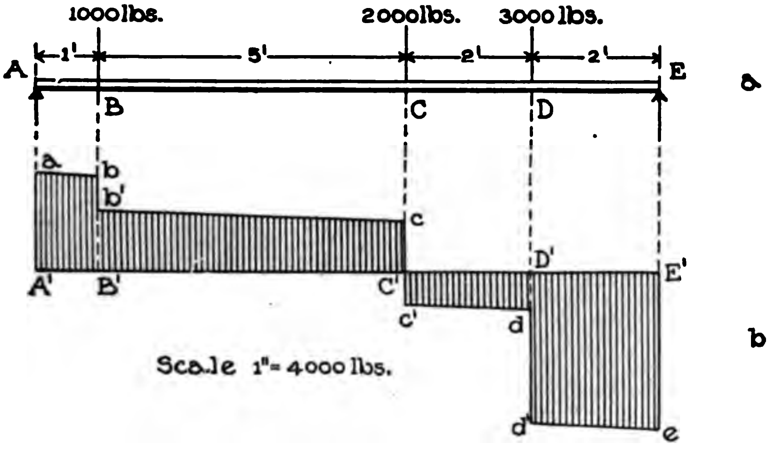


Fig. 14.

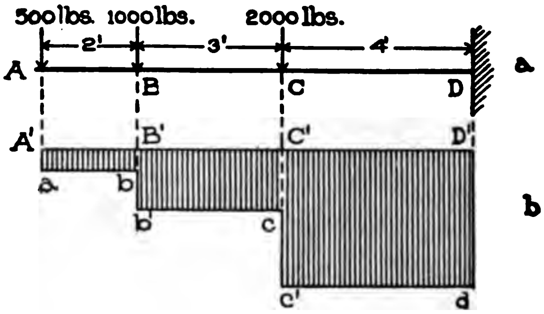
3. It is required to construct the shear diagram for the cantilever beam represented in Fig. 15, *a*, neglecting the weight of the beam.

The value of the shear for any section in AB is -500 pounds; for any section in BC, $-1{,}500$ pounds; and for any section in CD, $-3{,}500$ pounds. Hence the shear lines are *ab*, *b'c*, *c'd*. The scale being 5,000 pounds to an inch,

$A'a = 500 \div 5{,}000 = 0.1$ inch,
$B'b' = 1{,}500 \div 5{,}000 = 0.3$ "
$C'c' = 3{,}500 \div 5{,}000 = 0.7$ "

The shear lines are all below the base because all the values of the shear are negative.

4. Suppose that the cantilever of the preceding illustration sustains also a uniform load of 200 pounds per foot (see Fig. 16, *a*). Construct a shear diagram.



Scale 1"=5000 lbs.

Fig. 15.

First, we compute the values of the shear at several sections. Thus

$$
\begin{aligned}
V_0'' &= -500 \text{ pounds},\\
V_1 &= -500 - 200 = -700,\\
V_2' &= -500 - 200 \times 2 = -900,\\
V_2'' &= -500 - 200 \times 2 - 1{,}000 = -1{,}900,\\
V_3 &= -500 - 1{,}000 - 200 \times 3 = -2{,}100,\\
V_4 &= -500 - 1{,}000 - 200 \times 4 = -2{,}300,\\
V_5' &= -500 - 1{,}000 - 200 \times 5 = -2{,}500,\\
V_5'' &= -500 - 1{,}000 - 200 \times 5 - 2{,}000 = -4{,}500,\\
V_6 &= -500 - 1{,}000 - 2{,}000 - 200 \times 6 = -4{,}700,\\
V_7 &= -500 - 1{,}000 - 2{,}000 - 200 \times 7 = -4{,}900,\\
V_8 &= -500 - 1{,}000 - 2{,}000 - 200 \times 8 = -5{,}100,\\
V_9 &= -500 - 1{,}000 - 2{,}000 - 200 \times 9 = -5{,}300.
\end{aligned}
$$

The values, being negative, should be plotted downward. To a scale of 5,000 pounds to the inch they give the shear lines *ab*, *b'c*, *c'd* (Fig. 16, *b*).

EXAMPLES FOR PRACTICE.

1. Construct a shear diagram for the beam represented in Fig. 10, neglecting the weight of the beam (see example 1, Art. 37).

2. Construct the shear diagram for the beam represented in Fig. 11, neglecting the weight of the beam (see example 3, Art. 37).

3. Construct the shear diagram for the beam of Fig. 12 when it sustains, in addition to the loads represented, its own weight, 800 pounds, and a uniform load of 500 pounds per foot (see example 4, Art. 37).

4. Figs. *a*, cases 1 and 2, Table B (page 55), represent two cantilever beams, the first bearing a concentrated load P at the free end, and the second a uniform load W. Figs. *b* are the corresponding shear diagrams. Take P and W equal to 1,000 pounds, and satisfy yourself that the diagrams are correct.

5. Figs. *a*, cases 3 and 4, same table, represent simple beams supported at their ends, the first bearing a concentrated load P at the middle, and the second a uniform load W. Figs. *b* are the corresponding shear diagrams. Take P and W equal to 1,000 pounds, and satisfy yourself that they are correct.

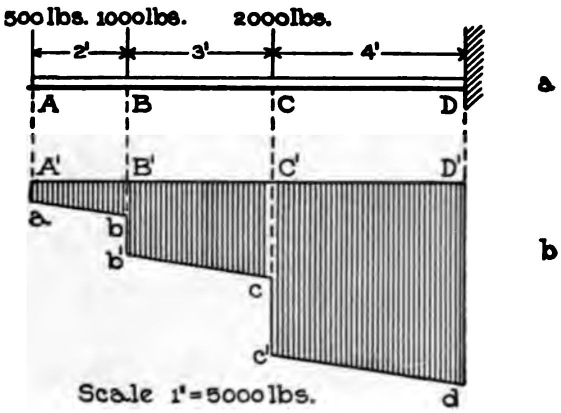


Fig. 16.

39. Maximum Shear. It is sometimes desirable to know the greatest or maximum value of the shear in a given case. This value can always be found with certainty by constructing the shear diagram, from which the maximum value of the shear is evident at a glance. In any case it can most readily be computed if one knows the section for which the shear is a maximum. The student should examine all the shear diagrams in the preceding articles and those that he has drawn, and see that

1. *In cantilevers fixed in a wall, the maximum shear occurs at the wall.*

2. *In simple beams, the maximum shear occurs at a section next to one of the supports.*

By the use of these propositions one can determine the value of the maximum shear without constructing the whole shear diagram. Thus, it is easily seen (referring to the diagrams, page 55) that for a

Cantilever, end load P, maximum shear $=P$
" , uniform load W, " " $=W$
Simple beam, middle load P, " " $=\frac{1}{2}P$
" " , uniform " W, " " $=\frac{1}{2}W$

40. Bending Moment. By bending moment at (or for) a section of a loaded beam, is meant the algebraic sum of the moments of all the loads (including weight of beam) and reactions to the left or right of the section with respect to any point in the section.

41. Rule of Signs. We follow the rule of signs previously stated (Art. 29) that the moment of a force which tends to produce clockwise rotation is plus, and that of a force which tends to produce counter-clockwise rotation is minus; but in order to get the same sign for the bending moment whether computed from the right or left, we *change the sign* of the sum of the moments when computed from the loads and reactions *on the right*. Thus for section *a*, Fig. 8, the algebraic sums of the moments of the forces are:

when computed from the left,

$$-1{,}000\times5+3{,}000\times1=-2{,}000 \text{ foot-pounds;}$$

and when computed from the right,

$$1{,}000\times19-3{,}000\times15+2{,}000\times13+2{,}000\times1=+2{,}000 \text{ foot-pounds.}$$

The bending moment at section *a* is $-2{,}000$ foot-pounds.

Again, for section *b*, the algebraic sums of the moments of the forces are:

when computed from the left,

$$-1{,}000\times22+3{,}000\times18-2{,}000\times16-2{,}000\times4+3{,}000\times2= -2{,}000 \text{ foot-pounds;}$$

and when computed from the right,

$$1{,}000\times2=+2{,}000 \text{ foot-pounds.}$$

The bending moment at the section is -2.000 foot-pounds.

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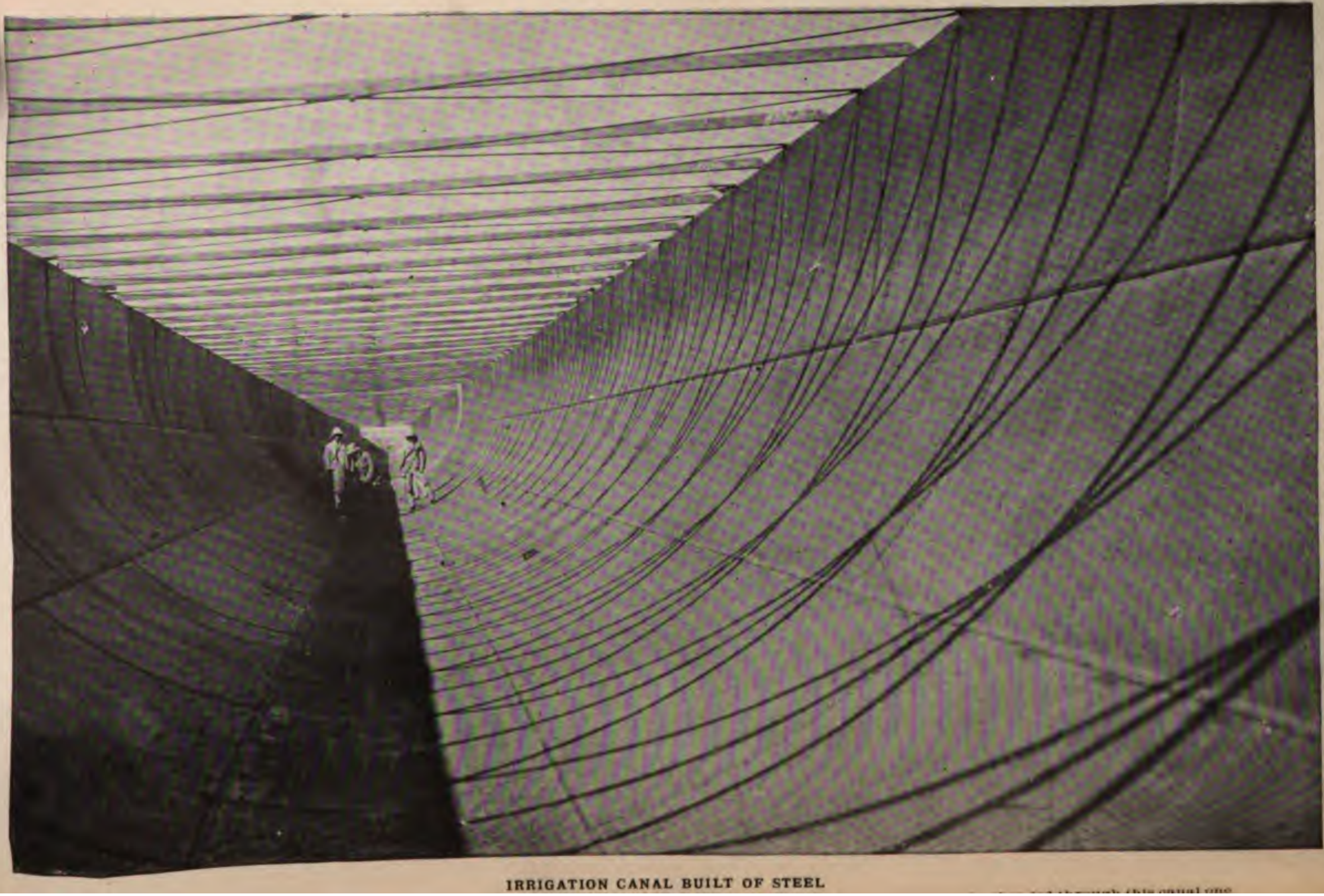


IRRIGATION CANAL BUILT OF STEEL

It is usually convenient to compute the bending moment for a section from the forces to the right or left according as there are fewer forces (loads and reactions) on the right or left side of the section.

42. Units. It is customary to express bending moments in inch-pounds, but often the foot-pound unit is more convenient. *To reduce foot-pounds to inch-pounds, multiply by twelve.*

43. Notation. We shall use M to denote bending moment at any section, and the bending moment at a particular section will be denoted by that letter subscripted; thus M_1, M_2, etc., denote values of the bending moment for sections one, two, etc., feet from the left end of the beam.

Examples. 1. Compute the bending moments for sections one foot apart in the beam represented in Fig. 9, neglecting the weight of the beam. (The right and left reactions are 3,700 and 2,300 pounds respectively. See example 1, Art. 33.)

Since there are no forces acting on the beam to the left of the right support, $M_0=0$. To the left of the section one foot from the left end there is but one force, the left reaction, and its arm is one foot; hence $M_1=+2{,}300\times1=2{,}300$ foot-pounds. To the left of a section two feet from the left end there are two forces, 2,300 and 1,000 pounds, and their arms are 2 feet and 1 foot respectively; hence $M_2=+2{,}300\times2-1{,}000\times1=3{,}600$ foot-pounds. At the left of all sections between B and C there are only two forces, 2,300 and 1,000 pounds; hence

$$
\begin{aligned}
M_3&=+2{,}300\times3-1{,}000\times2=+4{,}900 \text{ foot-pounds,}\\
M_4&=+2{,}300\times4-1{,}000\times3=+6{,}200 \quad \text{"}\\
M_5&=+2{,}300\times5-1{,}000\times4=+7{,}500 \quad \text{"}\\
M_6&=+2{,}300\times6-1{,}000\times5=+8{,}800 \quad \text{"}
\end{aligned}
$$

To the right of a section seven feet from the left end there are two forces, the 3,000-pound load and the right reaction (3,700 pounds), and their arms with respect to an origin in that section are respectively one foot and three feet; hence

$$M_7=-(-3{,}700\times3+3{,}000\times1)=+8{,}100 \text{ foot-pounds.}$$

To the right of any section between E and D there is only one force, the right reaction; hence

$$M_8 = -(-3{,}700 \times 2) = 7{,}400 \text{ foot-pounds.}$$
$$M_9 = -(-3{,}700 \times 1) = 3{,}700 \quad \text{''}$$

Clearly $M_{10} = 0$.

2. A simple beam 10 feet long and supported at its ends weighs 400 pounds, and bears a uniformly distributed load of 1,600 pounds. Compute the bending moments for sections two feet apart.

Each reaction equals one-half the whole load, that is, ½ of (1,600+400) = 1,000 pounds, and the load per foot including weight of the beam is 200 pounds. The forces acting on the beam to the left of the first section, two feet from the left end, are the left reaction (1,000 pounds) and the load (including weight) on the part of the beam to the left of the section (400 pounds). The arm of the reaction is 2 feet and that of the 400-pound force is 1 foot (the distance from the middle of the 400-pound load to the section). Hence

$$M_2 = 1{,}000 \times 2 - 400 \times 1 = 1{,}600 \text{ foot-pounds.}$$

The forces to the left of the next section, 4 feet from the left end, are the left reaction and all the load (including weight of beam) to the left (800 pounds). The arm of the reaction is 4 feet, and that of the 800-pound force is 2 feet; hence

$$M_4 = 1{,}000 \times 4 - 800 \times 2 = 2{,}400 \text{ foot-pounds.}$$

Without further explanation the student should see that

$$M_6 = 1{,}000 \times 6 - 1{,}200 \times 3 = 2{,}400 \text{ foot-pounds,}$$
$$M_8 = 1{,}000 \times 8 - 1{,}600 \times 4 = 1{,}600 \quad \text{''}$$

Evidently $M_0 = M_{10} = 0$.

3. Compute the values of the bending moment in example 1, taking into account the weight of the beam, 400 pounds. (The right and left reactions are respectively 3,900 and 2,500 pounds; see example 3, Art. 33.)

We proceed as in example 1, except that the moment of the weight of the beam to the left of each section (or to the right when computing from forces to the right) must be included in the respective moment equations. Thus, computing from the left,

$M_0 = 0$

$M_1 = +2{,}500 \times 1 - 40 \times \tfrac{1}{2} = +2{,}480$ foot-pounds,

$M_2 = +2{,}500 \times 2 - 1{,}000 \times 1 - 80 \times 1 = +3{,}920$,

$M_3 = +2{,}500 \times 3 - 1{,}000 \times 2 - 120 \times 1\tfrac{1}{2} = +5{,}320$,

$M_4 = +2{,}500 \times 4 - 1{,}000 \times 3 - 160 \times 2 = +6{,}680$,

$M_5 = +2{,}500 \times 5 - 1{,}000 \times 4 - 200 \times 2\tfrac{1}{2} = +8{,}000$,

$M_6 = +2{,}500 \times 6 - 1{,}000 \times 5 - 240 \times 3 = +9{,}280$.

Computing from the right,

$M_7 = -(-3{,}900 \times 3 + 3{,}000 \times 1 + 120 \times 1\tfrac{1}{2}) = +8{,}520$,

$M_8 = -(-3{,}900 \times 2 + 80 \times 1) = +7{,}720$,

$M_9 = -(-3{,}900 \times 1 + 40 \times \tfrac{1}{2}) = +3{,}880$,

$M_{10} = 0$.

EXAMPLES FOR PRACTICE.

1. Compute the values of the bending moment for sections one foot apart, beginning one foot from the left end of the beam represented in Fig. 10, neglecting the weight of the beam. (The right and left reactions are 3,300 and 4,000 pounds respectively; see example 2, Art. 33.)

Ans. (in foot-pounds)

$M_1 = -2{,}100$ $M_6 = +3{,}400$ $M_{11} = +2{,}100$ $M_{16} = -6{,}400$
$M_2 = -4{,}200$ $M_7 = +5{,}300$ $M_{12} = +400$ $M_{17} = -4{,}800$
$M_3 = -2{,}300$ $M_8 = +7{,}200$ $M_{13} = -1{,}300$ $M_{18} = -3{,}200$
$M_4 = -400$ $M_9 = +5{,}500$ $M_{14} = -3{,}000$ $M_{19} = -1{,}600$
$M_5 = +1{,}500$ $M_{10} = +3{,}800$ $M_{15} = -4{,}700$ $M_{20} = 0$

2. Solve the preceding example, taking into account the weight of the beam, 42 pounds per foot. (The right and left reactions are 3,780 and 4,360 pounds respectively; see example 4, Art. 33.)

Ans. (in foot-pounds)

$M_1 = -2{,}121$ $M_6 = +4{,}084$ $M_{11} = +2{,}799$ $M_{16} = -6{,}736$
$M_2 = -4{,}284$ $M_7 = +6{,}071$ $M_{12} = +976$ $M_{17} = -4{,}989$
$M_3 = -2{,}129$ $M_8 = +8{,}016$ $M_{13} = -889$ $M_{18} = -3{,}284$
$M_4 = -16$ $M_9 = +6{,}319$ $M_{14} = -2{,}796$ $M_{19} = -1{,}621$
$M_5 = +2{,}055$ $M_{10} = +4{,}580$ $M_{15} = -4{,}745$ $M_{20} = 0$

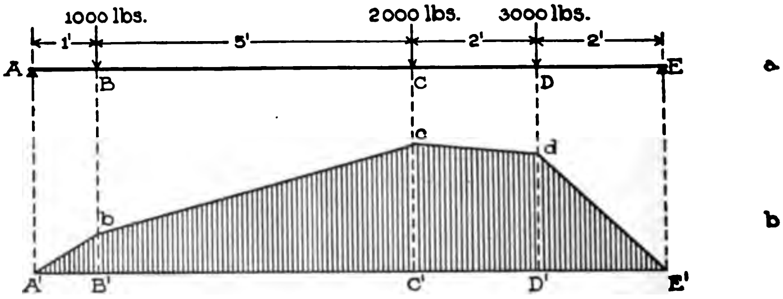
3. Compute the bending moments for sections one foot apart, of the beam represented in Fig. 11, neglecting the weight. (The right and left reactions are 1,444 and 1,556 pounds respectively; see example 1, Art. 33.)

Ans. (in foot-pounds) $\left\{\begin{array}{llll} M_1=+1{,}556 & M_5=+5{,}980 & M_9=+6{,}104 & M_{13}=+4{,}328 \\ M_2=+3{,}112 & M_6=+6{,}936 & M_{10}=+5{,}660 & M_{14}=+2{,}884 \\ M_3=+4{,}068 & M_7=+6{,}992 & M_{11}=+5{,}216 & M_{15}=+1{,}440 \\ M_4=+5{,}024 & M_8=+6{,}548 & M_{12}=+4{,}772 & M_{16}=0 \end{array}\right.$

4 Compute the bending moments at sections one foot apart in the beam of Fig. 12, taking into account the weight of the beam, 800 pounds, and a uniform load of 500 pounds per foot. (The right and left reactions are 4,870 and 11,930 pounds respectively; see Exs. 3 and 4, Art. 33.)

Ans. (in foot-pounds) $\left\{\begin{array}{llll} M_1=-270 & M_6=-19{,}720 & M_{11}=+3{,}980 & M_{16}=12{,}180 \\ M_2=-3{,}080 & M_7=-13{,}300 & M_{12}=+6{,}700 & M_{17}=12{,}200 \\ M_3=-6{,}430 & M_8=-7{,}420 & M_{13}=+8{,}880 & M_{18}=8{,}680 \\ M_4=-10{,}320 & M_9=-3{,}080 & M_{14}=+10{,}520 & M_{19}=4{,}620 \\ M_5=-14{,}750 & M_{10}=+720 & M_{15}=+11{,}620 & M_{20}=0 \end{array}\right.$

44. Moment Diagrams. The way in which the bending moment varies from section to section in a loaded beam can be well represented by means of a diagram called a *moment diagram*. To construct such a diagram for any loaded beam,



Scale: 1 = 10000 ft.-lbs

Fig. 17.

1. Lay off a base-line just as for a shear diagram (see Art. 38).

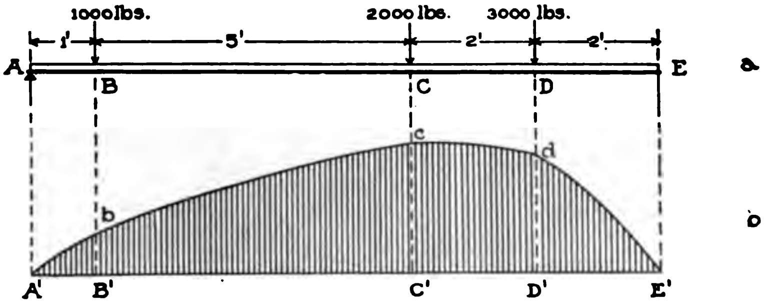
2. Draw a line such that the distance from any point of it to the base-line equals (by some scale) the value of the bending moment at the corresponding section of the beam, and so that the line is above the base where the bending moment is positive and below it where it is negative. (This line is called a "moment line.")

Examples. 1. It is required to construct a moment diagram for the beam of Fig. 17, *a* (a copy of Fig. 9), loaded as there shown.

Lay off A'E' (Fig. 17, *b*) as a base. In example 1, Art. 43, we computed the values of the bending moment for sections one foot apart, so we erect ordinates at points of A'E' one foot apart, to represent the bending moments.

We shall use a scale of 10,000 foot-pounds to the inch; then the ordinates (see example 1, Art. 43, for values of M) will be:

One foot from left end, 2,300÷10,000 = 0.23 inch,
Two feet " " " 3,600÷10,000 = 0.36 "
Three " " " " 4,900÷10,000 = 0.49 "
Four " " " " 6,200÷10,000 = 0.62 "
etc., etc.



Scale: 1"= 10000 ft.-lbs.

Fig. 18.

Laying these ordinates off, and joining their ends in succession, we get the line A'*bcd*E', which is the bending moment line. Fig. 17, *b*, is the moment diagram.

2. It is required to construct the moment diagram for the beam, Fig. 18, *a* (a copy of Fig. 9), taking into account the weight of the beam, 400 pounds.

The values of the bending moment for sections one foot apart were computed in example 3, Art. 43. So we have only to lay off ordinates equal to those values, one foot apart, on the base A'E' (Fig. 18, *b*).

To a scale of 10,000 foot-pounds to the inch the ordinates (see example 3, Art. 43, for values of M) are:

At left end, 0
One foot from left end, $2,480 \div 10,000 = 0.248$ inch
Two feet " " " $3,920 \div 10,000 = 0.392$ "
Three " " " " $5,320 \div 10,000 = 0.532$ "
Four " " " " $6,680 \div 10,000 = 0.668$ "

Laying these ordinates off at the proper points, we get A'*bcd*E as the moment line.

3. It is required to construct the moment diagram for the cantilever beam represented in Fig. 19, *a*, neglecting the weight of the beam. The bending moment at B equals

$$-500 \times 2 = -1,000 \text{ foot-pounds};$$

at C,

$$-500 \times 5 - 1,000 \times 3 = -5,500;$$

and at D,

$$-500 \times 9 - 1,000 \times 7 - 2,000 \times 4 = -19,500.$$

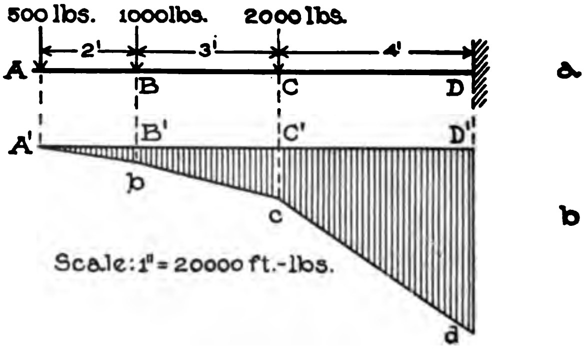


Fig. 19.

Using a scale of 20,000 foot-pounds to one inch, the ordinates in the bending moment diagram are:

At B, $1,000 \div 20,000 = 0.05$ inch,
" C, $5,500 \div 20,000 = 0.275$ "
" D, $19,500 \div 20,000 = 0.975$ "

Hence we lay these ordinates off, and downward because the bending moments are negative, thus fixing the points *b*, *c* and *d*. The bending moment at A is zero; hence the moment line connects A *b*, *c* and *d*. Further, the portions A*b*, *bc* and *cd* are straight, as can be shown by computing values of the bending moment for sections in AB, BC and CD, and laying off the corresponding ordinates in the moment diagram.

4. Suppose that the cantilever of the preceding illustration sustains also a uniform load of **100** pounds per foot (see Fig. 20, *a*). Construct a moment diagram.

First, we compute the values of the bending moment at several sections; thus,

$M_1 = -500 \times 1 - 100 \times \frac{1}{2} = -550$ foot-pounds,
$M_2 = -500 \times 2 - 200 \times 1 = -1{,}200$,
$M_3 = -500 \times 3 - 1{,}000 \times 1 - 300 \times 1\frac{1}{2} = -2{,}950$,
$M_4 = -500 \times 4 - 1{,}000 \times 2 - 400 \times 2 = -4{,}800$,
$M_5 = -500 \times 5 - 1{,}000 \times 3 - 500 \times 2\frac{1}{2} = -6{,}750$,
$M_6 = -500 \times 6 - 1{,}000 \times 4 - 2{,}000 \times 1 - 600 \times 3 = -10{,}800$,
$M_7 = -500 \times 7 - 1{,}000 \times 5 - 2{,}000 \times 2 - 700 \times 3\frac{1}{2} = -14{,}950$,
$M_8 = -500 \times 8 - 1{,}000 \times 6 - 2{,}000 \times 3 - 800 \times 4 = -19{,}200$,
$M_9 = -500 \times 9 - 1{,}000 \times 7 - 2{,}000 \times 4 - 900 \times 4\frac{1}{2} = -23{,}550$.

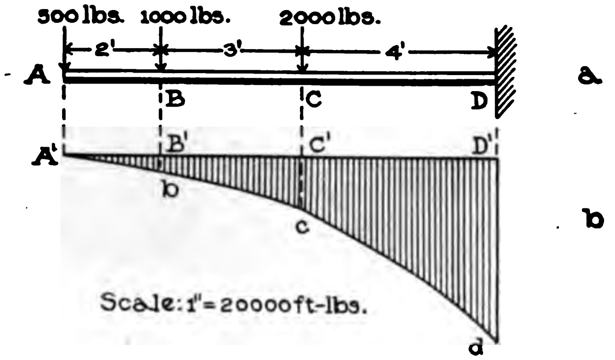


Fig. 20.

These values all being negative, the ordinates are all laid off downwards. To a scale of 20,000 foot-pounds to one inch, they fix the moment line *A'bcd*.

EXAMPLES FOR PRACTICE.

1. Construct a moment diagram for the beam represented in Fig. 10, neglecting the weight of the beam. (See example 1, Art. 43).

2. Construct a moment diagram for the beam represented in Fig. 11, neglecting the weight of the beam. (See example 3, Art. 43).

3. Construct the moment diagram for the beam of Fig. 12

when it sustains, in addition to the loads represented and its own weight (800 pounds), a uniform load of 500 pounds per foot. (See example 4, Art. 43.)

4. Figs. *a*, cases 1 and 2, page 55, represent two cantilever beams, the first bearing a load P at the free end, and the second a uniform load W. Figs. *b* are the corresponding moment diagrams. Take P and W equal to 1,000 pounds, and l equal to 10 feet, and satisfy yourself that the diagrams are correct.

5. Figs. *a*, cases 3 and 4, page 55, represent simple beams on end supports, the first bearing a middle load P, and the other a uniform load W. Figs. *b* are the corresponding moment diagrams. Take P and W equal to 1,000 pounds, and l equal to 10 feet, and satisfy yourself that the diagrams are correct.

45. Maximum Bending Moment. It is sometimes desirable to know the greatest or maximum value of the bending moment in a given case. This value can always be found with certainty by constructing the moment diagram, from which the maximum value of the bending-moment is evident at a glance. But in any case, it can be most readily computed if one knows the section for which the bending moment is greatest. If the student will compare the corresponding shear and moment diagrams which have been constructed in foregoing articles (Figs. 13 and 17, 14 and 18, 15 and 19; 16 and 20), and those which he has drawn, he will see that—*The maximum bending moment in a beam occurs where the shear changes sign.*

By the help of the foregoing principle we can readily compute the maximum moment in a given case. We have only to construct the shear line, and observe from it where the shear changes sign; then compute the bending moment for that section. If a simple beam has one or more overhanging ends, then the shear changes sign more than once—twice if there is one overhanging end, and three times if two. In such cases we compute the bending moment for each section where the shear changes sign; the largest of the values of these bending moments is the maximum for the beam.

The section of maximum bending moment in a cantilever fixed at one end (as when built into a wall) is always at the wall.

Thus, without reference to the moment diagrams, it is readily seen that,

for a cantilever whose length is l,

with an end load P, the maximum moment is Pl,
" a uniform " W, " " " " $\frac{1}{2}Wl$.

Also by the principle, it is seen that,

for a beam whose length is l, on end supports,

with a middle load P, the maximum moment is $\frac{1}{4}Pl$,
" uniform " W, " " " " $\frac{1}{8}Wl$.

46. Table of Maximum Shears, Moments, etc. Table B on page 55 shows the shear and moment diagrams for eight simple cases of beams. The first two cases are built-in cantilevers; the next four, simple beams on end supports; and the last two, restrained beams built in walls at each end. In each case l denotes the length.

CENTER OF GRAVITY AND MOMENT OF INERTIA.

It will be shown later that the strength of a beam depends partly on the form of its cross-section. The following discussion relates principally to cross-sections of beams, and the results reached (like shear and bending moment) will be made use of later in the subject of strength of beams.

47. Center of Gravity of an Area. The student probably knows what is meant by, and how to find, the center of gravity of any flat disk, as a piece of tin. Probably his way is to balance the piece of tin on a pencil point, the point of the tin at which it so balances being the center of gravity. (Really it is midway between the surfaces of the tin and over the balancing point.) The center of gravity of the piece of tin, is also that point of it through which the resultant force of gravity on the tin (that is, the weight of the piece) acts.

By "center of gravity" of a plane area of any shape we mean that point of it which corresponds to the center of gravity of a piece of tin when the latter is cut out in the shape of the area. The center of gravity of a quite irregular area can be found most readily by balancing a piece of tin or stiff paper cut in the shape of the area. But when an area is simple in shape, or consists of parts which are simple, the center of gravity of the whole can be found readily by computation, and such a method will now be described.

48. Principle of Moments Applied to Areas. Let Fig. 21 represent a piece of tin which has been divided off into any number of parts in any way, the weight of the whole being W, and that of the parts W_1, W_2, W_3, etc. Let C_1, C_2, C_3, etc., be the centers of gravity of the parts, C that of the whole, and c_1, c_2, c_3, etc., and c the distances from those centers of gravity respectively to some line (L L) in the plane of the sheet of tin. When the tin is lying in a horizontal position, the moment of the weight of the entire piece about L L is Wc, and the moments of the parts are W_1c_1, W_2c_2, etc. Since the weight of the whole is the resultant of the weights of the parts, the moment of the weight of the whole equals the sum of the moments of the weights of the parts; that is,

$$Wc = W_1c_1 + W_2c_2 + \text{etc.} \ldots$$

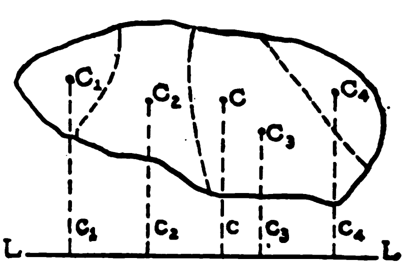


Fig. 21.

Now let A_1, A_2, etc. denote the areas of the parts of the pieces of tin, and A the area of the whole; then since the weights are proportional to the areas, we can replace the W's in the preceding equation by corresponding A's, thus:

$$Ac = A_1c_1 + A_2c_2 + \text{etc.} \ldots\ldots \qquad (4)$$

If we call the product of an area and the distance of its center of gravity from some line in its plane, the "moment" of the area with respect to that line, then the preceding equation may be stated in words thus:

The moment of an area with respect to any line equals the algebraic sum of the moments of the parts of the area.

If all the centers of gravity are on one side of the line with respect to which moments are taken, then all the moments should be given the plus sign; but if some centers of gravity are on one side and some on the other side of the line, then the moments of the areas whose centers of gravity are on one side should be given the same sign, and the moments of the others the opposite sign. The foregoing is the principle of moments for areas, and it is the basis of all rules for finding the center of gravity of an area.

To find the center of gravity of an area which can be divided up into simple parts, we write the principle in forms of equations for two different lines as "axes of moments," and then solve the equations for the unknown distances of the center of gravity of the whole from the two lines. We explain further by means of specific examples.

Examples. 1. It is required to find the center of gravity of Fig. 22, *a*, the width being uniformly one inch.

The area can be divided into two rectangles. Let C_1 and C_2 be the centers of gravity of two such parts, and C the center of gravity of the whole. Also let a and b denote the distances of C from the two lines OL′ and OL″ respectively.

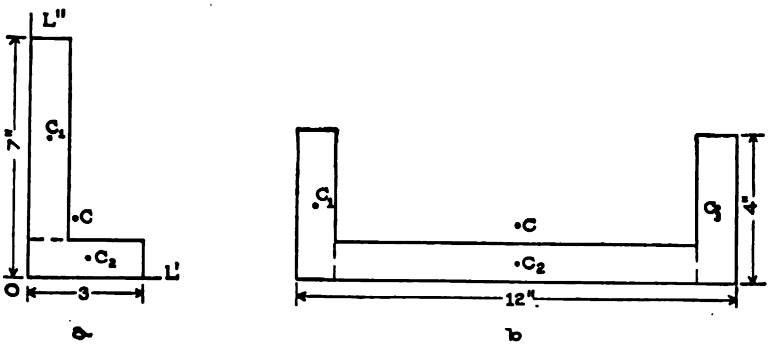


Fig. 22.

The areas of the parts are 6 and 3 square inches, and their arms with respect to OL′ are 4 inches and ½ inch respectively, and with respect to OL″ ½ inch and 1½ inches. Hence the equations of moments with respect to OL′ and OL″ (the whole area being 9 square inches) are:

$$9 \times a = 6 \times 4 + 3 \times \tfrac{1}{2} = 25.5,$$
$$9 \times b = 6 \times \tfrac{1}{2} + 3 \times 1\tfrac{1}{2} = 7.5.$$

Hence,
$$a = 25.5 \div 9 = 2.83 \text{ inches},$$
$$b = 7.5 \div 9 = 0.83 \text{ " }.$$

2. It is required to locate the center of gravity of Fig. 22, *b*, the width being uniformly one inch.

The figure can be divided up into three rectangles. Let C_1, C_2 and C_3 be the centers of gravity of such parts, C the center of gravity of the whole; and let a denote the (unknown) distance of C from the base. The areas of the parts are 4, 10 and 4 square inches, and their "arms" with respect to the base are 2, ½ and 2 inches respectively; hence the equation of moments with respect to the base (the entire area being 18 square inches) is:

$$18 \times a = 4 \times 2 + 10 \times \tfrac{1}{2} + 4 \times 2 = 21.$$

Hence, $a = 21 \div 18 = 1.17$ inches.

From the symmetry of the area it is plain that the center of gravity is midway between the sides.

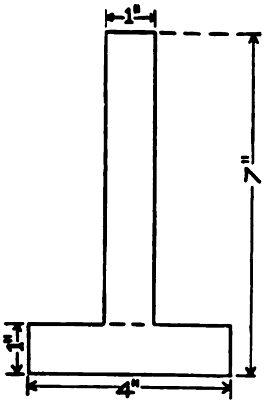


Fig. 23.

EXAMPLE FOR PRACTICE.

1. Locate the center of gravity of Fig. 23.

Ans. 2.6 inches above the base.

49. Center of Gravity of Built-up Sections. In Fig. 24 there are represented cross-sections of various kinds of rolled steel, called "shape steel," which is used extensively in steel construction. Manufacturers of this material publish "handbooks" giving full information in regard thereto, among other things, the position of the center of gravity of each cross section. With such a handbook available, it is therefore not necessary actually to compute the position of the center of gravity of any section, as we did in the preceding article; but sometimes several shapes are riveted together to

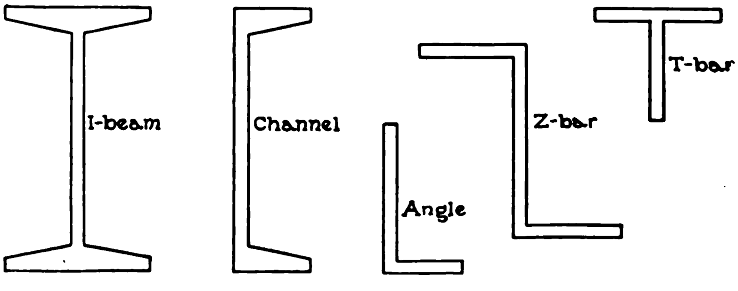


Fig. 24.

make a "built-up" section (see Fig. 25), and then it may be necessary to compute the position of the center of gravity of the section.

Example. It is desired to locate the center of gravity of the section of a built-up beam represented in Fig. 25. The beam consists of two channels and a plate, the area of the cross-section of a channel being 6.03 square inches.

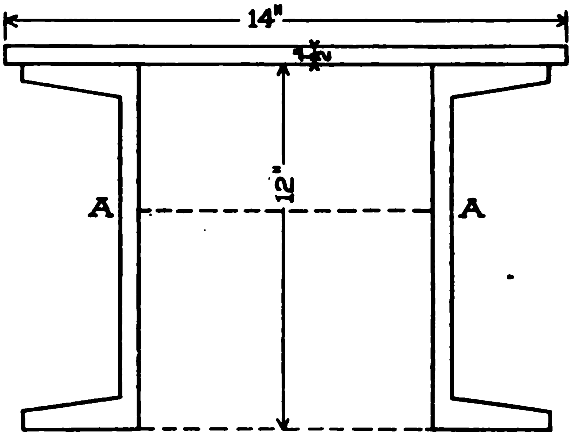


Fig. 25.

Evidently the center of gravity of each channel section is 6 inches, and that of the plate section is $12\frac{1}{4}$ inches, from the bottom.

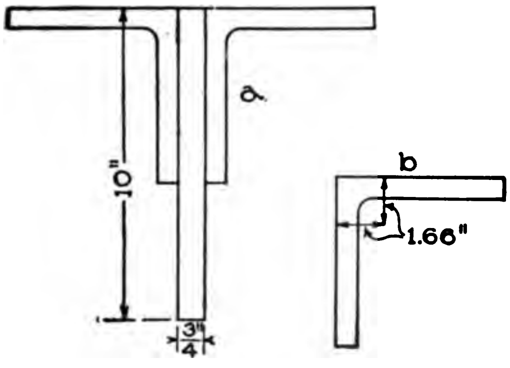


Fig. 26.

Let c denote the distance of the center of gravity of the whole section from the bottom; then since the area of the plate section is 7 square inches, and that of the whole section is 19.06,

$$19.06 \times c = 6.03 \times 6 + 6.03 \times 6 + 7 \times 12\tfrac{1}{4} = 158.11.$$

Hence, $c = 158.11 \div 19.06 = 8.30$ inches.

EXAMPLES FOR PRACTICE.

1. Locate the center of gravity of the built-up section of

Fig. 26, *a*, the area of each "angle" being 5.06 square inches, and the center of gravity of each being as shown in Fig. 26, *b*.

Ans. Distance from top, 3.08 inches.

2. Omit the left-hand angle in Fig. 26, *a*, and locate the center of gravity of the remainder.

$$\text{Ans.}\begin{cases}\text{Distance from top, 3.65 inches,}\\ \text{`` \quad `` \quad left side, 1.19 inches.}\end{cases}$$

50. Moment of Inertia. If a plane area be divided into an infinite number of infinitesimal parts, then the sum of the products obtained by multiplying the area of each part by the square of its distance from some line is called the *moment of inertia* of the area with respect to the line. The line to which the distances are measured is called the *inertia-axis;* it may be taken anywhere in the plane of the area. In the subject of beams (where we have sometimes to compute the moment of inertia of the cross-section of a beam), the inertia-axis is taken through the center of gravity of the section and horizontal.

An approximate value of the moment of inertia of an area can be obtained by dividing the area into small parts (not infinitesimal), and adding the products obtained by multiplying the area of each part by the square of the distance from its center to the inertia-axis.

Example. If the rectangle of Fig. 27, *a*, is divided into 8 parts as shown, the area of each is one square inch, and the distances from the axis to the centers of gravity of the parts are $\frac{1}{2}$ and $1\frac{1}{2}$ inches. For the four parts lying nearest the axis the product (area times distance squared) is:

$1\times(\frac{1}{2})^2=\frac{1}{4}$; and for the other parts it is
$1\times(1\frac{1}{2})^2=\frac{9}{4}$.

Hence the approximate value of the moment of inertia of the area with respect to the axis, is

$$4(\tfrac{1}{4})+4(\tfrac{9}{4})=10.$$

If the area is divided into 32 parts, as shown in Fig. 27, *b*, the area of each part is $\frac{1}{4}$ square inch. For the eight of the little squares farthest away from the axis, the distance from their centers of gravity to the axis is $1\frac{3}{4}$ inches; for the next eight it is $1\frac{1}{4}$; for the next eight $\frac{3}{4}$; and for the remainder $\frac{1}{4}$ inch. Hence an approximate value of the moment of inertia of the rectangle with respect to the axis is:

$$8\times\tfrac{1}{4}\times(1\tfrac{3}{4})^2+8\times\tfrac{1}{4}\times(1\tfrac{1}{4})^2+8\times\tfrac{1}{4}\times(\tfrac{3}{4})^2+8\times\tfrac{1}{4}\times(\tfrac{1}{4})^2=10\tfrac{1}{2}.$$

If we divide the rectangle into still smaller parts and form the products

$$(\text{small area})\times(\text{distance})^2,$$

and add the products just as we have done, we shall get a larger answer than 10½. The smaller the parts into which the rectangle is divided, the larger will be the answer, but it will never be larger than 10⅔. This 10⅔ is the sum corresponding to a division of the rectangle into an infinitely large number of parts (infinitely small) and it is the exact value of the moment of inertia of the rectangle with respect to the axis selected.

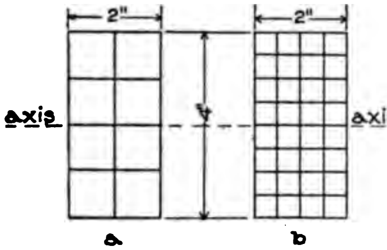


Fig. 27.

There are short methods of computing the exact values of the moments of inertia of simple figures (rectangles, circles, etc.,), but they cannot be given here since they involve the use of difficult mathematics. The foregoing method to obtain approximate values of moments of inertia is used especially when the area is quite irregular in shape, but it is given here to explain to the student the *meaning* of the moment of inertia of an area. He should understand now that the moment of inertia of an area is simply a name for such sums as we have just computed. The name is not a fitting one, since the sum has nothing whatever to do with inertia. It was first used in this connection because the sum is very similar to certain other sums which had previously been called moments of inertia.

51. Unit of Moment of Inertia. The product (area × distance²) is really the product of four lengths, two in each factor; and since a moment of inertia is the sum of such products, a moment of inertia is also the product of four lengths. Now the product of two lengths is an area, the product of three is a volume, and the product of four is moment of inertia—unthinkable in the way in which we can think of an area or volume, and therefore the source of much difficulty to the student. The units of these quantities (area, volume, and moment of inertia) are respectively:

the square inch, square foot, etc.,
" cubic " , cubic " " ,
" biquadratic inch, biquadratic foot, etc.;

but the biquadratic inch is almost exclusively used in this connection; that is, the inch is used to compute values of moments of inertia, as in the preceding illustration. It is often written thus: Inches^4.

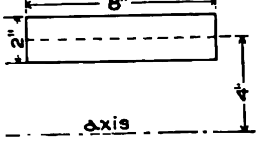


Fig 28.

52. Moment of Inertia of a Rectangle. Let b denote the base of a rectangle, and a its altitude; then by higher mathematics it can be shown that the moment of inertia of the rectangle with respect to a line through its center of gravity and parallel to its base, is $\frac{1}{12} ba^3$.

Example. Compute the value of the moment of inertia of a rectangle 4×12 inches with respect to a line through its center of gravity and parallel to the long side.

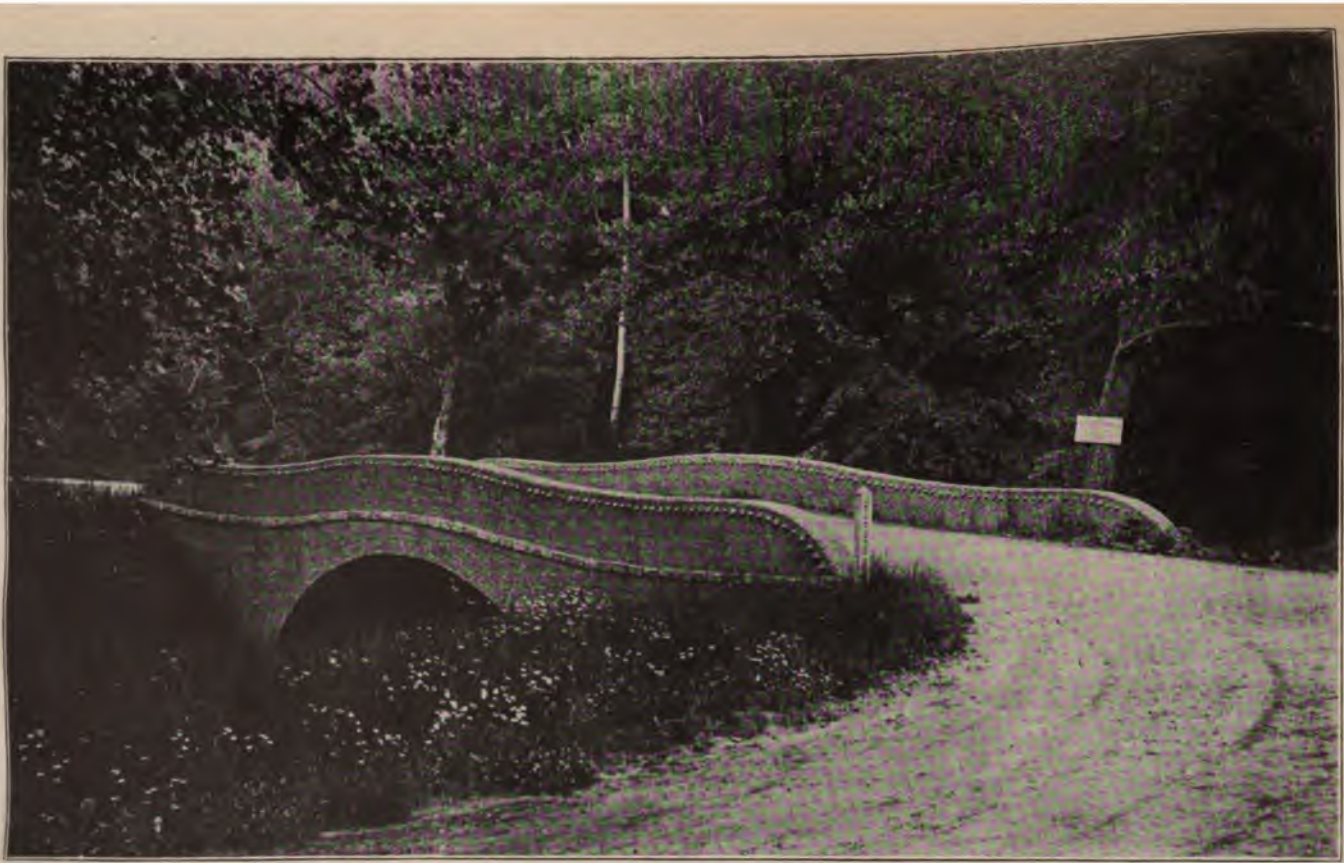
Here $b = 12$, and $a = 4$ inches; hence the moment of inertia desired equals

$$\tfrac{1}{12}(12 \times 4^3) = 64 \text{ inches}^4.$$

EXAMPLE FOR PRACTICE.

1. Compute the moment of inertia of a rectangle 4×12 inches with respect to a line through its center of gravity and parallel to the short side. Ans. 576 inches^4.

53. Reduction Formula. In the previously mentioned "handbooks" there can be found tables of moments of inertia of all the cross-sections of the kinds and sizes of rolled shapes made. The inertia-axes in those tables are always taken through the center of gravity of the section, and usually parallel to some edge of the section. Sometimes it is necessary to compute the moment of inertia of a "rolled section" with respect to some other axis, and if the two axes (that is, the one given in the tables, and the other) are parallel, then the desired moment of inertia can be easily computed from the one given in the tables by the following rule:



CEMENT AND PEBBLE BRIDGE IN ROCK CREEK PARK, WASHINGTON, D. C.

The moment of inertia of an area with respect to any axis equals the moment of inertia with respect to a parallel axis through the center of gravity, plus the product of the area and the square of the distance between the axes.

Or, if I denotes the moment of inertia with respect to any axis; I_0 the moment of inertia with respect to a parallel axis through the center of gravity; A the area; and d the distance between the axes, then

$$I = I_0 + Ad^2 \ldots \quad (5)$$

Example. It is required to compute the moment of inertia of a rectangle 2×8 inches with respect to a line parallel to the long side and 4 inches from the center of gravity.

Let I denote the moment of inertia sought, and I_0 the moment of inertia of the rectangle with respect to a line parallel to the long side and through the center of gravity (see Fig. 28). Then

$I_0 = \frac{1}{12} ba^3$ (see Art. 52); and, since $b = 8$ inches and $a = 2$ inches,

$I_0 = \frac{1}{12}(8 \times 2^3) = 5\frac{1}{3}$ biquadratic inches.

The distance between the two inertia-axes is 4 inches, and the area of the rectangle is 16 square inches, hence equation 5 becomes

$$I = 5\tfrac{1}{3} + 16 \times 4^2 = 261\tfrac{1}{3} \text{ biquadratic inches.}$$

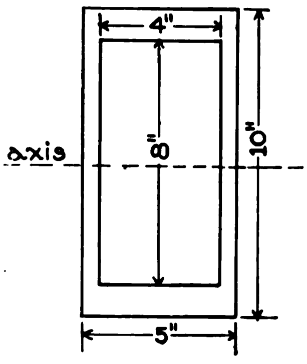


Fig. 29.

EXAMPLE FOR PRACTICE.

1. The moment of inertia of an "angle" $2\frac{1}{2} \times 2 \times \frac{1}{4}$ inches (lengths of sides and width respectively) with respect to a line through the center of gravity and parallel to the long side, is 0.64 inches⁴. The area of the section is 2 square inches, and the distance from the center of gravity to the long side is 0.63 inches. (These values are taken from a "handbook".) It is required to compute the moment of inertia of the section with respect to a line parallel to the long side and 4 inches from the center of gravity. Ans. 32.64 inches⁴.

54. Moment of Inertia of Built-up Sections. As before stated, beams are sometimes "built up" of rolled shapes (angles, channels, etc.). The moment of inertia of such a section with respect to a definite axis is computed by adding the moments of inertia of the parts, *all with respect to that same axis.* This is the method for computing the moment of any area which can be divided into simple parts.

The moment of inertia of an area which may be regarded as consisting of a larger area *minus* other areas, is computed by subtracting from the moment of inertia of the large area those of the "minus areas."

Examples. 1. Compute the moment of inertia of the built-up section represented in Fig. 30 (in part same as Fig. 25) with respect to a horizontal axis passing through the center of gravity, it being given that the moment of inertia of each channel section with respect to a horizontal axis through its center of gravity is 128.1 inches^4, and its area 6.03 square inches.

The center of gravity of the whole section was found in the example of Art. 49 to be 8.30 inches from the bottom of the section; hence the distances from the inertia-axis to the centers of gravity of the channel section and the plate are 2.30 and 3.95 inches respectively (see Fig. 30).

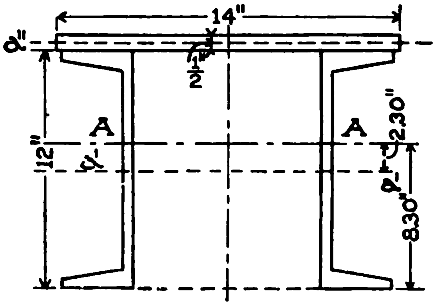


Fig. 30.

The moment of inertia of one channel section with respect to the axis AA (see equation 5, Art. 53) is:

$$128.1+6.03\times 2.30^2=160.00 \text{ inches}^4.$$

The moment of inertia of the plate section (rectangle) with respect to the line $a''a''$ (see Art. 52) is:

$$\tfrac{1}{12}\, ba^3=\tfrac{1}{12}[14\times(\tfrac{1}{2})^3]=0.15 \text{ inches}^4;$$

and with respect to the axis AA (the area being 7 square inches) it is:

$$0.15+7\times 3.95^2=109.37 \text{ inches}^4.$$

Therefore the moment of inertia of the whole section with respect to AA is:

$$2\times 160.00+109.37=429.37 \text{ inches}^4.$$

2. It is required to compute the moment of inertia of the "hollow rectangle" of Fig. 29 with respect to a line through the center of gravity and parallel to the short side.

The amount of inertia of the large rectangle with respect to the named axis (see Art. 52) is:

$$\tfrac{1}{12}(5\times10^3) = 416\tfrac{2}{3};$$

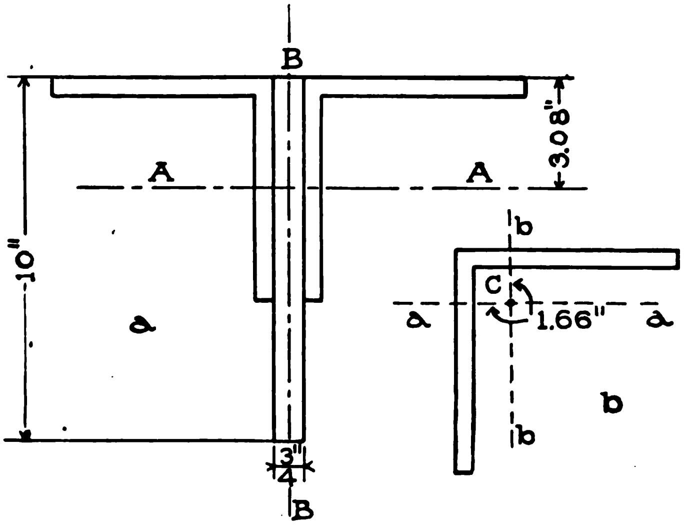


Fig. 31.

and the moment of inertia of the smaller one with respect to the same axis is:

$$\tfrac{1}{12}(4\times8^3) = 170\tfrac{2}{3};$$

hence the moment of inertia of the hollow section with respect to the axis is:

$$416\tfrac{2}{3} - 170\tfrac{2}{3} = 246 \text{ inches}^4.$$

EXAMPLES FOR PRACTICE.

1. Compute the moment of inertia of the section represented in Fig. 31, *a*, about the axis AA, it being 3.08 inches from the top. Given also that the area of one angle section is 5.06 square inches, its center of gravity C (Fig. 31, *b*) 1.66 inches from the top, and its moment of inertia with respect to the axis *aa* 17.68 inches⁴. Ans. 145.8 inches⁴.

2. Compute the moment of inertia of the section of Fig. 31, *a*,

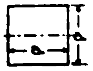
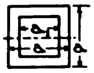
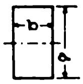
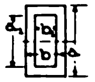
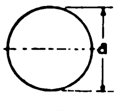
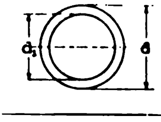
with respect to the axis BB. Given that distance of the center of gravity of one angle from one side is 1.66 inches (see Fig. 31,*b*), and its moment of inertia with respect to *bb* 17.68 inches.

Ans. 77.5 inches^4.

55. Table of Centers of Gravity and Moments of Inertia. Column 2 in Table A below gives the formula for moment of inertia with respect to the horizontal line through the center of gravity. The numbers in the third column are explained in Art. 62; and those in the fourth, in Art. 80.

TABLE A.

Moments of Inertia, Section Moduli, and Radii of Gyration.
In each case the axis is horizontal and passes through the center of gravity.

Section.	Moment of Inertia.	Section Modulus.	Radius of Gyration.
	$\frac{a^4}{12}$	$\frac{a^3}{6}$	$\frac{a}{\sqrt{12}}$
	$\frac{a^4-a_1^4}{12}$	$\frac{a^4-a_1^4}{6a}$	$\sqrt{\frac{a^2+a_1^2}{12}}$
	$\frac{ba^3}{12}$	$\frac{ba^2}{6}$	$\frac{a}{\sqrt{12}}$
	$\frac{ba^3-b_1a_1^3}{12}$	$\frac{ba^3-b_1a_1^3}{6a}$	$\sqrt{\frac{ba^3-b_1a_1^3}{12(ba-b_1a_1)}}$
	$0.049d^4$	$0.098d^3$	$\frac{d}{4}$
	$0.049(d^4-d_1^4)$	$0.098\frac{d^4-d_1^4}{d}$	$\frac{\sqrt{d^2+d_1^2}}{4}$

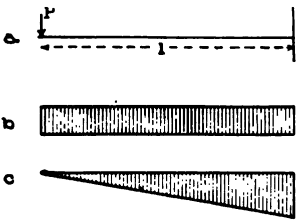
STRENGTH OF BEAMS.

56. Kinds of Loads Considered. The loads that are applied to a horizontal beam are usually vertical, but sometimes forces are applied otherwise than at right angles to beams. Forces acting on beams at right angles are called **transverse forces**; those applied

TABLE B.

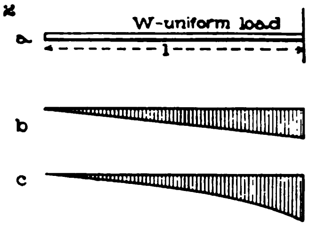
Shear Diagrams (b) and Moment Diagrams (c) for Eight Different Cases (a). Also Values of Maximum Shear (V), Bending Moment (M), and Deflection (d).

1



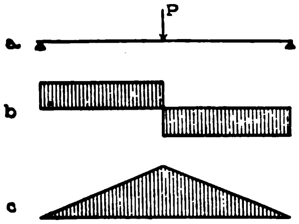
$V=P$, $M=Pl$, $d=Pl^3 \div 3EI$.

2



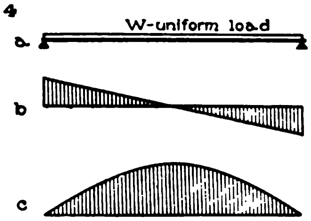
$V=W$, $M=\frac{1}{2}Wl$, $d=Wl^3 \div 8EI$.

3



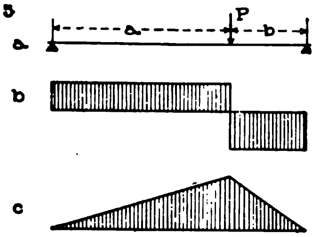
$V=\frac{1}{2}P$, $M=\frac{1}{4}Pl$, $d=Pl^3 \div 48EI$.

4



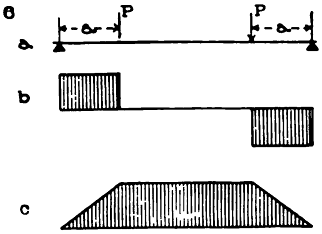
$V=\frac{1}{2}W$, $M=\frac{1}{8}Wl$, $d=5Wl^3 \div 384EI$.

5



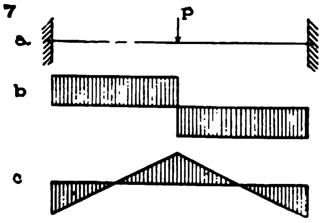
$V=Pa \div l$, $M=Pab \div l$.

6



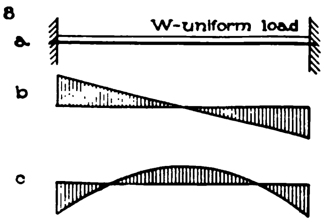
$V=P$, $M=Pa$, $d=Pa\,(3l^2-4a^2) \div 24EI$.

7



$V=\frac{1}{2}P$, $M=\frac{1}{8}Pl$, $d=Pl^3 \div 192EI$.

8



$V=\frac{1}{2}W$, $M=\frac{1}{12}Wl$, $d=Wl^3 \div 384EI$.

parallel to a beam are called **longitudinal forces**; and others are called **inclined forces.** For the present we deal only with beams subjected to transverse forces (loads and reactions).

57. Neutral Surface, Neutral Line, and Neutral Axis. When a beam is loaded it may be wholly convex up (concave down), as a cantilever; wholly convex down (concave up), as a simple beam on end supports; or partly convex up and partly convex down, as a simple beam with overhanging ends, a restrained beam, or a continuous beam. Two vertical parallel lines drawn close together on the side of a beam before it is loaded will not be parallel after it is loaded and bent. If they are on a convex-down portion of a beam, they will be closer at the top and farther apart below than when drawn (Fig. 32*a*), and if they are on a convex-up portion, they will be closer below and farther apart above than when drawn (Fig. 32*b*).

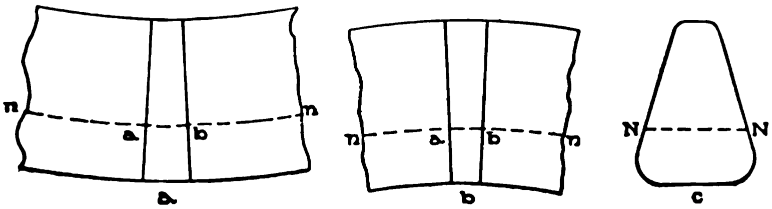


Fig. 32.

The "fibres" on the convex side of a beam are stretched and therefore under tension, while those on the concave side are shortened and therefore under compression. Obviously there must be some intermediate fibres which are neither stretched nor shortened, *i. e.*, under neither tension nor compression. These make up a sheet of fibres and define a surface in the beam, which surface is called the **neutral surface** of the beam. The intersection of the neutral surface with either side of the beam is called the **neutral line,** and its intersection with any cross-section of the beam is called the **neutral axis** of that section. Thus, if *ab* is a fibre that has been neither lengthened nor shortened with the bending of the beam, then *nn* is a portion of the neutral line of the beam; and, if Fig. 32*c* be taken to represent a cross-section of the beam, NN is the neutral axis of the section.

It can be proved that *the neutral axis of any cross-section of* *a loaded beam passes through the center of gravity of that section*, provided that all the forces applied to the beam are transverse, and that the tensile and compressive stresses at the cross-section are all within the elastic limit of the material of the beam.

58. Kinds of Stress at a Cross-section of a Beam. It has already been explained in the preceding article that there are tensile and compressive stresses in a beam, and that the tensions are on the convex side of the beam and the compressions on the concave (see Fig. 33). The forces T and C are exerted upon the portion of the beam represented by the adjoining portion to the right (not shown). These, the student is reminded, are often called **fibre stresses.**

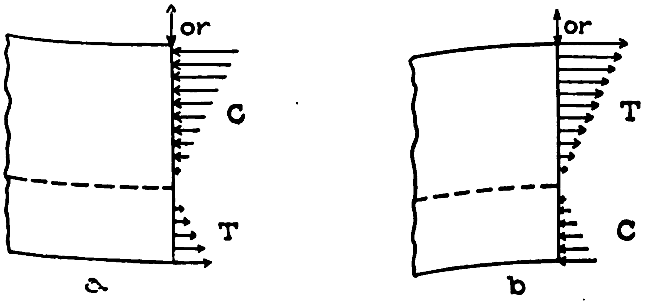


Fig. 33.

Besides the fibre stresses there is, in general, a shearing stress at every cross-section of a beam. This may be proved as follows:

Fig. 34 represents a simple beam on end supports which has actually been cut into two parts as shown. The two parts can maintain loads when in a horizontal position, if forces are applied at the cut ends equivalent to the forces that would act there if the beam were not cut. Evidently in the solid beam there are at the section a compression above and a tension below, and such forces can be applied in the cut beam by means of a short block C and a chain or cord T, as shown. The block furnishes the compressive forces and the chain the tensile forces. At first sight it appears as if the beam would stand up under its load after the block and chain have been put into place. Except in certain cases*, however, it would not remain in a horizontal position, as would the

---

* When the external shear for the section is zero.

61. Value of the Resisting Moment. If S denotes the unit-fibre stress in the fibre farthest from the neutral axis (the greatest unit-fibre stress on the cross-section), and c the distance from the neutral axis to the remotest fibre, while S_1, S_2, S_3, etc., denote the unit-fibre stresses at points whose distances from the neutral axis are, respectively, y_1, y_2, y_3, etc. (see Fig. 36b), then

$$S : S_1 :: c : y_1; \text{ or } S_1 = \frac{S}{c} y_1.$$

Also,
$$S_2 = \frac{S}{c} y_2; \; S_3 = \frac{S}{c} y_3, \text{ etc.}$$

Let a_1, a_2, a_3, etc., be the areas of the cross-sections of the fibres

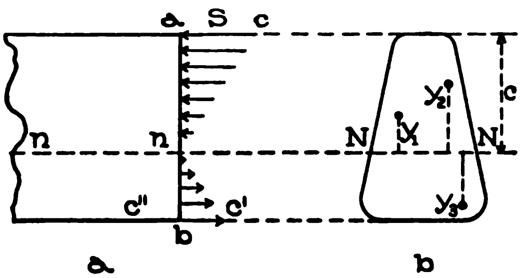


Fig. 36.

whose distances from the neutral axis are, respectively, y_1, y_2, y_3, etc. Then the stresses on those fibres are, respectively,

$$S_1 a_1, \; S_2 a_2, \; S_3 a_3, \text{ etc.};$$

or,
$$\frac{S}{c} y_1 a_1, \; \frac{S}{c} y_2 a_2, \; \frac{S}{c} y_3 a_3, \text{ etc.}$$

The arms of these forces or stresses with respect to the neutral axis are, respectively, y_1, y_2, y_3, etc.; hence their moments are

$$\frac{S}{c} a_1 y_1^2, \; \frac{S}{c} a_2 y_2^2, \; \frac{S}{c} a_3 y_3^2, \text{ etc.},$$

and the sum of the moments (that is, the resisting moment) is

$$\frac{S}{c} a_1 y_1^2 + \frac{S}{c} a_2 y_2^2 + \text{etc.} = \frac{S}{c}(a_1 y_1^2 + a_2 y_2^2 + \text{etc.})$$

Now $a_1 y_1^2 + a_2 y_2^2 +$ etc. is the sum of the products obtained by multiplying each infinitesimal part of the area of the cross-section by the square of its distance from the neutral axis; hence, it is the moment of inertia of the cross-section with respect to the neutral axis. If this moment is denoted by I, then the value of the resisting moment is $\frac{SI}{c}$.

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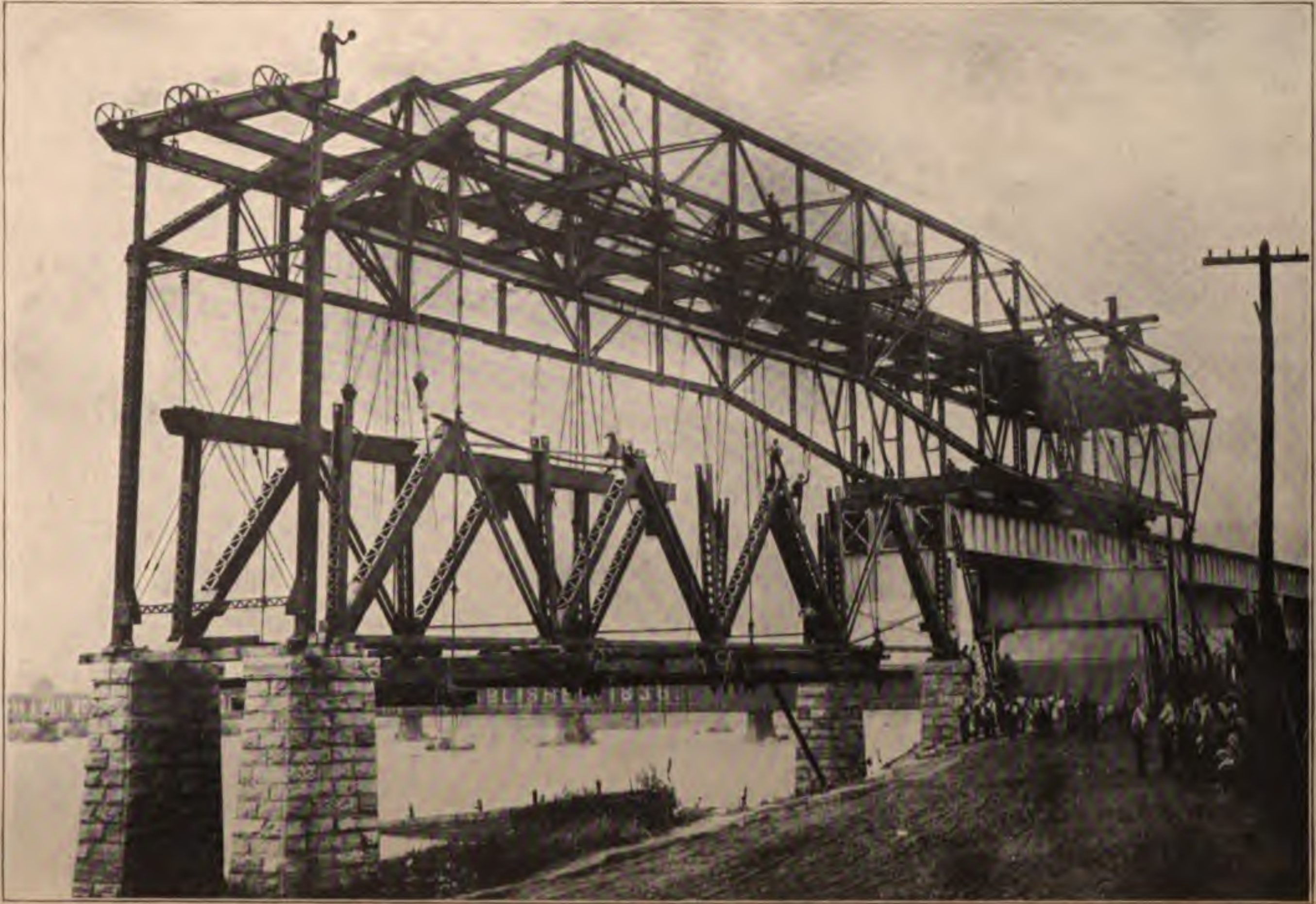

RICHMOND VIADUCT AND TERMINAL RAILWAY AT RICHMOND, VIRGINIA

On line of Chesapeake & Ohio Railway. Built in 1900-03. Total length of viaduct, 14,569 ft.

Courtesy of American Bridge Company.

STRENGTH OF MATERIALS.

PART II.

STRENGTH OF BEAMS---(Concluded).

62. First Beam Formula. As shown in the preceding article, the resisting and bending moments for any section of a beam are equal; hence

$$\frac{SI}{c} = M, \qquad (6)$$

all the symbols referring to the same section. This is the most important formula relating to beams, and will be called the "first beam formula."

The ratio $I \div c$ is now quite generally called the **section modulus.** Observe that for a given beam it depends only on the dimensions of the cross-section, and not on the material or anything else. Since I is the product of four lengths (see article 51), $I \div c$ is the product of three; and hence a section modulus can be expressed in units of volume. The cubic inch is practically always used; and in this connection it is written thus, inches^3. See Table A, page 54, for values of the section moduli of a few simple sections.

63. Applications of the First Beam Formula. There are three principal applications of equation 6, which will now be explained and illustrated.

64. *First Application.* The dimensions of a beam and its manner of loading and support are given, and it is required to compute the greatest unit-tensile and compressive stresses in the beam.

This problem can be solved by means of equation 6, written in this form,

$$S = \frac{Mc}{I} \text{ or } \frac{M}{I \div c}. \qquad (6')$$

Unless otherwise stated, we assume that the beams are uniform in cross-section, as they usually are; then the section modulus $(I \div c)$ is the same for all sections, and S (the unit-fibre stress on

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the remotest fibré) varies just as M varies, and is therefore greatest where M is a maximum.* Hence, to compute the value of the greatest unit-fibre stress in a given case, *substitute the values of the section modulus and the maximum bending moment in the preceding equation, and reduce.*

If the neutral axis is equally distant from the highest and lowest fibres, then the greatest tensile and compressive unit-stresses are equal, and their value is S. If the neutral axis is unequally distant from the highest and lowest fibres, let c' denote its distance from the nearer of the two, and S′ the unit-fibre stress there. Then, since the unit-stresses in a cross-section are proportional to the distances from the neutral axis,

$$\frac{S'}{S} = \frac{c'}{c}, \text{ or } S' = \frac{c'}{c}S.$$

If the remotest fibre is on the convex side of the beam, S is tensile and S′ compressive; if the remotest fibre is on the concave side, S is compressive and S′ tensile.

Examples. 1. A beam 10 feet long is supported at its ends, and sustains a load of 4,000 pounds two feet from the left end (Fig. 37, *a*). If the beam is 4 × 12 inches in cross-section (the long side vertical as usual), compute the maximum tensile and compressive unit-stresses.

The section modulus of a rectangle whose base and altitude are b and a respectively (see Table A, page 54), is $\frac{1}{6}ba^2$; hence, for the beam under consideration, the modulus is

$$\frac{1}{6} \times 4 \times 12^2 = 96 \text{ inches}^3.$$

To compute the maximum bending moment, we have, first, to find the dangerous section. This section is where the shear changes sign (see article 45); hence, we have to construct the shear diagram, or as much thereof as is needed to find where the change of sign occurs. Therefore we need the values of the reaction. Neglecting the weight of the beam, the moment equation with origin at C (Fig. 37, *a*) is

$$R_1 \times 10 - 4{,}000 \times 8 = 0, \text{ or } R_1 = 3{,}200 \text{ pounds}$$

---

* NOTE. Because S is greatest in the section where M is maximum, this section is usually called the "dangerous section" of the beam.

Then, constructing the shear diagram, we see (Fig. 37, *b*) that the change of sign of the shear (also the dangerous section) is at the load. The value of the bending moment there is

$$3{,}200 \times 2 = 6{,}400 \text{ foot-pounds,}$$

or

$$6{,}400 \times 12 = 76{,}800 \text{ inch-pounds.}$$

Substituting in equation 6′, we find that

$$S = \frac{76{,}800}{96} = 800 \text{ pounds per square inch.}$$

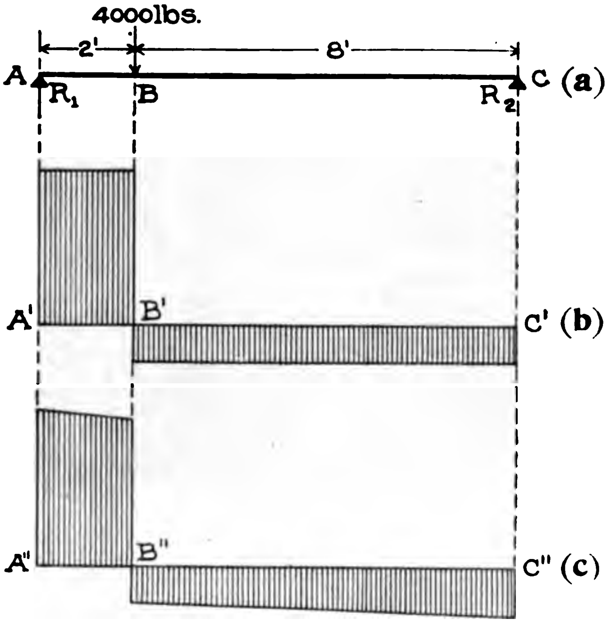


Fig. 37.

2. It is desired to take into account the weight of the beam in the preceding example, supposing the beam to be wooden.

The volume of the beam is

$$\frac{4 \times 12}{144} \times 10 = 3\tfrac{1}{3} \text{ cubic feet;}$$

and supposing the timber to weigh 45 pounds per cubic foot, the beam weighs 150 pounds (insignificant compared to the load). The left reaction, therefore, is

$$3{,}200 + \left(\frac{1}{2} \times 150\right) = 3{,}275;$$

and the shear diagram looks like Fig. 37, *c*, the shear changing sign at the load as before. The weight of the beam to the left of the dangerous section is 30 pounds; hence the maximum bending moment equals

$$3{,}275 \times 2 - 30 \times 1 = 6{,}520 \text{ foot-pounds,}$$

or

$$6{,}520 \times 12 = 78{,}240 \text{ inch-pounds.}$$

Substituting in equation 6′, we find that

$$S = \frac{78{,}240}{96} = 815 \text{ pounds per square inch.}$$

The weight of the beam therefore increases the unit-stress produced by the load at the dangerous section by 15 pounds per square inch.

3. A T-bar (see Fig. 38) 8 feet long and supported at each end, bears a uniform load of 1,200 pounds. The moment of inertia of its cross-section with respect to the neutral axis being 2.42 inches⁴, compute the maximum tensile and compressive unit-stresses in the beam.

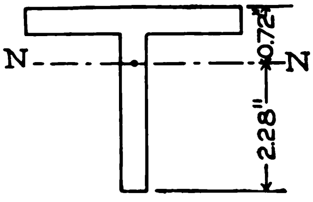


Fig. 38.

Evidently the dangerous section is in the middle, and the value of the maximum bending moment (see Table B, page 55, Part I) is $\frac{1}{8}$ W*l*, W and *l* denoting the load and length respectively. Here

$$\frac{1}{8} Wl = \frac{1}{8} \times 1{,}200 \times 8 = 1{,}200 \text{ foot-pounds,}$$

or

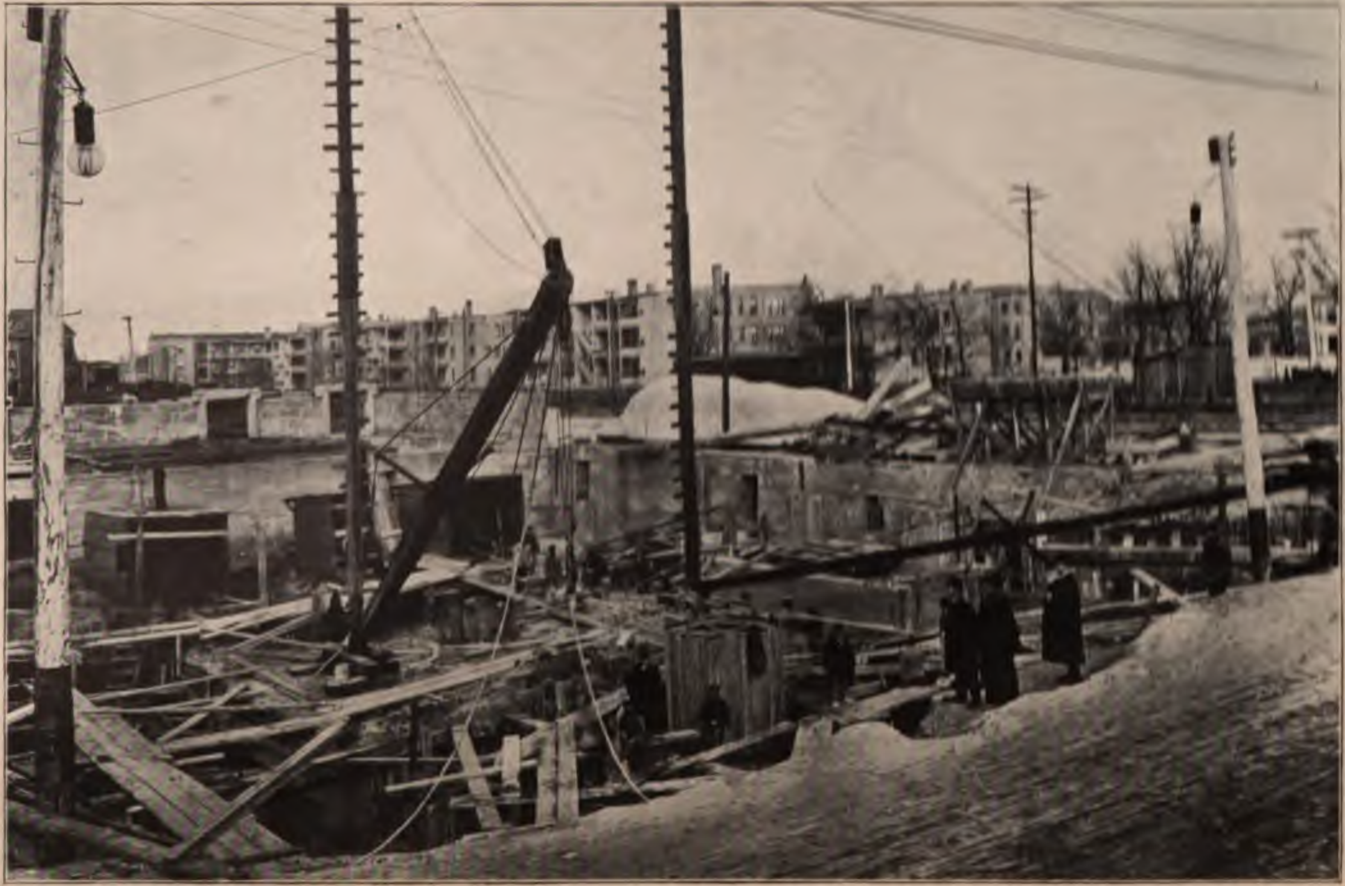
$$1{,}200 \times 12 = 14{,}400 \text{ inch-pounds.}$$

The section modulus equals 2.42 ÷ 2.28 = 1.06; hence

$$S = \frac{14{,}400}{1.06} = 13{,}585 \text{ pounds per square inch.}$$

This is the unit-fibre stress on the lowest fibre at the middle section, and hence is tensile. On the highest fibre at the middle section the unit-stress is compressive, and equals (see page 62):

$$S' = \frac{c'}{c} S = \frac{0.72}{2.28} \times 13{,}585 = 4{,}290 \text{ pounds per square inch.}$$



LAWRENCE AVENUE PUMPING STATION, SYSTEM OF INTERCEPTING SEWERS, CHICAGO, ILL., UNDER CONSTRUCTION

EXAMPLES FOR PRACTICE.

1. A beam 12 feet long and 6 × 12 inches in cross-section rests on end supports, and sustains a load of 3,000 pounds in the middle. Compute the greatest tensile and compressive unit-stresses in the beam, neglecting the weight of the beam.

Ans. 750 pounds per square inch.

2. Solve the preceding example taking into account the weight of the beam, 300 pounds

Ans. 787.5 pounds per square inch.

3. Suppose that a built-in cantilever projects 5 feet from the wall and sustains an end load of 250 pounds. The cross-section of the cantilever being represented in Fig. 38, compute the greatest tensile and compressive unit-stresses, and tell at what places they occur. (Neglect the weight.)

Ans. { Tensile, 4,469 pounds per square inch.
{ Compressive, 14,150 " " " "

4. Compute the greatest tensile and compressive unit-stresses in the beam of Fig. 18, *a*, due to the loads and the weight of beam (400 pounds). (A moment diagram is represented in Fig. 18, *b*; for description see example 2, Art. 44, p. 39.) The section of the beam is a rectangle 8 × 12 inches.

Ans. 580 pounds per square inch.

5. Compute the greatest tensile and compressive unit-stresses in the cantilever beam of Fig. 19, *a*, it being a steel I-beam whose section modulus is 20.4 inches^3. (A bending moment diagram for it is represented in Fig. 19, *b*; for description, see Ex. 3, Art. 44.)

Ans. 11,470 pounds per square inch.

6. Compute the greatest tensile and compressive unit-stresses in the beam of Fig. 10, neglecting its weight, the cross-sections being rectangular 6 × 12 inches. (See example for practice 1, Art. 43.)

Ans. 600 pounds per square inch.

65. *Second Application.* The dimensions and the working strengths of a beam are given, and it is required to determine its safe load (the manner of application being given).

This problem can be solved by means of equation 6 written in this form,

$$M = \frac{SI}{c} \qquad (6'')$$

We substitute for S the given working strength for the material of the beam, and for I and c their values as computed from the given dimensions of the cross-section; then reduce, thus obtaining the value of the safe resisting moment of the beam, which equals the greatest safe bending moment that the beam can stand. We next compute the value of the maximum bending moment in terms of the unknown load; equate this to the value of the resisting moment previously found; and solve for the unknown load.

In cast iron, the tensile and compressive strengths are very different; and the smaller (the tensile) should always be used if the neutral surface of the beam is midway between the top and bottom of the beam; but if it is unequally distant from the top and bottom, proceed as in example 4, following.

Examples. 1. A wooden beam 12 feet long and 6 × 12 inches in cross-section rests on end supports. If its working strength is 800 pounds per square inch, how large a load uniformly distributed can it sustain?

The section modulus is $\frac{1}{6}bd^2$, b and d denoting the base and altitude of the section (see Table A, page 54); and here

$$\frac{1}{6}bd^2 = \frac{1}{6} \times 6 \times 12^2 = 144 \text{ inches}^3.$$

Hence $$S\frac{I}{c} = 800 \times 144 = 115{,}200 \text{ inch-pounds}.$$

For a beam on end supports and sustaining a uniform load, the maximum bending moment equals $\frac{1}{8}Wl$ (see Table B, page 55), W denoting the sum of the load and weight of beam, and l the length. If W is expressed in pounds, then

$$\frac{1}{8}Wl = \frac{1}{8}W \times 12 \text{ foot-pounds} = \frac{1}{8}W \times 144 \text{ inch-pounds}.$$

Hence, equating the two values of maximum bending moment and the safe resisting moment, we get

$$\frac{1}{8}W \times 144 = 115{,}200;$$

or, $$W = \frac{115{,}200 \times 8}{144} = 6{,}400 \text{ pounds}.$$

The safe load for the beam is 6,400 pounds minus the weight of the beam.

2. A steel I-beam whose section modulus is 20.4 inches^3 rests on end supports 15 feet apart. Neglecting the weight of the beam, how large a load may be placed upon it 5 feet from one end, if the working strength is 16,000 pounds per square inch?

The safe resisting moment is

$$\frac{SI}{c} = 16,000 \times 20.4 = 326,400 \text{ inch-pounds};$$

hence the bending moment must not exceed that value. The dangerous section is under the load; and if P denotes the unknown value of the load in pounds, the maximum moment (see Table B, page 55, Part I) equals $\frac{2}{3}$ P × 5 foot-pounds, or $\frac{2}{3}$ P × 60 inch-pounds. Equating values of bending and resisting moments, we get

$$\frac{2}{3} P \times 60 = 326,400;$$

or, $$P = \frac{326,400 \times 3}{2 \times 60} = 8,160 \text{ pounds}.$$

3. In the preceding example, it is required to take into account the weight of the beam, 375 pounds.

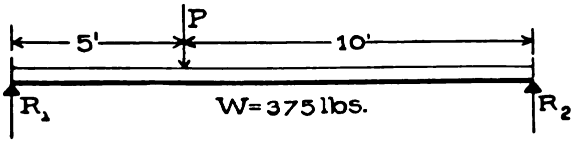


Fig. 39.

As we do not know the value of the safe load, we cannot construct the shear diagram and thus determine where the dangerous section is. But in cases like this, where the distributed load (the weight) is small compared with the concentrated load, the dangerous section is practically always where it is under the concentrated load alone; in this case, at the load. The reactions due to the weight equal $\frac{1}{2}$ × 375 = 187.5; and the reactions due to the load equal $\frac{1}{3}$ P and $\frac{2}{3}$ P, P denoting the value of the load. The larger reaction R_1 (Fig. 39) hence equals $\frac{2}{3}$ P + 187.5. Since the weight of the beam per foot is 375 ÷ 15 = 25 pounds, the maximum bending moment (at the load) equals

$$\left(\frac{2}{3}P + 187.5\right)5 - (25 \times 5)\,2\tfrac{1}{2} =$$

$$\frac{10}{3}P + 937.5 - 312.5 = \frac{10}{3}P + 625.$$

This is in foot-pounds if P is in pounds.

The safe resisting moment is the same as in the preceding illustration, 326,400 inch-pounds; hence

$$\left(\frac{10}{3}P + 625\right)12 = 326{,}400.$$

Solving for P, we have

$$\frac{10}{3}P + 625 = \frac{326{,}400}{12};$$

$$10\,P + 625 \times 3 = \frac{326{,}400 \times 3}{12} = 81{,}600;$$

$$10\,P = 79{,}725;$$

or, $$P = 7{,}972.5 \text{ pounds.}$$

It remains to test our assumption that the dangerous section is at the load. This can be done by computing R_1 (with P = 7,972.5), constructing the shear diagram, and noting where the shear changes sign. It will be found that the shear changes sign at the load, thus verifying the assumption.

4. A cast-iron built-in cantilever beam projects 8 feet from the wall. Its cross-section is represented in Fig. 40, and the moment of inertia with respect to the neutral axis is 50 inches^4; the working strengths in tension and compression are 2,000 and 9,000 pounds per square inch respectively. Compute the safe uniform load which the beam can sustain, neglecting the weight of the beam.

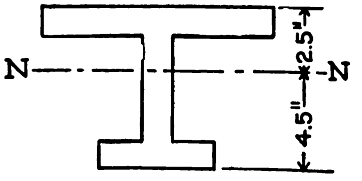


Fig. 40.

The beam being convex up, the upper fibres are in tension and the lower in compression. The resisting moment (SI ÷ *c*), as determined by the compressive strength, is

$$\frac{9,000 \times 50}{4.5} = 100,000 \text{ inch-pounds};$$

and the resisting moment, as determined by the tensile strength, is

$$\frac{2,000 \times 50}{2.5} = 40.000 \text{ inch-pounds}.$$

Hence the safe resisting moment is the lesser of these two, or 40,000 inch-pounds. The dangerous section is at the wall (see Table B, page 55), and the value of the maximum bending moment is $\frac{1}{2}$ Wl, W denoting the load and l the length. If W is in pounds, then

$$M = \tfrac{1}{2} W \times 8 \text{ foot-pounds} = \tfrac{1}{2} W \times 96 \text{ inch-pounds}.$$

Equating bending and resisting moments, we have

$$\frac{1}{2} W \times 96 = 40,000;$$

or,

$$W = \frac{40,000 \times 2}{96} = 833 \text{ pounds}.$$

EXAMPLES FOR PRACTICE.

1. An 8 × 8-inch timber projects 8 feet from a wall. If its working strength is 1,000 pounds per square inch, how large an end load can it safely sustain?

Ans. 890 pounds.

2. A beam 12 feet long and 8 × 16 inches in cross-section, on end supports, sustains two loads P, each 3 feet from its ends respectively. The working strength being 1,000 pounds per square inch, compute P (see Table B, page 55).

Ans. 9,480 pounds.

3. An I-beam weighing 25 pounds per foot rests on end supports 20 feet apart. Its section modulus is 20.4 inches^3, and its working strength 16,000 pounds per square inch. Compute the safe uniform load which it can sustain.

Ans. 10,880 pounds-

66. *Third Application.* The loads, manner of support, and working strength of beam are given, and it is required to determine the size of cross-section necessary to sustain the load safely, that is, to "design the beam."

To solve this problem, we use the first beam formula (equation 6), written in this form,

$$\frac{I}{c} = \frac{M}{S}. \qquad (6''')$$

We first determine the maximum bending moment, and then substitute its value for M, and the working strength for S. Then we have the value of the section modulus $(I \div c)$ of the required beam. Many cross-sections can be designed, all having a given section modulus. Which one is to be selected as most suitable will depend on the circumstances attending the use of the beam and on considerations of economy.

Examples. 1. A timber beam is to be used for sustaining a uniform load of 1,500 pounds, the distance between the supports being 20 feet. If the working strength of the timber is 1,000 pounds per square inch, what is the necessary size of cross-section ?

The dangerous section is at the middle of the beam; and the maximum bending moment (see Table B, page 55) is

$$\frac{1}{8}Wl = \frac{1}{8} \times 1,500 \times 20 = 3,750 \text{ foot-pounds,}$$

or $$3,750 \times 12 = 45,000 \text{ inch-pounds.}$$

Hence $$\frac{I}{c} = \frac{45,000}{1,000} = 45 \text{ inches}^3.$$

Now the section modulus of a rectangle is $\frac{1}{6}ba^2$ (see Table A, page 54, Part I); therefore, $\frac{1}{6}ba^2 = 45$, or $ba^2 = 270$.

Any wooden beam (safe strength 1,000 pounds per square inch) whose breadth times its depth square equals or exceeds 270, is strong enough to sustain the load specified, 1,500 pounds.

To determine a size, we may choose any value for b or a, and solve the last equation for the unknown dimension. It is best, however, to select a value of the breadth, as 1, 2, 3, or 4 inches, and solve for a. Thus, if we try $b = 1$ inch, we have

$$a^2 = 270, \text{ or } a = 16.43 \text{ inches.}$$

This would mean a board 1 × 18 inches, which, if used, would have to be supported sidewise so as to prevent it from tipping or "buckling." Ordinarily, this would not be a good size.

Next try $b = 2$ inches; we have

$$2 \times a^2 = 270; \text{ or } a = \sqrt{270 \div 2} = 11.62 \text{ inches.}$$

This would require a plank 2 × 12, a better proportion than the first. Trying $b = 3$ inches, we have

$$3 \times a^2 = 270; \text{ or } a = \sqrt{270 \div 3} = 9.49 \text{ inches.}$$

This would require a plank 3 × 10 inches; and a choice between a 2 × 12 and a 3 × 10 plank would be governed by circumstances in the case of an actual construction.

It will be noticed that we have neglected the weight of the beam. Since the dimensions of wooden beams are not fractional, and we have to select a commercial size next larger than the one computed (12 inches instead of 11.62 inches, for example), the additional depth is usually sufficient to provide strength for the weight of the beam. If there is any doubt in the matter, we can settle it by computing the maximum bending moment including the weight of the beam, and then computing the greatest unit-fibre stress due to load and weight. If this is less than the safe strength, the section is large enough; if greater, the section is too small.

Thus, let us determine whether the 2 × 12-inch plank is strong enough to sustain the load and its own weight. The plank will weigh about 120 pounds, making a total load of

$$1,500 + 120 = 1,620 \text{ pounds.}$$

Hence the maximum bending moment is

$$\frac{1}{8}Wl = \frac{1}{8}1,620 \times 20 \times 12 = 48,600 \text{ inch-pounds.}$$

Since $\frac{I}{c} = \frac{1}{6}ba^2 = \frac{1}{6} \times 2 \times 12^2 = 48$, and $S = \frac{M}{I \div c}$,

$$S = \frac{48,600}{48} = 1,013 \text{ pounds per square inch.}$$

Strictly, therefore, the 2 × 12-inch plank is not large enough; but as the greatest unit-stress in it would be only 13 pounds per square inch too large, its use would be permissible.

2. What size of steel I-beam is needed to sustain safely the loading of Fig. 9 if the safe strength of the steel is 16,000 pounds per square inch?

The maximum bending moment due to the loads was found in example 1, Art. 43, to be 8,800 foot-pounds, or 8,800 × 12 = 105,600 inch-pounds.

Hence $$\frac{I}{c} = \frac{105,600}{16,000} = 6.6 \text{ inches}^3.$$

That is, an I-beam is needed whose section modulus is a little larger than 6.6, to provide strength for its own weight.

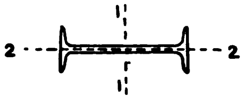
To select a size, we need a descriptive table of I-beams, such as is published in handbooks on structural steel.

Below is an abridged copy of such a table. (The last two columns contain information for use later.) The figure illustrates a cross-section of an I-beam, and shows the axes referred to in the table.

It will be noticed that two sizes are given for each depth; these are the lightest and heaviest of each size that are made, but intermediate sizes can be secured. In column 5 we find 7.3 as the next larger section modulus than the one required (6.6); and this corresponds to a 12¼-pound 6-inch I-beam, which is probably the proper size. To ascertain whether the excess (7.3 – 6.6 = 0.70) in the section modulus is sufficient to provide for the weight of the beam, we might proceed as in example 1. In this case, however, the excess is quite large, and the beam selected is doubtless safe.

TABLE C.

Properties of Standard I-Beams



Section of beam, showing axes 1-1 and 2-2.

1	2	3	4	5	6
Depth of Beam, in inches.	Weight per foot, in pounds.	Area of cross-section, in square inches.	Moment of inertia, axis 1 - 1.	Section modulus, axis 1--1.	Moment of inertia, axis 2- 2.
3	5.50	1.63	2.5	1.7	0.46
3	7.50	2.21	2.9	1.9	.60
4	7.50	2.21	6.0	3.0	.77
4	10.50	3.09	7.1	3.6	1.01
5	9.75	2.87	12.1	4.8	1.23
5	14.75	4.34	15.1	6.1	1.70
6	12.25	3.61	21.8	7.3	1.85
6	17.25	5.07	26.2	8.7	2.36
7	15.00	4.42	36.2	10.4	2.67
7	20.00	5.88	42.2	12.1	3.24
8	18.00	5.33	56.9	14.2	3.78
8	25.25	7.43	68.0	17.0	4.71
9	21.00	6.31	84.9	18.9	5.16
9	35.00	10.29	111.8	24.8	7.31
10	25.00	7.37	122.1	24.4	6.89
10	40.00	11.76	158.7	31.7	9.50
12	31.50	9.26	215.8	36.0	9.50
12	40.00	11.76	245.9	41.0	10.95
15	42.00	12.48	441.8	58.9	14.62
15	60.00	17.65	538.6	71.8	18.17
18	55.00	15.93	795.6	88.4	21.19
18	70.00	20.59	921.2	102.4	24.62
20	65.00	19.08	1,169.5	117.0	27.86
20	75.00	22.06	1,268.8	126.9	30.25
24	80.00	23.32	2,087.2	173.9	42.86
24	100.00	29.41	2,379.6	198.3	48.55

EXAMPLES FOR PRACTICE.

1. Determine the size of a wooden beam which can safely sustain a middle load of 2,000 pounds, if the beam rests on end supports 16 feet apart, and its working strength is 1,000 pounds per square inch. Assume width 6 inches.

Ans. 6 × 10 inches.

2. What sized steel I-beam is needed to sustain safely a uniform load of 200,000 pounds, if it rests on end supports 10 feet apart, and its working strength is 16,000 pounds per square inch?

Ans. 100-pound 24-inch.

3. What sized steel I-beam is needed to sustain safely the loading of Fig. 10, if its working strength is 16,000 pounds per square inch?

Ans. 14.75-pound 5-inch.

67. Laws of Strength of Beams. The strength of a beam is measured by the bending moment that it can safely withstand; or, since bending and resisting moments are equal, by its safe resisting moment ($SI \div c$). Hence the **safe strength** of a beam varies (1) directly as the working fibre strength of its material, and (2) directly as the section modulus of its cross-section. For beams rectangular in cross-section (as wooden beams), the section modulus is $\frac{1}{6}ba^2$, b and a denoting the breadth and altitude of the rectangle. Hence the strength of such beams varies also directly as the breadth, and as the square of the depth. Thus, doubling the breadth of the section for a rectangular beam doubles the strength, but doubling the depth quadruples the strength.

The **safe load** that a beam can sustain varies directly as its resisting moment, and depends on the way in which the load is distributed and how the beam is supported. Thus, in the first four and last two cases of the table on page 55,

$$\begin{array}{lcl}
M = Pl, & \text{hence} & P = SI \div lc, \\
M = \frac{1}{2}Wl, & \text{“} & W = 2SI \div lc, \\
M = \frac{1}{4}Pl, & \text{“} & P = 4SI \div lc, \\
M = \frac{1}{8}Wl, & \text{“} & W = 8SI \div lc, \\
M = \frac{1}{8}Pl, & \text{“} & P = 8SI \div lc, \\
M = \frac{1}{12}Wl, & \text{“} & W = 12SI \div lc,
\end{array}$$

Therefore the safe load in all cases varies inversely with the length: and for the different cases the safe loads are as 1, 2, 4, 8, 8, and 12 respectively.

Example. What is the ratio of the strengths of a plank 2 × 10 inches when placed edgewise and when placed flatwise on its supports?

When placed edgewise, the section modulus of the plank is $\frac{1}{6} \times 2 \times 10^2 = 33\frac{1}{3}$, and when placed flatwise it is $\frac{1}{6} \times 10 \times 2^2 = 6\frac{2}{3}$; hence its strengths in the two positions are as $33\frac{1}{3}$ to $6\frac{2}{3}$ respectively, or as 5 to 1.

EXAMPLE FOR PRACTICE.

What is the ratio of the safe loads for two beams of wood, one being 10 feet long, 3 × 12 inches in section, and having its load in the middle; and the other 8 feet long and 2 × 8 inches in section, with its load uniformly distributed.

Ans. As 135 to 100.

68. Modulus of Rupture. If a beam is loaded to destruction, and the value of the bending moment for the rupture stage is computed and substituted for M in the formula $SI \div c = M$, then the value of S computed from the equation is the **modulus of rupture** for the material of the beam. Many experiments have been performed to ascertain the moduli of rupture for different materials and for different grades of the same material. The following are fair values, all in pounds per square inch:

TABLE D.

Moduli of Rupture.

Timber.				
Spruce	4,000	- 7,000,	average	5,000
Hemlock	3,500	7,000,	"	4,500
White pine	5,500	10,500,	"	8,000
Long leaf pine	10,000	16,000,	"	12,500
Short leaf pine	8,000	14,000,	"	10,000
Douglas spruce	4,000	12,000,	"	8,000
White oak	7,500	18,500,	"	13,000
Red oak	9,000	15,000,	"	11,500
Stone.				
Sandstone	400	1,200,		
Limestone	400	1,000,		
Granite	800	1,400,		
Cast iron.	One and one half to two and one quarter times its ultimate tensile strength			
Hard steel	Varies from 100,000 to 150,000			

Wrought iron and structural steels have no modulus of rupture, as specimens of those materials will "bend double," but not break. The modulus of rupture of a material is used principally as a basis for determining its working strength. *The factor of safety of a loaded beam is computed by dividing the modulus of rupture of its material by the greatest unit-fibre stress in the beam.*

69. The Resisting Shear. The shearing stress on a cross-section of a loaded beam is not a uniform stress; that is, it is not uniformly distributed over the section. In fact the intensity or unit-stress is actually zero on the highest and lowest fibres of a cross-section, and is greatest, in such beams as are used in practice, on fibres at the neutral axis. In the following article we explain how to find the maximum value in two cases—cases which are practically important.

70. Second Beam Formula. Let S_s denote the average value of the unit-shearing stress on a cross-section of a loaded beam, and A the area of the cross-section. Then the value of the whole shearing stress on the section is :

$$\text{Resisting shear} = S_s A.$$

Since the resisting shear and the external shear at any section of a beam are equal (see Art. 59),

$$S_s A = V. \qquad (7)$$

This is called the "second beam formula" It is used to investigate and to design for shear in beams.

In beams uniform in cross-section, A is constant, and S_s is greatest in the section for which V is greatest. Hence the greatest unit-shearing stress in a loaded beam is at the neutral axis of the section at which the external shear is a maximum. There is a formula for computing this maximum value in any case, but it is not simple, and we give a simpler method for computing the value in the two practically important cases:

1. In wooden beams (rectangular or square in cross-section), the greatest unit-shearing stress in a section is 50 per cent larger than the average value S_s.

2. In I-beams, and in others with a thin vertical web, the greatest unit-shearing stress in a section practically equals S_s, as given by equation 7, if the area of the web is substituted for A.

Examples. 1. What is the greatest value of the unit-shearing stress in a wooden beam 12 feet long and 6 × 12 inches in cross-section when resting on end supports and sustaining a uniform load of 6,400 pounds? (This is the safe load as determined by working fibre stress; see example 1, Art. 65.)

The maximum external shear equals one-half the load (see Table B, page 55), and comes on the sections near the supports. Since

$$A = 6 \times 12 = 72 \text{ square inches;}$$

$$S_s = \frac{3,200}{72} = 44 \text{ pounds per square inch,}$$

and the greatest unit-shearing stress equals

$$\frac{3}{2} S_s = \frac{3}{2}\, 44 = 66 \text{ pounds per square inch.}$$

Apparently this is very insignificant; but it is not negligible, as is explained in the next article.

2. A steel I-beam resting on end supports 15 feet apart sustains a load of 8,000 pounds 5 feet from one end. The weight of the beam is 375 pounds, and the area of its web section is 3.2 square inches. (This is the beam and load described in examples 2 and 3, Art. 65.) What is the greatest unit-shearing stress?

The maximum external shear occurs near the support where the reaction is the greater, and its value equals that reaction. Calling that reaction R, and taking moments about the other end of the beam, we have

$$R \times 15 - 375 \times 7\frac{1}{2} - 8,000 \times 10 = 0;$$

therefore $15\ R = 80,000 + 2,812.5 = 82,812.5;$

or, $R = 5,520.8$ pounds.

Hence $S_s = \frac{5,520.8}{3.2} = 1,725$ pounds per square inch.

EXAMPLES FOR PRACTICE.

1. A wooden beam 10 feet long and 2 × 10 inches in cross-section sustains a middle load of 1,000 pounds. Neglecting the weight of the beam, compute the value of the greatest unit-shearing stress.

Ans. 37.5 pounds per square inch.

2. Solve the preceding example taking into account the weight of the beam, 60 pounds.

Ans. 40 pounds per square inch.

3. A wooden beam 12 feet long and 4 × 12 inches in cross-section sustains a load of 3,000 pounds 4 feet from one end. Neglecting the weight of the beam, compute the value of the greatest shearing unit-stress.

Ans. 62.5 pounds per square inch.

71. Horizontal Shear. It can be proved that there is a shearing stress on every horizontal section of a loaded beam. An experimental explanation will have to suffice here. Imagine a pile of six boards of equal length supported so that they do not bend. If the intermediate supports are removed, they will bend and their ends will not be flush but somewhat as represented in Fig. 41. This indicates that the boards slid over each other during the bending, and hence there was a rubbing and a frictional resistance exerted by the boards upon each other. Now, when a solid beam is being bent, there is an exactly similar tendency for the horizontal layers to slide over each other; and, instead of a frictional resistance, there exists shearing stress on all horizontal sections of the beam.

In the pile of boards the amount of slipping is different at different places between any two boards, being greatest near the supports and zero midway between them. Also, in any cross-section the slippage is least between the upper two and lower two boards, and is greatest between the middle two. These facts indicate that the shearing unit-stress on horizontal sections of a solid beam is greatest in the neutral surface at the supports.

It can be proved that at any place in a beam the shearing unit-stresses on a horizontal and on a vertical section are equal.

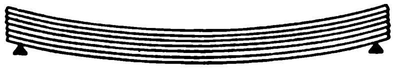


Fig. 41.

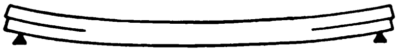


Fig. 42.

It follows that the horizontal shearing unit-stress is greatest at the neutral axis of the section for which the external shear (V) is a maximum. Wood being very weak in shear along the grain, timber beams sometimes fail under shear, the "rupture" being two horizontal cracks along the neutral surface somewhat as represented in Fig. 42. It is therefore necessary, when dealing with timber beams, to give due attention to their strength as determined by the working strength of the material in shear along the grain.

Example. A wooden beam 3 × 10 inches in cross-section rests on end supports and sustains a uniform load of 4,000 pounds Compute the greatest horizontal unit-stress in the beam.

The maximum shear equals one-half the load (see Table B, page 55), or 2,000 pounds. Hence, by equation 7, since $A = 3 \times 10 = 30$ square inches,

$$S_s = \frac{2{,}000}{30} = 66\frac{2}{3} \text{ pounds per square inch.}$$

This is the average shearing uni -stress on the cross-sections near the supports; and the greatest value equals

$$\frac{3}{2} \times 66\frac{2}{3} = 100 \text{ pounds per square inch.}$$

According to the foregoing, this is also the value of the greatest horizontal shearing unit-stress. (If of white pine, for example, the beam would not be regarded as safe, since the ultimate shearing strength along the grain of selected pine is only about 400 pounds per square inch.)

72. Design of Timber Beams. In any case we may proceed as follows:—(1) Determine the dimensions of the cross-section of the beam from a consideration of the fibre stresses as explained in Art. 66. (2) With dimensions thus determined, compute the value of the greatest shearing unit-stress from the formula,

$$\text{Greatest shearing unit-stress} = \frac{3}{2} V \div ab,$$

where V denotes the maximum external shear in the beam, and b and a the breadth and depth of the cross-section.

If the value of the greatest shearing unit-stress so computed does not exceed the working strength in shear along the grain, then the dimensions are large enough; but if it exceeds that value, then a or b, or both, should be increased until $\frac{3}{2}$ V ÷ ab is less than the working strength. Because timber beams are very often "season checked" (cracked) along the neutral surface, it is advisable to take the working strength of wooden beams, in shear along the grain, quite low. One-twentieth of the working fibre strength has been recommended* for all pine beams.

If the working strength in shear is taken equal to one-twentieth the working fibre strength, then it can be shown that,

1. For a beam on end supports loaded in the middle, the safe load depends on the shearing or fibre strength according as the ratio of length to depth ($l \div a$) is less or greater than 10.
2. For a beam on end supports uniformly loaded, the safe load depends on the shearing or fibre strength according as $l \div a$ is less or greater than 20.

Examples. 1. It is required to design a timber beam to sustain loads as represented in Fig. 11, the working fibre strength being 550 pounds and the working shearing strength 50 pounds per square inch.

The maximum bending moment (see example for practice 3, Art. 43; and example for practice 2, Art. 44) equals practically 7,000 foot-pounds or, 7,000 × 12 = 84,000 inch-pounds. Hence, according to equation 6‴,

$$\frac{I}{c} = \frac{84{,}000}{550} = 152.7 \text{ inches}^3.$$

Since for a rectangle

$$\frac{I}{c} = \frac{1}{6} ba^2,$$

$$\frac{1}{6} ba^2 = 152.7, \text{ or } ba^2 = 916.2.$$

Now, if we let $b = 4$, then $a^2 = 229$;

or, $a = 15.1$ (practically 16) inches.

If, again, we let $b = 6$, then $a^2 = 152.7$;

or $a = 12.4$ (practically 13) inches.

Either of these sizes will answer so far as fibre stress is concerned, but there is more "timber" in the second.

The maximum external shear in the beam equals 1,556 pounds, neglecting the weight of the beam (see example 3, Art. 37; and example 2, Art. 38). Therefore, for a 4 × 16-inch beam,

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[*] See "Materials of Construction."—Johnson. Page 55.

$$\text{greatest shearing unit-stress} = \frac{3}{2} \cdot \frac{1{,}556}{4 \cdot 16}$$

$$= 36.5 \text{ pounds per s}$$

and for the 14-inch beam it equals

$$\frac{3}{2} \cdot \frac{1{,}556}{[illegible] 14} = 37.7 \text{ pounds per square in}$$

Since these values are less than the working strength either size of beam is safe as regards shear.

If it is desired to allow for weight of beam, one o should be selected. First its weight should be comp the new reactions, and then the unit-fibre stress ma [illegible] as in Art. [illegible], and the greatest shearing unit-stres [illegible]. If these values are within the working v the size is large enough to sustain safely the load and t of the beam.

2. What is the safe load for a white pine beam [illegible] and 2 × 12 inches in cross-section, if the beam rests on en and the load is at the middle of the beam, the wor strength being 1,000 pounds and the shearing strength per square inch?

The ratio of the length to the depth is less than the safe load depends on the shearing strength of th [illegible] the load P, the maximum external shear (see [illegible]) [illegible] formula for greatest she [illegible]

$$[illegible] \frac{3}{2} \cdot \frac{\frac{1}{2}P}{[illegible]\,12} \text{ or } P = 1{,}600 \text{ pou}$$

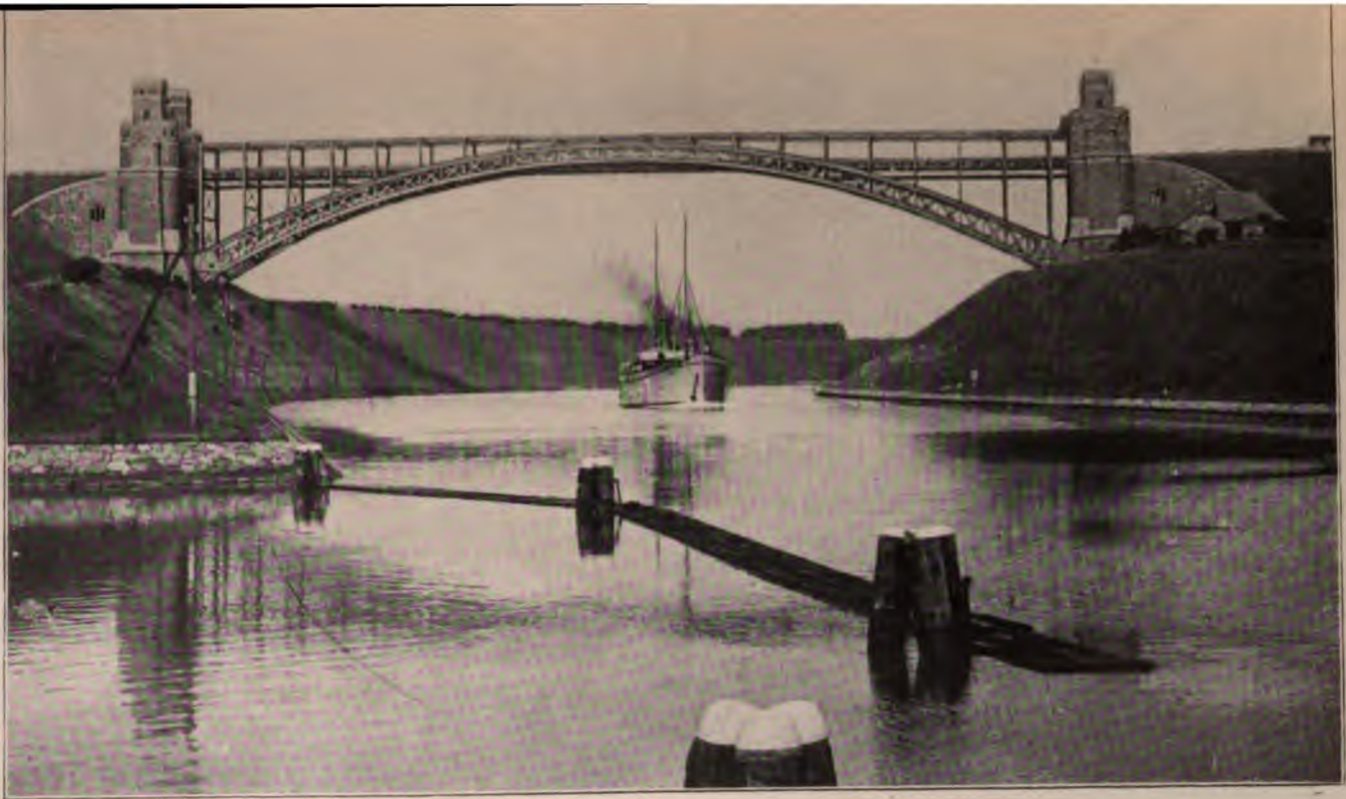
EXAMPLES FOR PRACTICE.

1. What size of wooden beam can safely sustain Fig. 12, with shearing and fibre working strength equal 1,000 pounds per square inch respectively?

Ans. 6 ×

2. What is the safe load for a wooden beam 4 × and 18 feet long, if the beam rests on end supports an is uniformly distributed, with working strengths as in e

Ans. 3,73



STEEL ARCH SPAN OVER KAISER WILHELM CANAL, GERMANY

View looking east, near terminus at Kiel. This 61-mile sea-level waterway connecting the Baltic and North Seas, and commonly called the "Kiel Canal," was opened in 1895, practically doubling the efficiency of the German Navy, and cutting 2½ days off the journey around the Danish Peninsula. It is the longest canal in Europe and the deepest in the world. Minimum depth, 29½ ft., or 3½ ft. more than the Suez and Manchester Ship Canals; width at bottom, 72 ft., increasing to a navigable width of 118 ft. in a depth of 20 ft. 6 in. Total excavation, 104,630,000 cu. yds.; cost, $39,000,000. On account of tides in the North Sea, regulating locks are necessary at each end of the canal.

73. Kinds of Loads and Beams. We shall now discuss the strength of beams under **longitudinal** forces (acting parallel to the beam) and **transverse loads.** The longitudinal forces are supposed to be applied at the ends of the beams and along the axis* of the beam in each case. We consider only beams resting on end supports.

The transverse forces produce **bending** or **flexure,** and the longitudinal or end forces, if pulls, produce **tension** in the beam; if pushes, they produce **compression.** Hence the cases to be considered may be called "Combined Flexure and Tension" and "Combined Flexure and Compression."

74. Flexure and Tension. Let Fig. 43, *a*, represent a beam subjected to the transverse loads L_1, L_2 and L_3, and to two equal end pulls P and P. The reactions R_1 and R_2 are due to the transverse loads and can be computed by the methods of moments just as though there were no end pulls. To find the stresses at any cross-section, we determine those due to the transverse forces (L_1, L_2, L_3, R_1 and R_2) and those due to the longitudinal; then combine these stresses to get the total effect of all the applied forces.

The stress due to the transverse forces consists of a shearing stress and a fibre stress; it will be called the **flexural stress.** The fibre stress is compressive above and tensile below. Let M denote the value of the bending moment at the section considered; c_1 and c_2 the distances from the neutral axis to the highest and the lowest fibre in the section; and S_1 and S_2 the corresponding unit-fibre stresses due to the transverse loads. Then

$$S_1 = \frac{Mc_1}{I}; \text{ and } S_2 = \frac{Mc_2}{I}.$$

The stress due to the end pulls is a simple tension, and it equals P; this is sometimes called the **direct stress.** Let S_0 denote the unit-tension due to P, and A the area of the cross-section; then

$$S_0 = \frac{P}{A}.$$

Both systems of loads to the left of a section between L_1 and

---

* NOTE. By "axis of a beam" is meant the line through the centers of gravity of all the cross-sections.

$$S_t = S_2 - S_0 \text{ (tensile)};$$

and that on the upper fibre is

$$S_c = S_1 + S_0 \text{ (compressive)}.$$

The combined fibre stress is represented by Fig. 44, *a*, and is part tensile and part compressive.

(2) The flexural unit-stress on the lower fibre is less than the direct unit-stress; that is, S_2 is less than S_0. Then the combined unit-stress on the lower fibre is

$$S_t = S_0 - S_2 \text{ (compressive)};$$

and that on the upper fibre is

$$S_c = S_0 + S_1 \text{ (compressive)}.$$

The combined fibre stress is represented by Fig. 44, *b*, and is all compressive.

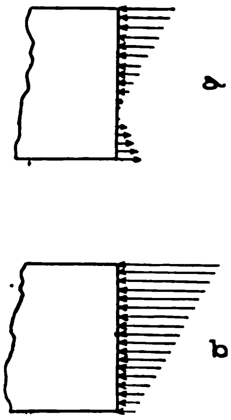


Fig. 44.

Example. A piece of timber 6 × 6 inches, and 10 feet long, is subjected to end pushes of 9,000 pounds. It is supported in a horizontal position at its ends, and sustains a middle load of 400 pounds. Compute the combined fibre stresses.

Evidently the dangerous section is at the middle, and $M = \frac{1}{4} Pl$; that is,

$$M = \frac{1}{4} \times 400 \times 10 = 1{,}000 \text{ foot-pounds},$$

or $\quad 1{,}000 \times 12 = 12{,}000$ inch-pounds.

Since $\quad c_1 = c_2 = 3$ inches, and

$$I = \frac{1}{12} bd^3 = \frac{1}{12} \times 6 \times 6^3 = 108 \text{ inches}^4,$$

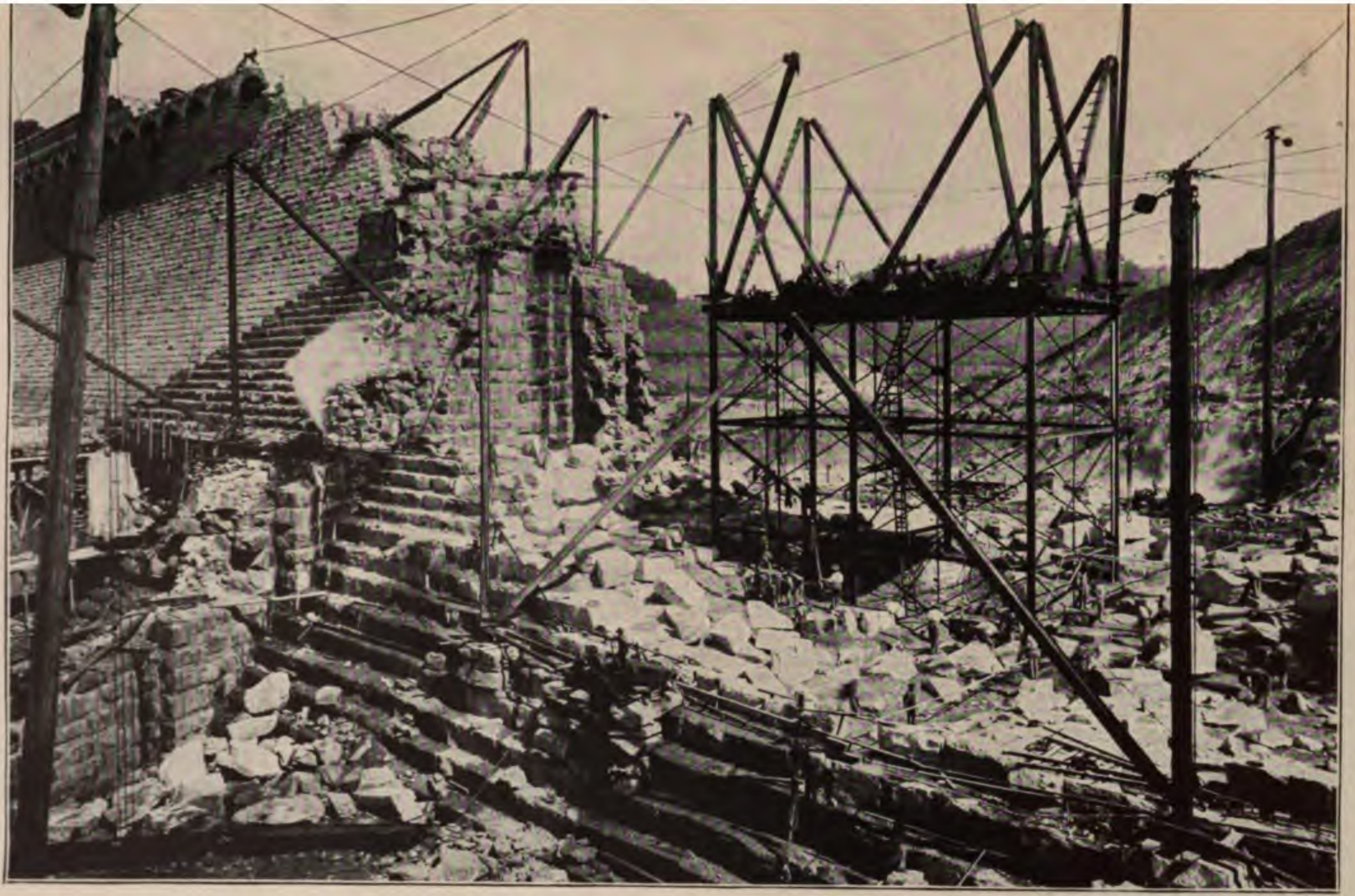
$$S_1 = S_2 = \frac{12{,}000 \times 3}{108} = 333\frac{1}{3} \text{ pounds per square inch.}$$

Since $\quad A = 6 \times 6 = 36$ square inches,

$$S_0 = \frac{9{,}000}{36} = 250 \text{ pounds per square inch.}$$

Hence the greatest value of the combined compressive stress is

$$S_0 + S_1 = 333\frac{1}{3} + 250 = 583\frac{1}{3} \text{ pounds per square inch.}$$



CONSTRUCTION WORK IN PROGRESS ON NEW CROTON DAM, PART OF WATERWORKS SYSTEM OF NEW YORK CITY

View showing steel towers used in the work and afterwards sealed into the reservoir wall.

It occurs on the upper fibres of the middle section. The greatest value of the combined tensile stress is

$$S_2 - S_0 = 333\frac{1}{3} - 250 = 83\frac{1}{3} \text{ pounds per square inch.}$$

It occurs on the lowest fibres of the middle section.

EXAMPLE FOR PRACTICE.

Change the load of the preceding illustration to a uniform load and solve.

$$\text{Ans.} \begin{cases} S_c = 417 \text{ pounds per square inch.} \\ S_t = 83 \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{(compression).} \end{cases}$$

76. Combined Flexural and Direct Stress by More Exact Formulas. The results in the preceding articles are only approximately correct. Imagine the beam represented in Fig. 45, *a*, to be first loaded with the transverse loads alone. They cause the beam to bend more or less, and produce certain flexural stresses at each section of the beam. ~~Now,~~ if end pulls are applied they tend to straighten the beam and hence diminish the flexural stresses. This effect of the end pulls was omitted in the discussion of Art. 74, and the results there given are therefore only approximate, the value of the greatest combined fibre unit-stress (S_t) being too large. On the other hand, if the end forces are pushes, they increase the bending, and therefore increase the flexural fibre stresses already caused by the transverse forces (see Fig. 45, *b*). The results indicated in Art. 75 must therefore in this case also be regarded as only approximate, the value of the greatest unit-fibre stress (S_c) being too small.

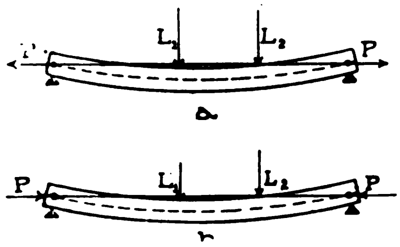


Fig. 45.

For beams loaded in the middle or with a uniform load, the following formulas, which take into account the flexural effect of the end forces, may be used:

M denotes bending moment at the middle section of the beam;

I denotes the moment of inertia of the middle section with respect to the neutral axis;

S_1, S_2, c_1 and c_2 have the same meanings as in Arts. 74 and 75, but refer always to the middle section;

l denotes length of the beam;

E is a number depending on the stiffness of the material, the average values of which are, for timber, 1,500,000; and for structural steel 30,000,000.*

$$S_1 = \frac{Mc_1}{I \pm \dfrac{Pl^2}{10E}}, \text{ and } S_2 = \frac{Mc_2}{I \pm \dfrac{Pl^2}{10E}}. \qquad (8)$$

The plus sign is to be used when the end forces P are pulls, and the minus sign when they are pushes.

It must be remembered that S_1 and S_2 are flexural unit-stresses. The combination of these and the direct unit-stress is made exactly as in articles 74 and 75.

Examples. 1. It is required to apply the formulas of this article to the example of article 74.

As explained in the example referred to, $M = 108{,}000$ inch-pounds; $c_1 = c_2 = 3$ inches; and $I = 36$ inches^4.

Now, since $l = 12$ feet $= 144$ inches,

$$S_1 = S_2 = \frac{108{,}000 \times 3}{36 + \dfrac{45{,}000 \times 144^2}{10 \times 30{,}000{,}000}} = \frac{324{,}000}{36 + 3.11} = 8{,}284 \text{ pounds}$$

per square inch, as compared with 9,000 pounds per square inch, the result reached by the use of the approximate formula.

As before, $S_0 = 3{,}750$ pounds per square inch; hence

$S_c = 8{,}284 - 3{,}750 = 4{,}534$ pounds per square inch;

and $S_t = 8{,}284 + 3{,}750 = 12{,}034$ " " " "

2. It is required to apply the formulas of this article to the example of article 75.

As explained in that example,

$M = 12{,}000$ inch-pounds;

$c_1 = c_2 = 3$ inches, and $I = 108$ inches^4.

Now, since $l = 120$ inches,

$$S_1 = S_2 = \frac{12{,}000 \times 3}{108 - \dfrac{9{,}000 \times 120^2}{10 \times 1{,}500{,}000}} = \frac{36{,}000}{108 - 8.64} = 362 \text{ pounds}$$

---

* NOTE. This quantity "E" is more fully explained in Article 95.

per square inch, as compared with $333\frac{1}{3}$ pounds per square inch, the result reached by use of the approximate method.

As before, $S_o = 250$ pounds per square inch; hence

$S_c = 362 + 250 = 612$ pounds per square inch; and
$S_t = 362 - 250 = 112$ " " " " .

EXAMPLES FOR PRACTICE.

1. Solve the example for practice of Art. 74 by the formulas of this article.

Ans. $\begin{cases} S_c = 12,820 \text{ pounds per square inch.} \\ S_t = 20,320 \text{ " " " "} \end{cases}$

2. Solve the example for practice of Art. 75 by the formulas of this article.

Ans. $\begin{cases} S_c = 430 \text{ pounds per square inch.} \\ S_t = 70 \text{ " " " " (compression).} \end{cases}$

STRENGTH OF COLUMNS.

A stick of timber, a bar of iron, etc., when used to sustain end loads which act lengthwise of the pieces, are called **columns, posts,** or **struts** if they are so long that they would bend before breaking. When they are so short that they would not bend before breaking, they are called **short blocks,** and their compressive strengths are computed by means of equation 1. The strengths of columns cannot, however, be so simply determined, and we now proceed to explain the method of computing them.

77. End Conditions. The strength of a column depends in part on the way in which its ends bear, or are joined to other parts of a structure, that is, on its "end conditions." There are practically but three kinds of end conditions, namely:

1. "Hinge" or "pin" ends,
2. "Flat" or "square" ends, and
3. "Fixed" ends.

(1) When a column is fastened to its support at one end by means of a pin about which the column could rotate if the other end were free, it is said to be "hinged" or "pinned" at the former end. Bridge posts or columns are often hinged at the ends.

(2) A column either end of which is flat and perpendicular to its axis and bears on other parts of the structure at that surface, is said to be "flat" or "square" at that end.

(3) Columns are sometimes riveted near their ends directly to other parts of the structure and do not bear directly on their ends; such are called "fixed ended." A column which bears on its flat ends is often fastened near the ends to other parts of the structure, and such an end is also said to be "fixed." The fixing of an end of a column stiffens and therefore strengthens it more or less, but the strength of a column with fixed ends is computed as though its ends were flat. Accordingly we have, so far as strength is concerned, the following classes of columns:

78. Classes of Columns. (1) Both ends hinged or pinned; (2) one end hinged and one flat; (3) both ends flat.

Other things being the same, columns of these three classes are unequal in strength. Columns of the first class are the weakest, and those of the third class are the strongest.

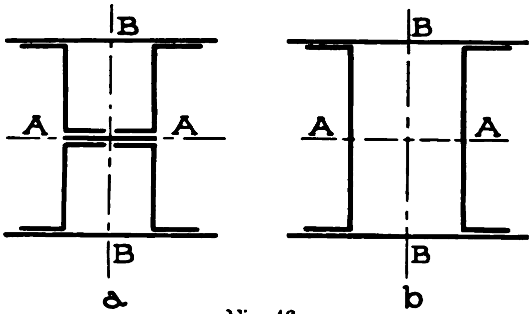


Fig. 46.

79. Cross-sections of Columns. Wooden columns are usually solid, square, rectangular, or round in section; but sometimes they are "built up" hollow. Cast-iron columns are practically always made hollow, and rectangular or round in section. Steel columns are made of single rolled shapes—angles, zees, channels, etc.; but the larger ones are usually "built up" of several shapes. Fig. 46, *a*, for example, represents a cross-section of a "Z-bar" column; and Fig. 46, *b*, that of a "channel" column.

80. Radius of Gyration. There is a quantity appearing in almost all formulas for the strength of columns, which is called "radius of gyration." It depends on the form and extent of the cross-section of the column, and may be defined as follows:

The radius of gyration of any plane figure (as the section of a column) with respect to any line, is such a length that the square of this length multiplied by the area of the figure equals the moment of inertia of the figure with respect to the given line.

Thus, if A denotes the area of a figure; I, its moment of inertia with respect to some line; and r, the radius of gyration with respect to that line; then

$$r^2\mathrm{A} = \mathrm{I}; \text{ or } r = \sqrt{\mathrm{I} \div \mathrm{A}}. \qquad (9)$$

In the column formulas, the radius of gyration always refers to an axis through the center of gravity of the cross-section, and usually to that axis with respect to which the radius of gyration (and moment of inertia) is least. (For an exception, see example 3, Art. 83.) Hence the radius of gyration in this connection is often called for brevity the "least radius of gyration," or simply the "least radius."

Examples. 1. Show that the value of the radius of gyration given for the square in Table A, page 54, is correct.

The moment of inertia of the square with respect to the axis is $\frac{1}{12}a^4$. Since A $= a^2$, then, by formula 9 above,

$$r = \sqrt{\frac{1}{12}a^4 \div a^2} = \sqrt{\frac{1}{12}a^2} = a\sqrt{\frac{1}{12}}.$$

2. Prove that the value of the radius of gyration given for the hollow square in Table A, page 54, is correct.

The value of the moment of inertia of the square with respect to the axis is $\frac{1}{12}(a^4 - a_1^4)$. Since A $= a^2 - a_1^2$,

$$r = \sqrt{\frac{\frac{1}{12}(a^4 - a_1^4)}{a^2 - a_1^2}} = \sqrt{\frac{1}{12}(a^2 + a_1^2)}.$$

EXAMPLE FOR PRACTICE.

Prove that the values of the radii of gyration of the other figures given in Table A, page 54, are correct. The axis in each case is indicated by the line through the center of gravity.

81. Radius of Gyration of Built-up Sections. The radius of gyration of a built-up section is computed similarly to that of any other figure. First, we have to compute the moment of inertia of the section, as explained in Art. 54; and then we use formula 9, as in the examples of the preceding article.

Example. It is required to compute the radius of gyration of the section represented in Fig. 30 (page 52) with respect to the axis AA.

In example 1, Art. 54, it is shown that the moment of inertia of the section with respect to the axis AA is 429 inches^4. The area of the whole section is

$$2 \times 6.03 + 7 = 19.06;$$

hence the radius of gyration r is

$$r = \sqrt{\frac{429}{19.06}} = 4.74 \text{ inches.}$$

EXAMPLE FOR PRACTICE.

Compute the radii of gyration of the section represented in Fig. 31, *a*, with respect to the axes AA and BB. (See examples for practice 1 and 2, Art. 54.)

Ans. { 2.87 inches.
2.09 "

82. Kinds of Column Loads. When the loads applied to a column are such that their resultant acts through the center of gravity of the top section and along the axis of the column, the column is said to be **centrally loaded.** When the resultant of the loads does not act through the center of gravity of the top section, the column is said to be **eccentrically loaded.** All the following formulas refer to columns centrally loaded.

83. Rankine's Column Formula. When a perfectly straight column is centrally loaded, then, if the column does not bend and if it is homogeneous, the stress on every cross-section is a uniform compression. If P denotes the load and A the area of the cross-section, the value of the unit-compression is P ÷ A.

On account of lack of straightness or lack of uniformity in material, or failure to secure exact central application of the load, the load P has what is known as an "arm" or "leverage" and bends the column more or less. There is therefore in such a column a bending or flexural stress in addition to the direct compressive stress above mentioned; this bending stress is compressive on the concave side and tensile on the convex. The value of the stress per unit-area (unit-stress) on the fibre at the concave side, according to equation 6, is $Mc \div I$, where M denotes the bending moment at the section (due to the load on the column), c the distance from the neutral axis to the concave side, and I the moment of inertia of the cross-section with respect to the neutral axis. (Notice that this axis is perpendicular to the plane in which the column bends.)

The upper set of arrows (Fig. 47) represents the direct compressive stress; and the second set the bending stress if the load is not excessive, so that the stresses are within the elastic limit of the material. The third set represents the combined stress that actually exists on the cross-section. The greatest combined unit-stress evidently occurs on the fibre at the concave side and where the deflection of the column is greatest. The stress is compressive, and its value S per unit-area is given by the formula,

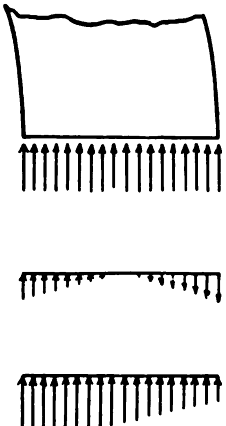


Fig. 47.

$$S = \frac{P}{A} + \frac{Mc}{I}.$$

Now, the bending moment at the place of greatest deflection equals the product of the load P and its arm (that is, the deflection). Calling the deflection d, we have $M = Pd$; and this value of M, substituted in the last equation, gives

$$S = \frac{P}{A} + \frac{Pdc}{I}.$$

Let r denote the radius of gyration of the cross-section with respec to the neutral axis. Then $I = Ar^2$ (see equation 9); and this value, substituted in the last equation, gives

$$S = \frac{P}{A} + \frac{Pdc}{Ar^2} = \frac{P}{A}\left(1 + \frac{dc}{r^2}\right).$$

According to the theory of the stiffness of beams on end supports, deflections vary directly as the square of the length l, and inversely as the distance c from the neutral axis to the remotest fibre of the cross-section. Assuming that the deflections of columns follow the same laws, we may write $d = k\ (l^2 \div c)$, where k is some constant depending on the material of the column and on the end conditions. Substituting this value for d in the last equation, we find that

$$\left.\begin{aligned} \mathrm{S} &= \frac{\mathrm{P}}{\mathrm{A}}\left(1 + k\frac{l^2}{r^2}\right); \\ \frac{\mathrm{P}}{\mathrm{A}} &= \frac{\mathrm{S}}{1 + k\dfrac{l^2}{r^2}}; \\ \text{and} \qquad \mathrm{P} &= \frac{\mathrm{SA}}{1 + k\dfrac{l^2}{r^2}}. \end{aligned}\right\} \quad (10)$$

Each of these (usually the last) is known as "Rankine's formula."

For *mild-steel* columns a certain large steel company uses $\mathrm{S} = 50{,}000$ pounds per square inch, and the following values of k:

1. Columns with two pin ends, $k = 1 \div 18{,}000$.
2. " " one flat and one pin end, $k = 1 \div 24{,}000$.
3. " " both ends flat, $k = 1 \div 36{,}000$.

With these values of S and k, P of the formula means the **ultimate load**, that is, the load causing failure. The safe load equals P divided by the selected factor of safety—a factor of 4 for steady loads, and 5 for moving loads, being recommended by the company referred to. The same unit is to be used for l and r.

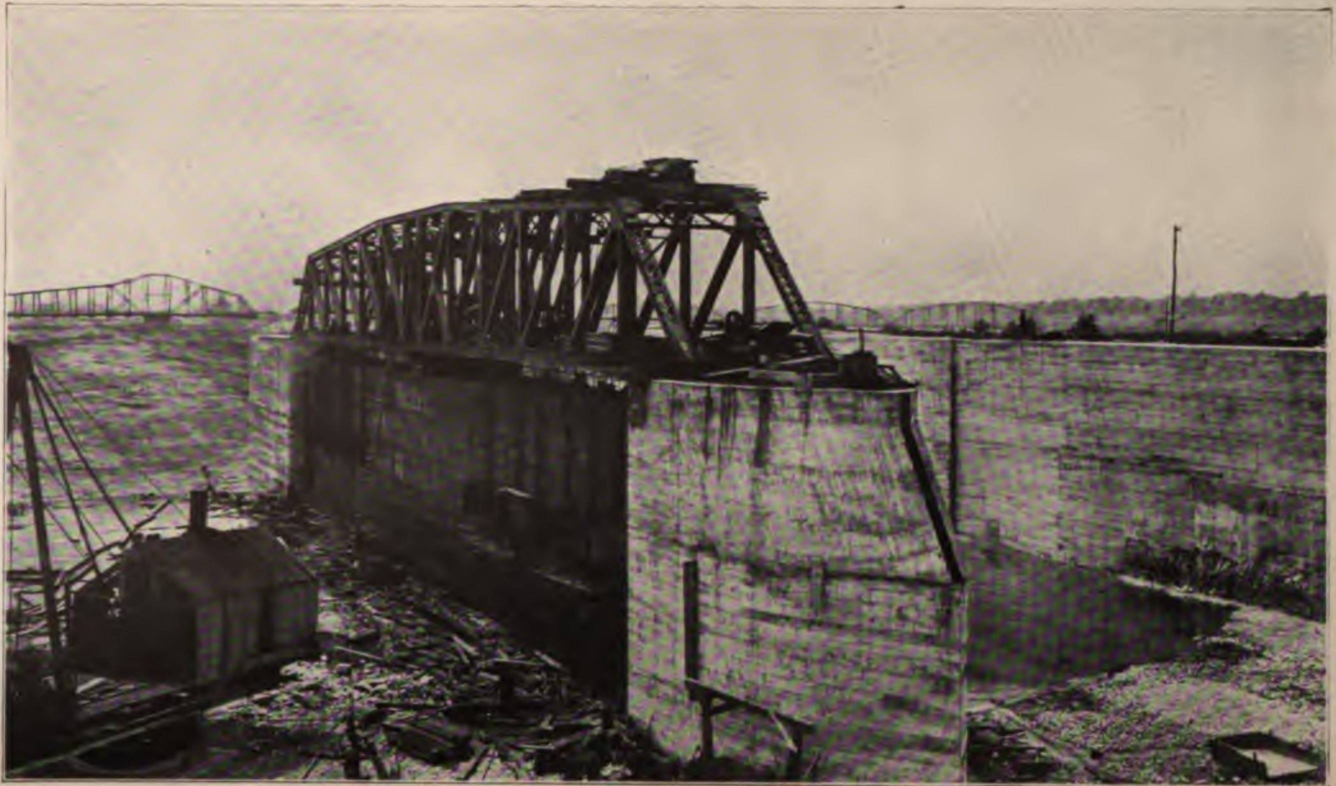
Cast-iron columns are practically always made hollow with comparatively thin walls, and are usually circular or rectangular in cross-section. The following modifications of Rankine's formula are sometimes used:

$$\left.\begin{aligned} &\text{For circular sections,} \qquad &\frac{\mathrm{P}}{\mathrm{A}} &= \frac{80.000}{1 + \dfrac{l^2}{800\,d^2}} \\ &\text{For rectangular sections,} \qquad &\frac{\mathrm{P}}{\mathrm{A}} &= \frac{80.000}{1 + \dfrac{l^2}{1{,}000\,d^2}} \end{aligned}\right\} \quad (10')$$

In these formulas d denotes the outside diameter of the circular sections or the length of the lesser side of the rectangular sections. The same unit is to be used for l and d.

Examples. 1. A 40-pound 10-inch steel I-beam 8 feet long is used as a flat-ended column. Its load being 100,000 pounds, what is its factor of safety?

Obviously the column tends to bend in a plane perpendicular to its web. Hence the radius of gyration to be used is the one



"BUTTERFLY" DAM, CHICAGO DRAINAGE CANAL, LOCKPORT, ILL.

Ordinarily the dam is open, the movable leaf, shown under the bridge, pointing up and down stream. This leaf swings on a central pivot and, when once started by rack and pinion throwing it into the current, swings across the channel, closing it automatically, the small gates in one arm of the leaf being closed while those in the other arm are left open, thus varying the pressure on the two arms. The leaf may be swung back automatically by simply reversing the operation of the small gates, closing those that were before open, and *vice versâ.*

Isham Randolph, Chief Engineer.

with respect to that central axis of the cross-section which is in the web, that is, axis 2 – 2 (see figure accompanying table, page 72). The moment of inertia of the section with respect to that axis, according to the table, is 9.50 inches^4; and since the area of the section is 11.76 square inches,

$$r^2 = \frac{9.50}{11.76} = 0.81.$$

Now, l = 8 feet = 96 inches; and since $k = 1 \div 36{,}000$, and $S = 50{,}000$, the breaking load for this column, according to Rankine's formula, is

$$P = \frac{50{,}000 \times 11.76}{1 + \frac{96^2}{36{,}000 \times 0.81}} = 446{,}790 \text{ pounds.}$$

Since the factor of safety equals the ratio of the breaking load to the actual load on the column, the factor of safety in this case is

$$\frac{446{,}790}{100{,}000} = 4.5 \text{ (approx.).}$$

2. What is the safe load for a cast-iron column 10 feet long with square ends and a hollow rectangular section, the outside dimensions being 5 × 8 inches; the inner, 4 × 7 inches; and the factor of safety, 6?

In this case l = 10 feet = 120 inches; $A = 5 \times 8 - 4 \times 7 = 12$ square inches; and d = 5 inches. Hence, according to formula 10', for rectangular sections, the breaking load is

$$P = \frac{80{,}000 \times 12}{1 + \frac{120^2}{1{,}000 \times 5^2}} = 610{,}000 \text{ pounds.}$$

Since the safe load equals the breaking load divided by the factor of safety, in this case the safe load equals

$$\frac{610{,}000}{6} = 101{,}700 \text{ pounds.}$$

3. A channel column (see Fig. 46, b) is pin-ended, the pins being perpendicular to the webs of the channel (represented by AA in the figure), and its length is 16 feet (distance between axes

of the pins). If the sectional area is 23.5 square inches, and the moment of inertia with respect to AA is 386 inches^4 and with respect to BB 214 inches^4, what is the safe load with a factor of safety of 4?

The column is liable to bend in one of two ways, namely, in the plane perpendicular to the axes of the two pins, or in the plane containing those axes.

(1) For bending in the first plane, the strength of the column is to be computed from the formula for a pin-ended column. Hence, for this case, $r^2 = 386 \div 23.5 = 16$; and the breaking load is

$$P = \frac{50{,}000 \times 23.5}{1 + \dfrac{(16 \times 12)^2}{18{,}000 \times 16}} = 1{,}041{,}600 \text{ pounds.}$$

The safe load for this case equals $\dfrac{1{,}041{,}600}{4} = 260{,}400$ pounds.

(2) If the supports of the pins are rigid, then the pins stiffen the column as to bending in the plane of their axes, and the strength of the column for bending in that plane should be computed from the formula for the strength of columns with flat ends. Hence, $r_1^2 = 214 \div 23.5 = 9.11$, and the breaking load is

$$P = \frac{50{,}000 \times 23.5}{1 + \dfrac{(16 \times 12)^2}{36{,}000 \times 9.11}} = 1{,}056{,}000 \text{ pounds.}$$

The safe load for this case equals $\dfrac{1{,}056{,}000}{4} = 264{,}000$ pounds.

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a column with flat ends sustaining a load of 100,000 pounds. What is its factor of safety?

Ans. 4.1

2. A cast-iron column 15 feet long sustains a load of 150,000 pounds. Its section being a hollow circle, 9 inches outside and 7 inches inside diameter, what is the factor of safety?

Ans. 8.9

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is

3.1 inches; and the area of the cross-section is 24.5 square inches. What is the safe load for the column with a factor of safety of 4?

Ans. 247,000 pounds.

4. A cast-iron column 13 feet long has a hollow circular cross-section 7 inches outside and $5\frac{1}{2}$ inches inside diameter. What is its safe load with a factor of safety of 6?

Ans. 121,142 pounds.

5. Compute the safe load for a 40-pound 12-inch steel I-beam used as a column with flat ends, its length being 17 feet. Use a factor of safety of 5.

Ans. 52,470 pounds.

84. Graphical Representation of Column Formulas. Column (and most other engineering) formulas can be represented graphically. To represent Rankine's formula for flat-ended mild-steel columns,

$$\frac{P}{A} = \frac{50,000}{1 + \frac{(l \div r)^2}{36,000}},$$

we first substitute different values of $l \div r$ in the formula, and solve for $P \div A$. Thus we find, when

$l \div r = 40$, $P \div A = 47,900$;
$l \div r = 80$, $P \div A = 42,500$;
$l \div r = 120$, $P \div A = 35,750$;
etc., etc.

Now, if these values of $l \div r$ be laid off by some scale on a line from O, Fig. 48, and the corresponding values of $P \div A$ be laid

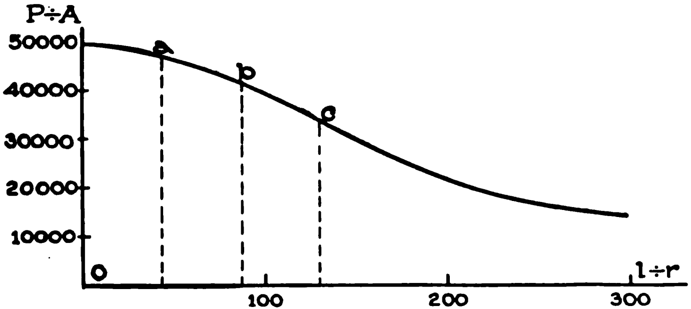


Fig. 48.

off vertically from the points on the line, we get a series of points as *a*, *b*, *c*, etc.; and a smooth curve through the points *a*, *b*, *c*, etc., represents the formula. Such a curve, besides representing the formula to one's eye, can be used for finding the value of P ÷ A for any value of *l* ÷ *r*; or the value of *l* ÷ *r* for any value of P ÷ A. The use herein made is in explaining other column formulas in succeeding articles.

85. Combination Column Formulas. Many columns have been tested to destruction in order to discover in a practical way the laws relating to the strength of columns of different kinds. The results of such tests can be most satisfactorily represented graphically by plotting a point in a diagram for each test. Thus, suppose that a column whose *l* ÷ *r* was 80 failed under a load of 276,000 pounds, and that the area of its cross-section was 7.12 square inches. This test would be represented by laying off O*a*, Fig. 49, equal to 80, according to some scale; and then *ab* equal to 276,000 ÷ 7.12 (P ÷ A), according to some other convenient scale. The point *b* would then represent the result of this particular test. All the dots in the figure represent the way in which the results of a series of tests appear when plotted.

It will be observed at once that the dots do not fall upon any one curve, as the curve of Rankine's formula. Straight lines and

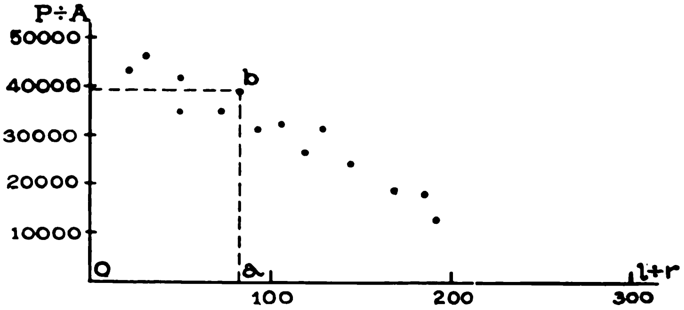
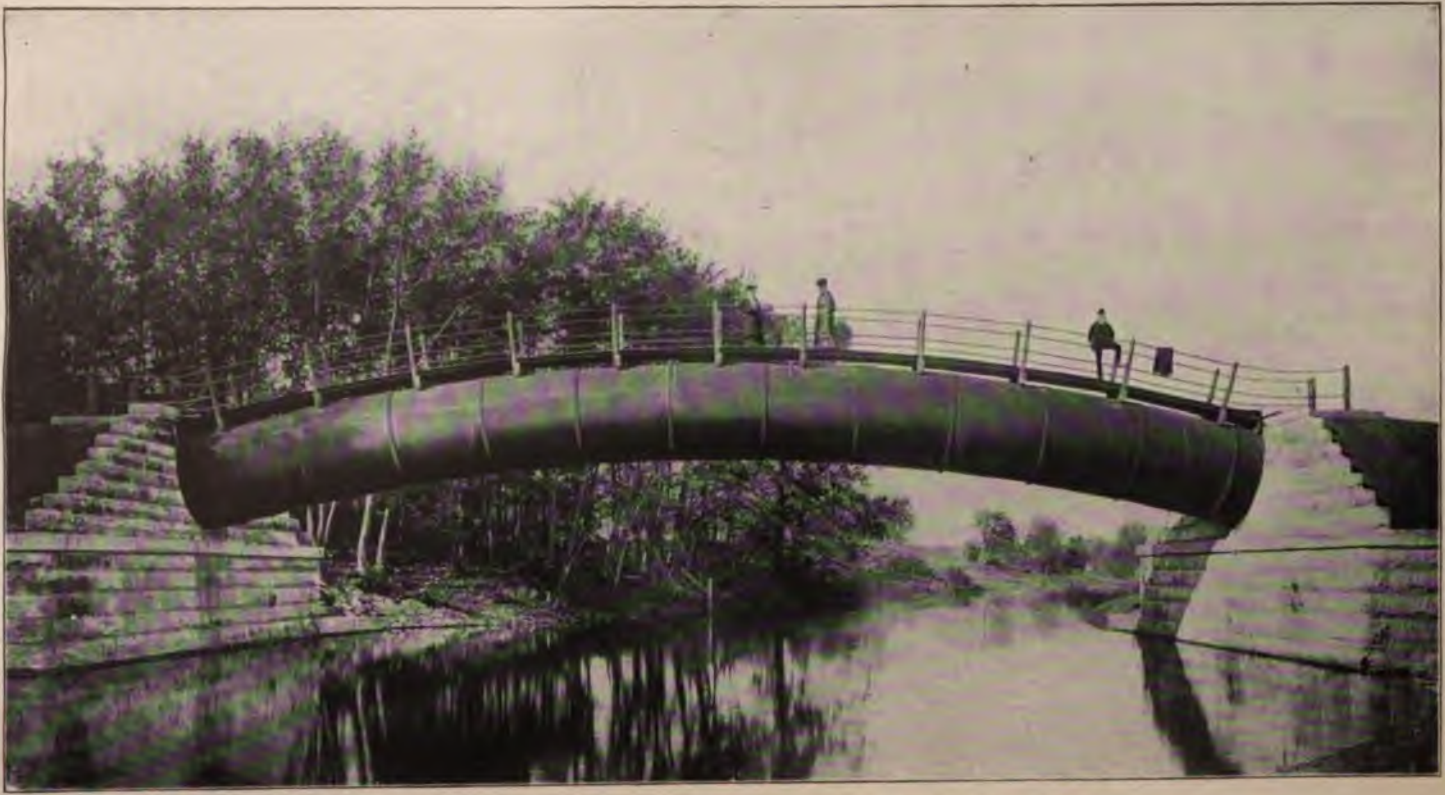


Fig. 49.

curves simpler than the curve of Rankine's formula have been fitted to represent the average positions of the dots as determined by actual tests, and the formulas corresponding to such lines have been deduced as column formulas. These are explained in the following articles.

86. Straight-Line and Euler Formulas. It occurred to Mr. T. H. Johnson that most of the dots corresponding to ordinary



PIPE-ARCH BRIDGE OVER SUDBURY RIVER, NEAR SAXONVILLE, MASS.

Part of the waterworks system of Boston. Width of span 80 feet, central part of arch rising 5½ feet above horizontal. Arch consists of double-riveted sections of steel pipe, ⅜ inch thick and 7½ feet in diameter, sustaining both its own weight and that of the water it contains. Stone abutment reinforced with a backing of 40 feet of solid concrete. A larger bridge built on same principle, supported by two arched mains, spans Rock Creek, between Washington and Georgetown, D. C.

lengths of columns agree with a straight line just as well as with a curve. He therefore, in 1886, made a number of such plats or diagrams as Fig. 49, fitted straight lines to them, and deduced the formula corresponding to each line. These have become known as "straight-line formulas," and their general form is as follows:

$$\frac{P}{A} = S - m\frac{l}{r}, \qquad (11)$$

P, A, l, and r having meanings as in Rankine's formula (Art. 83), and S and m being constants whose values according to Johnson are given in Table E below.

For the slender columns, another formula (Euler's, long since deduced) was used by Johnson. Its general form is—

$$\frac{P}{A} = \frac{n}{(l \div r)^2}, \qquad (12)$$

n being a constant whose values, according to Johnson, are given in the following table:

TABLE E.

Data for Mild-Steel Columns.

	S	m	Limit ($l \div r$)	n
Hinged ends......	52,500	220	160	444,000,000
Flat ends.........	52,500	180	195	666,000,000

The numbers in the fourth column of the table mark the point of division between columns of ordinary length and slender columns. For the former kind, the straight-line formula applies; and for the second, Euler's. That is, if the ratio $l \div r$ for a steel column with hinged end, for example, is less than 160, we must use the straight-line formula to compute its safe load, factor of safety, etc.; but if the ratio is greater than 160, we must use Euler's formula.

For *cast-iron columns* with flat ends, S = 34,000, and m = 88; and since they should never be used "slender," there is no use of Euler's formula for cast-iron columns.

The line AB, Fig. 50, represents Johnson's straight-line formula; and BC, Euler's formula. It will be noticed that the two lines are tangent; the point of tangency corresponds to the "limiting value" $l \div r$, as indicated in the table.

Examples. 1. A 40-pound 10-inch steel I-beam column 8 feet long sustains a load of 100,000 pounds, and the ends are flat. Compute its factor of safety according to the methods of this article.

The first thing to do is to compute the ratio $l \div r$ for the column, to ascertain whether the straight-line formula or Euler's formula should be used. From Table C, on page 72, we find that the moment of inertia of the column about the neutral axis of its cross-section is 9.50 inches^4, and the area of the section is 11.76 square inches. Hence

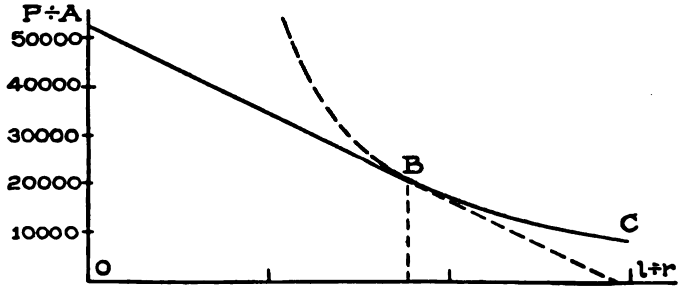


Fig. 50.

$$r^2 = \frac{9.50}{11.76} = 0.81; \text{ or } r = 0.9 \text{ inch.}$$

Since $l = 8$ feet $= 96$ inches,

$$\frac{l}{r} = \frac{96}{0.9} = 106\frac{2}{3}$$

This value of $l \div r$ is less than the limiting value (195) indicated by the table for steel columns with flat ends (Table E, p. 97), and we should therefore use the straight-line formula; hence

$$\frac{P}{11.76} = 52{,}500 - 180 \times 106\frac{2}{3};$$

or, $$P = 11.76\,(52{,}500 - 180 \times 106\frac{2}{3}) = 391{,}600 \text{ pounds.}$$

This is the breaking load for the column according to the straight-line formula; hence the factor of safety is

$$\frac{391{,}600}{100{,}000} = 3.9$$

2. Suppose that the length of the column described in the preceding example were 16 feet. What would its factor of safety be?

Since l = 16 feet = 192 inches; and, as before, r = 0.9 inch, $l \div r = 213\frac{1}{3}$. This value is greater than the limiting value (195) indicated by Table E (p. 97) for flat-ended steel columns; hence Euler's formula is to be used. Thus

$$\frac{P}{11.76} = \frac{666{,}000{,}000}{(213\frac{1}{3})^2};$$

or,
$$P = \frac{11.76 \times 666{,}000{,}000}{(213\frac{1}{3})^2} = 172{,}100 \text{ pounds.}$$

This is the breaking load; hence the factor of safety is

$$\frac{172{,}100}{100{,}000} = 1.7$$

3. What is the safe load for a cast-iron column 10 feet long with square ends and hollow rectangular section, the outside dimensions being 5×8 inches and the inside 4×7 inches, with a factor of safety of 6?

Substituting in the formula for the radius of gyration given in Table A, page 54, we get

$$r = \sqrt{\frac{8 \times 5^3 - 7 \times 4^3}{12\,(8 \times 5 - 7 \times 4)}} = 1.96 \text{ inches.}$$

Since l = 10 feet = 120 inches,

$$\frac{l}{r} = \frac{120}{1.96} = 61.22$$

According to the straight-line formula for cast iron, A being equal to 12 square inches,

$$\frac{P}{12} = 34{,}000 - 88 \times 61.22;$$

or,
$$P = 12\,(34{,}000 - 88 \times 61.22) = 343{,}360 \text{ pounds.}$$

This being the breaking load, the safe load is

$$\frac{343{,}360}{6} = 57{,}227 \text{ pounds.}$$

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EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a flat-ended column. Its load being 100,000 pounds, compute the factor of safety by the formulas of this article.

Ans. 3.5

2. A cast-iron column 15 feet long sustains a load of 150,000 pounds. Its section being a hollow circle of 9 inches outside and 7 inches inside diameter, compute the factor of safety by the straight-line formula.

Ans. 4.8

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches; and the area of the cross-section is 24.5 square inches. Compute the safe load for the column by the formulas of this article, using a factor of safety of 4.

Ans. 219,000 pounds.

4. A hollow cast-iron column 13 feet long has a circular cross-section, and is 7 inches outside and $5\frac{1}{2}$ inches inside in diameter. Compute its safe load by the formulas of this article, using a factor of safety of 6.

Ans. 68,500 pounds

5. Compute by the methods of this article the safe load for a 40-pound 12-inch steel I-beam used as a column with flat ends, if the length is 17 feet and the factor of safety 5.

Ans. 35,100 pounds.

87. Parabola-Euler Formulas. As better fitting the results of tests of the strength of columns of "ordinary lengths," Prof. J. B. Johnson proposed (1892) to use parabolas instead of straight lines. The general form of the "parabola formula" is

$$\frac{P}{A} = S - m\left(\frac{l}{r}\right)^2, \qquad (13)$$

P, A, l and r having the same meanings as in Rankine's formula, Art. 83; and S and m denoting constants whose values, according to Professor Johnson, are given in Table F below.

Like the straight-line formula, the parabola formula should not be used for slender columns, but the following (Euler's) is applicable:

$$\frac{P}{A} = \frac{n}{(l \div r)^2}, \qquad (14)$$

the values of n (Johnson) being given in the following table:

TABLE F.

Data for Mild Steel Columns.

	S	m	Limit $(l \div r)$	n
Hinged ends......	42,000	0.97	150	456,000,000
Flat ends..	42,000	0.62	190	712,000,000

The point of division between columns of ordinary length and slender columns is given in the fourth column of the table. That is, if the ratio $l \div r$ for a column with hinged ends, for example, is less than 150, the parabola formula should be used to compute the safe load, factor of safety, etc.; but if the ratio is greater than 150, then Euler's formula should be used.

The line AB, Fig. 51, represents the parabola formula; and the line BC, Euler's formula. The two lines are tangent, and the point of tangency corresponds to the "limiting value" $l \div r$ of the table.

For *wooden columns* square in cross-section, it is convenient to replace r by d, the latter denoting the length of the sides of the square. The formula becomes

$$\frac{P}{A} = S - m \left(\frac{l}{d}\right)^2,$$

S and m for flat-ended columns of various kinds of wood having the following values according to Professor Johnson:

For White pine, S=2,500, $m = 0.6$;
" Short-leaf yellow pine, S=3,300, $m = 0.7$;
" Long-leaf yellow pine, S=4,000, $m = 0.8$;
" White oak, S=3,500, $m = 0.8$.

The preceding formula applies to any wooden column whose ratio, $l \div d$, is less than 60, within which limit columns of practice are included.

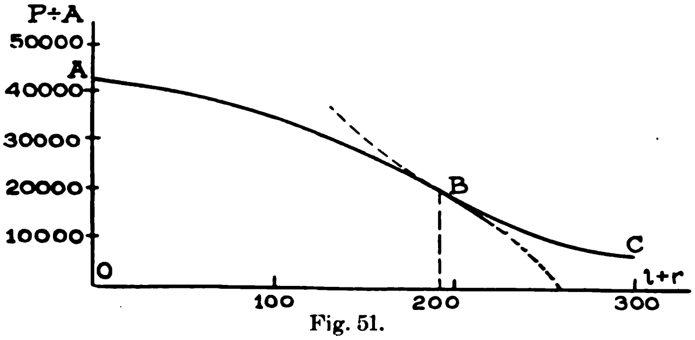

Fig. 51.

Examples. 1. A 40-pound 10-inch steel I-beam column

8 feet long sustains a load of 100,000 pounds, and its ends are flat. Compute its factor of safety according to the methods of this article.

The first thing to do is to compute the ratio $l \div r$ for the column, to ascertain whether the parabola formula or Euler's formula should be used. As shown in example 1 of the preceding article, $l \div r = 106\frac{2}{3}$. This ratio being less than the limiting value, 190, of the table, we should use the parabola formula. Hence, since the area of the cross-section is 11.76 square inches (see Table C, page 72),

$$\frac{P}{11.76} = 42{,}000 - 0.62\ (106\tfrac{2}{3})^2;$$

or, $$P = 11.76\ [42{,}000 - 0.62\ (106\tfrac{2}{3})^2] = 410{,}970 \text{ pounds.}$$

This is the breaking load according to the parabola formula; hence the factor of safety is

$$\frac{410{,}970}{100{,}000} = 4.1$$

2. A white pine column 10 × 10 inches in cross-section and 18 feet long sustains a load of 40,000 pounds. What is its factor of safety?

The length is 18 feet or 216 inches; hence the ratio $l \div d = 21.6$, and the parabola formula is to be applied.

Now, since $A = 10 \times 10 = 100$ square inches,

$$\frac{P}{100} = 2{,}500 - 0.6 \times 21.6^2;$$

or, $$P = 100\ (2{,}500 - 0.6 \times 21.6^2) = 222{,}000 \text{ pounds.}$$

This being the breaking load according to the parabola formula, the factor of safety is

$$\frac{222{,}000}{40{,}000} = 5.5$$

3. What is the safe load for a long-leaf yellow pine column 12 × 12 inches square and 30 feet long, the factor of safety being 5?

The length being 30 feet or 360 inches,

$$\frac{l}{d} = \frac{360}{12} = 30;$$

hence the parabola formula should be used. Since A = 12 × 12 = 144 square inches,

$$\frac{P}{144} = 4{,}000 - 0.8 \times 30^2;$$

or, $P = 144\ (4{,}000 - 0.8 \times 30^2) = 472{,}320$ pounds.

This being the breaking load according to the parabola formula, the safe load is

$$\frac{472{,}320}{5} = 94{,}465 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. A 40-pound 12-inch steel I-beam 10 feet long is used as a flat-ended column. Its load being 100,000 pounds, compute its factor of safety by the formulas of this article.

Ans. 3.8

2. A white oak column 15 feet long sustains a load of 30,000 pounds. Its section being 8 × 8 inches, compute the factor of safety by the parabola formula.

Ans. 6.6

3. A steel Z-bar column (see Fig. 46, *a*) is 24 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches; and the area of its cross-section is 24.5 square inches. Compute the safe load for the column by the formulas of this article, using a factor of safety of 4.

Ans. 224,500 pounds.

4. A short-leaf yellow pine column 14 × 14 inches in section is 20 feet long. What load can it sustain, with a factor of safety of 6?

Ans. 101,000 pounds.

88. "Broken Straight-Line" Formula. A large steel company computes the strength of its flat-ended steel columns by two formulas represented by two straight lines AB and BC, Fig. 52. The formulas are

$$\frac{P}{A} = 48{,}000,$$

and

$$\frac{P}{A} = 68{,}400 - 228\ \frac{l}{r},$$

P, A, l, and r having the same meanings as in Art. 83.

The point B corresponds very nearly to the ratio $l \div r = 90$. Hence, for columns for which the ratio $l \div r$ is less than 90, the first formula applies; and for columns for which the ratio is greater than 90, the second one applies. The point C corresponds to the ratio $l \div r = 200$, and the second formula does not apply to a column for which $l \div r$ is greater than that limit.

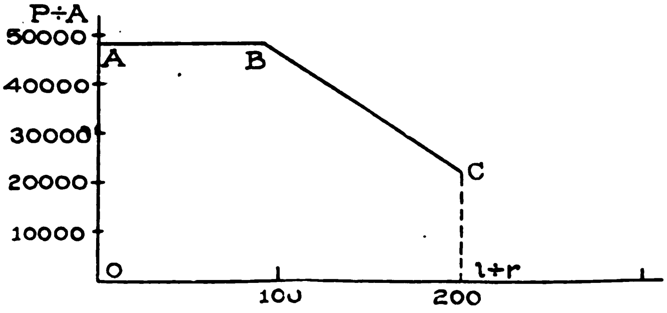

Fig. 52.

The ratio $l \div r$ for steel columns of practice rarely exceeds 150, and is usually less than 100.

Fig. 53 is a combination of Figs. 49, 50, 51 and 52, and represents graphically a comparison of the Rankine, straight-line, Euler, parabola-Euler, and broken straight-line formulas for flat-ended mild-steel columns. It well illustrates the fact that our knowledge of the strength of columns is not so exact as that, for example, of the strength of beams.

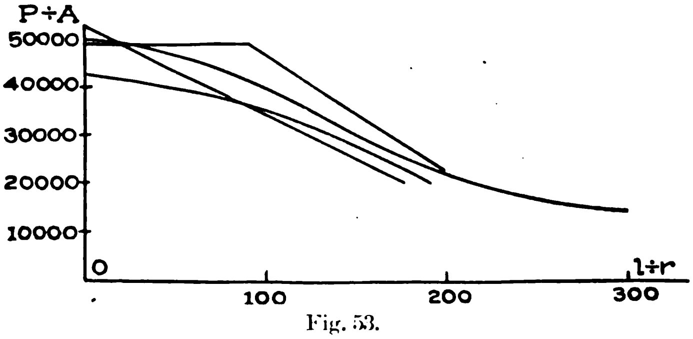

Fig. 53.

89. Design of Columns. All the preceding examples relating to columns were on either (1) computing the factor of safety of a given loaded column, or (2) computing the safe load for a given column. A more important problem is to design a column to sustain a given load under given conditions. A complete discussion of this problem is given in a later paper on design. We show here merely how to compute the *dimensions* of the cross-section of the column after the *form* of the cross-section has been decided upon.

In only a few cases can the dimensions be computed directly (see example 1 following), but usually, when a column formula is applied to a certain case, there will be two unknown quantities in it, A and r or d. Such cases can best be solved by trial (see examples 2 and 3 below).

Example. 1. What is the proper size of white pine column to sustain a load of 80,000 pounds with a factor of safety of 5, when the length of the column is 22 feet?

We use the parabola formula (equation 13). Since the safe load is 80,000 pounds and the factor of safety is 5, the breaking load P is

$$80{,}000 \times 5 = 400{,}000 \text{ pounds.}$$

The unknown side of the (square) cross-section being denoted by d, the area A is d^2. Hence, substituting in the formula, since $l = 22$ feet $= 264$ inches, we have

$$\frac{400{,}000}{d^2} = 2{,}500 - 0.6\,\frac{264^2}{d^2}.$$

Multiplying both sides by d^2 gives

$$400{,}000 = 2{,}500\,d^2 - 0.6 \times 264^2,$$

or $$2{,}500\,d^2 = 400{,}000 + 0.6 \times 264^2 = 441{,}817.6.$$

Hence $$d^2 = 176.73, \text{ or } d = 13.3 \text{ inches.}$$

2. What size of cast-iron column is needed to sustain a load of 100,000 pounds with a factor of safety of 10, the length of the column being 14 feet?

We shall suppose that it has been decided to make the cross-section circular, and shall compute by Rankine's formula modified for cast-iron columns (equation 10'). The breaking load for the column would be

$$100{,}000 \times 10 = 1{,}000{,}000 \text{ pounds.}$$

The length is 14 feet or 168 inches; hence the formula becomes

$$\frac{1{,}000{,}000}{A} = \frac{80{,}000}{1 + \frac{168^2}{800d^2}};$$

or, reducing by dividing both sides of the equation by 10,000, and then clearing of fractions, we have

$$100\left[1 + \frac{168^2}{800d^2}\right] = 8A.$$

There are two unknown quantities in this equation, *d* and A, and we cannot solve directly for them. Probably the best way to proceed is to assume or guess at a practical value of *d*, then solve for A, and finally compute the thickness or inner diameter. Thus, let us try *d* equal to 7 inches, first solving the equation for A as far as possible. Dividing both sides by 8 we have

$$A = \frac{100}{8}\left[1 + \frac{168^2}{800d^2}\right],$$

and, combining,

$$A = 12.5 + \frac{441}{d^2}.$$

Now, substituting 7 for *d*, we have

$$A = 12.5 + \frac{441}{49} = 21.5 \text{ square inches.}$$

The area of a hollow circle whose outer and inner diameters are d and d_1 respectively, is $0.7854\ (d^2 - d_1^2)$. Hence, to find the inner diameter of the column, we substitute 7 for d in the last expression, equate it to the value of A just found, and solve for d_1. Thus,

$$0.7854\ (49 - d_1^2) = 21.5\cdot$$

hence

$$49 - d_1^2 = \frac{21.5}{0.7854} = 27.37;$$

and $\qquad d_1^2 = 49 - 27.37 = 21.63 \text{ or } d_1 = 4.65.$

This value of d makes the thickness equal to

$$\tfrac{1}{2}\ (7 - 4.65) = 1.175 \text{ inches,}$$

which is safe. It might be advisable in an actual case to try *d* equal to 8 repeating the computation.*

EXAMPLE FOR PRACTICE.

1. What size of white oak column is needed to sustain a load of 45,000 pounds with a factor of safety of 6, the length of the column being 12 feet.

Ans. $d = 8\frac{1}{2}$, practically a 10 × 10-inch section

STRENGTH OF SHAFTS.

A **shaft** is a part of a machine or system of machines, and is used to transmit power by virtue of its torsional strength, or resistance to twisting. Shafts are almost always made of metal and are usually circular in cross-section, being sometimes made hollow.

90. Twisting Moment. Let AF, Fig. 54, represent a shaft with four pulleys on it. Suppose that D is the driving pulley and that B, C and E are pulleys from which power is taken off to drive machines. The portions of the shafts between the pulleys are twisted when it is transmitting power; and by the twisting moment at any cross-section of the shaft is meant the algebraic sum of the moments of all the forces acting on the shaft on either

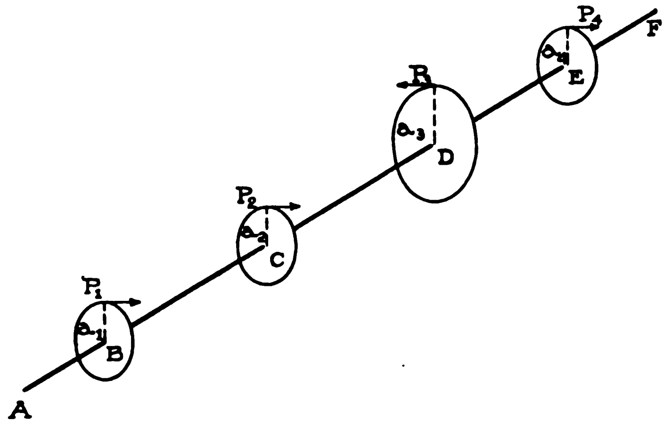

Fig. 54.

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* NOTE. The structural steel handbooks contain extensive tables by means of which the design of columns of steel or cast iron is much facilitated. The difficulties encountered in the use of formulæ are well illustrated in this example.

side of the section, the moments being taken with respect to the axis of the shaft. Thus, if the forces acting on the shaft (at the pulleys) are P_1, P_2, P_3, and P_4 as shown, and if the arms of the forces or radii of the pulleys are a_1, a_2, a_3, and a_4 respectively, then the twisting moment at any section in

BC is $P_1 a_1$,
CD is $P_1 a_1 + P_2 a_2$,
DE is $P_1 a_1 + P_2 a_2 - P_3 a_3$.

Like bending moments, twisting moments are usually expressed in inch-pounds.

Example. Let $a_1 = a_2 = a_4 = 15$ inches, $a_3 = 30$ inches, $P_1 = 400$ pounds, $P_2 = 500$ pounds, $P_3 = 750$ pounds, and $P_4 = 600$ pounds.* What is the value of the greatest twisting moment in the shaft?

At any section between the first and second pulleys, the twisting moment is

$$400 \times 15 = 6{,}000 \text{ inch-pounds};$$

at any section between the second and third it is

$$400 \times 15 + 500 \times 15 = 13{,}500 \text{ inch-pounds; and}$$

at any section between the third and fourth it is

$$400 \times 15 + 500 \times 15 - 750 \times 30 = -9{,}000 \text{ inch-pounds.}$$

Hence the greatest value is 13,500 inch-pounds.

91. Torsional Stress. The stresses in a twisted shaft are called "torsional" stresses. The torsional stress on a cross-section of a shaft is a shearing stress, as in the case illustrated by Fig. 55, which represents a flange coupling in a shaft. Were it not for the bolts, one flange would slip over the other when either part of the shaft is turned; but the bolts prevent the slipping. Obviously there is a tendency to shear the bolts off unless they are screwed up very tight; that is, the material of the bolts is subjected to shearing stress.

Just so, at any section of the solid shaft there is a tendency for one part to slip past the other, and to prevent the slipping or shearing of the shaft, there arise shearing stresses at all parts of the cross-section. The shearing stress on the cross-section of a shaft is not a uniform stress, its value per unit-area being zero at the center of the section, and increasing toward the circumference. In circular sections, solid or hollow, the shearing stress per unit-area (unit-stress) varies directly as the distance from the center of the section, provided the elastic limit is not exceeded. Thus, if the shearing unit-stress at the circumference of a section is 1,000 pounds per square inch, and the diameter of the shaft is 2 inches, then, at $\frac{1}{2}$ inch from the center, the unit-stress is 500 pounds per square inch; and at $\frac{1}{4}$ inch from the center it is 250 pounds per square inch. In Fig. 55 the arrows indicate the values and the directions of the shearing stresses on very small portions of the cross-section of a shaft there represented.

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* Note. These numbers were so chosen that the moment of P (driving moment) equals the sum of the moments of the other forces. This is always the case in a shaft rotating at constant speed; that is, the power given the shaft equals the power taken off.

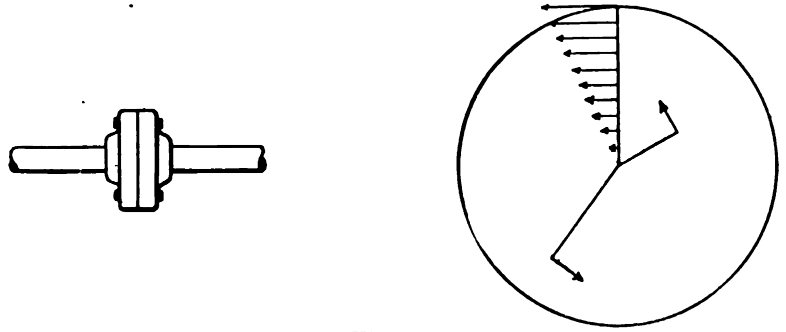


Fig. 55.

92. Resisting Moment. By "resisting moment" at a section of a shaft is meant the sum of the moments of the shearing stresses on the cross-section about the axis of the shaft.

Let S_s denote the value of the shearing stress per unit-area (unit-stress) at the outer points of a section of a shaft; d the diameter of the section (outside diameter if the shaft is hollow); and d_1 the inside diameter. Then it can be shown that the resisting moment is:

For a solid section, $0.1963\, S_s\, d^3$;

For a hollow section, $\dfrac{0.1963\, S_s\, (d^4 - d_1^4)}{d}$.

93. Formula for the Strength of a Shaft. As in the case of beams, the resisting moment equals the twisting moment at any section. If T be used to denote twisting moment, then we have the formulas:

$$\left.\begin{aligned}\text{For solid circular shafts, } & 0.1963\ S_s\ d^3 = T;\\ \text{For hollow circular shafts, } & \frac{0.1963\ S_s (d^4 - d_1^4)}{d} = T.\end{aligned}\right\}\ \textbf{(15)}$$

In any portion of a shaft of constant diameter, the unit-shearing stress S_s is greatest where the twisting moment is greatest. Hence, to compute the greatest unit-shearing stress in a shaft, we first determine the value of the greatest twisting moment, substitute its value in the first or second equation above, as the case may be, and solve for S_s. It is customary to express T in inch-pounds and the diameter in inches, S_s then being in pounds per square inch.

Examples. 1. Compute the value of the greatest shearing unit-stress in the portion of the shaft between the first and second pulleys represented in Fig. 54, assuming values of the forces and pulley radii as given in the example of article 90. Suppose also that the shaft is solid, its diameter being 2 inches.

The twisting moment T at any section of the portion between the first and second pulleys is 6,000 inch-pounds, as shown in the example referred to. Hence, substituting in the first of the two formulas 15 above, we have

$$0.1963\ S_s \times 2^3 = 6{,}000;$$

or, $$S_s = \frac{6{,}000}{0.1963 \times 8} = 3{,}820 \text{ pounds per square inch.}$$

This is the value of the unit-stress at the outside portions of all sections between the first and second pulleys.

2. A hollow shaft is circular in cross-section, and its outer and inner diameters are 16 and 8 inches respectively. If the working strength of the material in shear is 10,000 pounds per square inch, what twisting moment can the shaft safely sustain?

The problem requires that we merely substitute the values of S_s, d, and d_1 in the second of the above formulas 15, and solve for T. Thus,

$$T = \frac{0.1963 \times 10{,}000\ (16^4 - 8^4)}{16} = 7{,}537{,}920 \text{ inch-pounds.}$$

EXAMPLES FOR PRACTICE.

1. Compute the greatest value of the shearing unit-stress in the shaft represented in Fig. 54, using the values of the forces and pulley radii given in the example of article 90, the diameter of the shaft being 2 inches.

Ans. 8,595 pounds per square inch

2. A solid shaft is circular in cross-section and is 9.6 inches in diameter. If the working strength of the material in shear is 10,000 pounds per square inch, how large a twisting moment can the shaft safely sustain? (The area of the cross-section is practically the same as that of the hollow shaft of example 2 preceding.)

Ans. 1,736,736 inch-pounds.

94. Formula for the Power Which a Shaft Can Transmit. The power that a shaft can safely transmit depends on the shearing working strength of the material of the shaft, on the size of the cross-section, and on the speed at which the shaft rotates.

Let H denote the amount of horse-power; S_s the shearing working strength in pounds per square inch; d the diameter (outside diameter if the shaft is hollow) in inches; d_1 the inside diameter in inches if the shaft is hollow; and n the number of revolutions of the shaft per minute. Then the relation between power transmitted, unit-stress, etc., is:

$$\left.\begin{aligned} &\text{For solid shafts, } H = \frac{S_s d^3 n}{321{,}000}; \\ &\text{For hollow shafts, } H = \frac{S_s (d^4 - d_1^4)\, n}{321{,}000\, d}. \end{aligned}\right\} \qquad (16)$$

Examples. 1. What horse-power can a hollow shaft 16 inches and 8 inches in diameter safely transmit at 50 revolutions per minute, if the shearing working strength of the material is 10,000 pounds per square inch?

We have merely to substitute in the second of the two formulas 16 above, and reduce. Thus,

$$H = \frac{10{,}000\,(16^4 - 8^4)\,50}{321{,}000 \times 16} = 6{,}000 \text{ horse-power (nearly).}$$

2. What size of solid shaft is needed to transmit 6,000 horse-power at 50 revolutions per minute if the shearing working strength of the material is 10,000 pounds per square inch?

We have merely to substitute in the first of the two formulas 16, and solve for d. Thus,

$$6,000 = \frac{10,000 \times d^3 \times 50}{321,000};$$

therefore $$d^3 = \frac{6,000 \times 321,000}{10,000 \times 50} = 3,852;$$

or, $$d = \sqrt[3]{3,852} = 15.68 \text{ inches.}$$

(A solid shaft of this diameter contains over 25% more material than the hollow shaft of example 1 preceding. There is therefore considerable saving of material in the hollow shaft.)

3. A solid shaft 4 inches in diameter transmits 200 horsepower while rotating at 200 revolutions per minute. What is the greatest shearing unit-stress in the shaft?

We have merely to substitute in the first of the equations 16, and solve for S_s. Thus,

$$200 = \frac{S_s \times 4^3 \times 200}{321,000};$$

or, $$S_s = \frac{200 \times 321,000}{4^3 \times 200} = 5,016 \text{ pounds per square inch.}$$

EXAMPLES FOR PRACTICE.

1. What horse-power can a solid shaft 9.6 inches in diameter safely transmit at 50 revolutions per minute, if its shearing working strength is 10,000 pounds per square inch?

Ans. 1,378 horse-power.

2. What size of solid shaft is required to transmit 500 horsepower at 150 revolutions per minute, the shearing working strength of the material being 8,000 pounds per square inch.

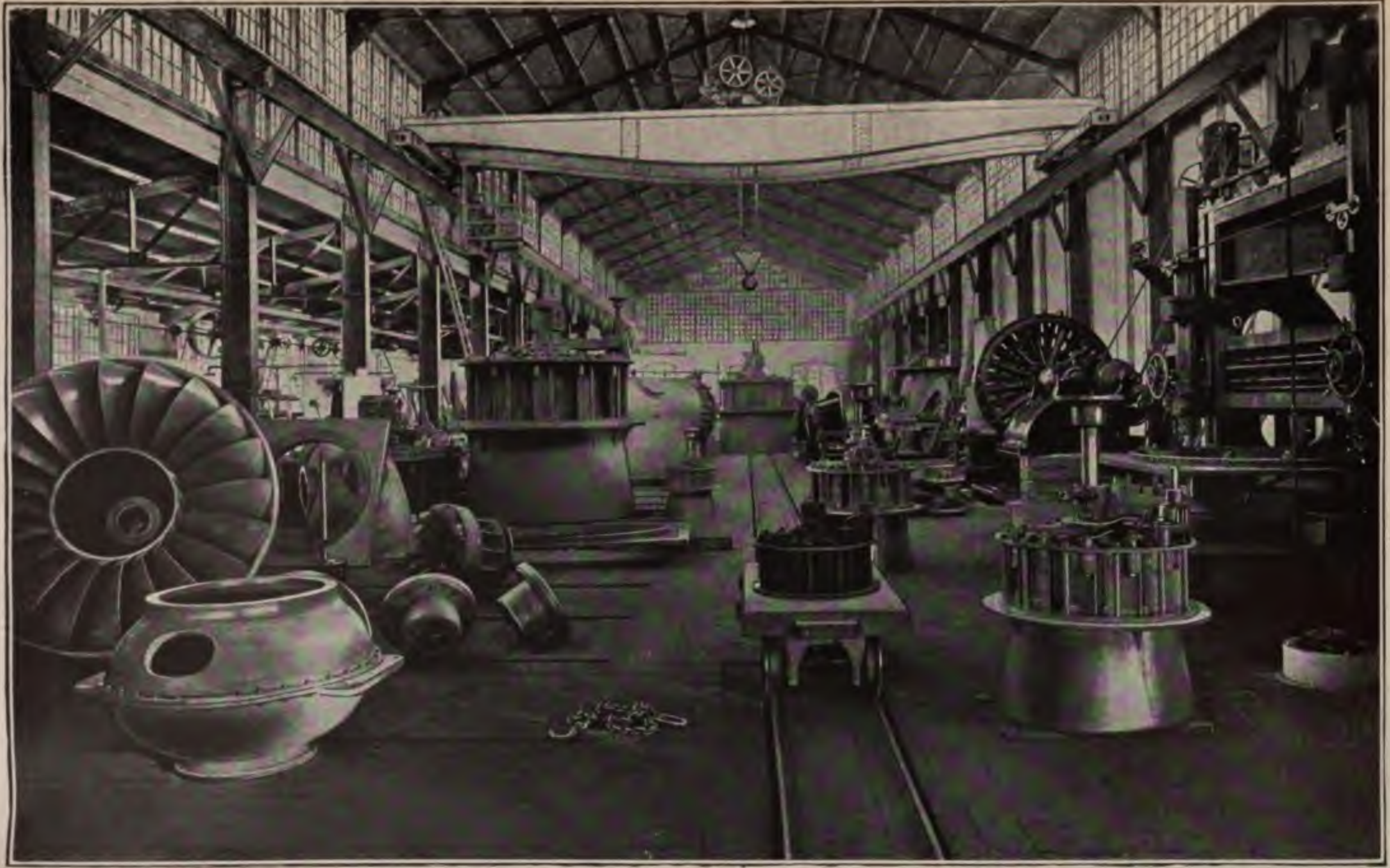
Ans. 5.1 inches.

3. A hollow shaft whose outer diameter is 14 and inner 6.7 inches transmits 5,000 horse-power at 60 revolutions per minute. What is the value of the greatest shearing unit-stress in the shaft?

Ans. 10,273 pounds per square inch.

STIFFNESS OF RODS, BEAMS, AND SHAFTS.

The preceding discussions have related to the *strength* of



PARTIAL INTERIOR VIEW OF MACHINE SHOP OF THE JAMES LEFFEL & CO. TURBINE WORKS, SPRINGFIELD, OHIO

materials. We shall now consider principally the *elongation of rods*, *deflection of beams*, and *twist of shafts*.

95. Coefficient of Elasticity. According to Hooke's Law (Art. 9, p. 7), the elongations of a rod subjected to an increasing pull are proportional to the pull, provided that the stresses due to the pull do not exceed the elastic limit of the material. Within the elastic limit, then, the ratio of the pull and the elongation is constant; hence the ratio of the unit-stress (due to the pull) to the unit-elongation is also constant. This last-named ratio is called "coefficient of elasticity." If E denotes this coefficient, S the unit-stress, and s the unit-deformation, then

$$E = \frac{S}{s}. \qquad (17)$$

Coefficients of elasticity are usually expressed in pounds per square inch.

The preceding remarks, definition, and formula apply also to a case of compression, provided that the material being compressed does not bend, but simply shortens in the direction of the compressing forces. The following table gives the *average* values of the coefficient of elasticity for various materials of construction:

TABLE 6.

Coefficients of Elasticity.

Material.	Average Coefficient of Elasticity.
Steel.............	30,000,000 pounds per square inch.
Wrought iron.....	27,500,000 " " " "
Cast iron.........	15,000,000 " " " "
Timber..........	1,800,000 " " " "

The coefficients of elasticity for steel and wrought iron, for different grades of those materials, are remarkably constant; but for different grades of cast iron the coefficients range from about 10,000,000 to 30,000,000 pounds per square inch. Naturally the coefficient has not the same value for the different kinds of wood; for the principal woods it ranges from 1,600,000 (for spruce) to 2,100,000 (for white oak).

Formula 17 can be put in a form more convenient for use, as follows:

Let P denote the force producing the deformation; A the area of the cross-section of the piece on which P acts; l the length of the piece; and D the deformation (elongation or shortening).

Then

$$S = P \div A \text{ (see equation 1),}$$

and

$$s = D \div l \text{ (see equation 2).}$$

Hence, substituting these values in equation 17, we have

$$E = \frac{Pl}{AD}; \text{ or } D = \frac{Pl}{AE}. \qquad (17')$$

The first of these two equations is used for computing the value of the coefficient of elasticity from measurements of a "test," and the second for computing the elongation or shortening of a given rod or bar for which the coefficient is known.

Examples. 1. It is required to compute the coefficient of elasticity of the material the record of a test of which is given on page 9.

Since the unit-stress S and unit-elongation *s* are already computed in that table, we can use equation 17 instead of the first of equations 17′. The elastic limit being between 40,000 and 45,000 pounds per square inch, we may use any value of the unit-stress less than that, and the corresponding unit-elongation.

Thus, with the first values given,

$$E = \frac{5,000}{0.00017} = 29,400,000.$$

With the second,

$$E = \frac{10,000}{0.00035} = 28,600,000.$$

This lack of constancy in the value of E as computed from different loads in a test of a given material, is in part due to errors in measuring the deformation, a measurement difficult to make. The value of the coefficient adopted from such a test, is the average of all the values of E which can be computed from the record.

2. How much will a pull of 5,000 pounds stretch a round steel rod 10 feet long and 1 inch in diameter?

We use the second of the two formulas 17′. Since $A = 0.7854 \times 1^2 = 0.7854$ square inches, $l = 120$ inches, and $E = 30,000,000$ pounds per square inch, the stretch is:

$$D = \frac{5,000 \times 120}{0.7854 \times 30,000,000} = 0.0254 \text{ inch.}$$

EXAMPLES FOR PRACTICE.

1. What is the coefficient of elasticity of a material if a pull of 20,000 pounds will stretch a rod 1 inch in diameter and 4 feet long 0.045 inch?

Ans. 27,000,000 pounds per square inch.

2. How much will a pull of 15,000 pounds elongate a round cast-iron rod 10 feet long and 1 inch in diameter?

Ans. 0.152 inch.

96. Temperature Stresses. In the case of most materials, when a bar or rod is heated, it lengthens; and when cooled, it shortens if it is free to do so. The **coefficient of linear expansion** of a material is the ratio which the elongation caused in a rod or bar of the material by a change of one degree in temperature bears to the length of the rod or bar. Its values for Fahrenheit degrees are about as follows:

For Steel,	0.0000065.
For Wrought iron,	.0000067.
For Cast iron,	.0000062.

Let K be used to denote this coefficient; t a change of temperature, in degrees Fahrenheit; l the length of a rod or bar; and D the change in length due to the change of temperature. Then

$$D = K\,t\,l. \qquad (18)$$

D and l are expressed in the same unit.

If a rod or bar is confined or restrained so that it cannot change its length when it is heated or cooled, then any change in its temperature produces a stress in the rod; such are called **temperature stresses.**

Examples. 1. A steel rod connects two solid walls and is screwed up so that the unit-stress in it is 10,000 pounds per square inch. Its temperature falls 10 degrees, and it is observed that the walls have not been drawn together. What is the temperature stress produced by the change of temperature, and what is the actual unit-stress in the rod at the new temperature?

Let l denote the length of the rod. Then the change in length which would occur if the rod were free, is given by formula 18, above, thus:

$$D = 0.0000065 \times 10 \times l = 0.000065\,l.$$

Now, since the rod could not shorten, it has a greater than normal length at the new temperature; that is, the fall in temperature has produced an effect equivalent to an elongation in the rod amounting to D, and hence a tensile stress. This tensile stress can be computed from the elongation D by means of formula 17. Thus,

$$S = E s;$$

and since s, the unit-elongation, equals

$$\frac{D}{l} = \frac{.0000065\ l}{l} = .0000035,$$

$S = 30{,}000{,}000 \times .0000035 = 195.0$ pounds per square inch. This is the value of the temperature stress; and the new unit-stress equals

$$10{,}000 + 195.0 = 10{,}195 \text{ pounds per square inch.}$$

Notice that the unit temperature stresses are independent of the length of the rod and the area of its cross-section.

2. Suppose that the change of temperature in the preceding example is a rise instead of a fall. What are the values of the temperature stress due to the change, and of the new unit-stress in the rod?

The temperature stress is the same as in example 1, that is, 1,950 pounds per square inch; but the rise in temperature releases, as it were, the stress in the rod due to its being screwed up, and the final unit stress is

$$10{,}000 - 1{,}950 = 8{,}050 \text{ pounds per square inch.}$$

EXAMPLE FOR PRACTICE.

1. The ends of a wrought-iron rod 1 inch in diameter are fastened to two heavy bodies which are to be drawn together, the temperature of the rod being 200 degrees when fastened to the objects. A fall of 120 degrees is observed not to move them. What is the temperature stress, and what is the pull exerted by the rod on each object?

Ans. { Temperature stress, 22,000 pounds per square inch.
Pull, 17,280 pounds.

97. Deflection of Beams. Sometimes it is desirable to know how much a given beam will deflect under a given load, or to design a beam which will not deflect more than a certain amount under a given load. In Table B, page 55, Part I, are given formulas for deflection in certain cases of beams and different kinds of loading.

In those formulas, d denotes deflection; I the moment of inertia of the cross-section of the beam with respect to the neutral axis, as in equation 6; and E the coefficient of elasticity of the material of the beam (for values, see Art. 95).

In each case, the load should be expressed in pounds, the length in inches, and the moment of inertia in biquadratic inches; then the deflection will be in inches.

According to the formulas for d, the deflection of a beam varies inversely as the coefficient of its material (E) and the moment of inertia of its cross-section (I); also, in the first four and last two cases of the table, the deflection varies directly as the cube of the length (l^3).

Example. What deflection is caused by a uniform load of 6,400 pounds (including weight of the beam) in a wooden beam on end supports, which is 12 feet long and 6 × 12 inches in cross-section? (This is the safe load for the beam; see example 1, Art. 65.)

The formula for this case (see Table B, page 55) is

$$d = \frac{5\ Wl^3}{384\ EI}.$$

Here $W = 6{,}400$ pounds; $l = 144$ inches; $E = 1{,}800{,}000$ pounds per square inch; and

$$I = \frac{1}{12}\ bd^3 = \frac{1}{12}\ 6 \times 12^3 = 864 \text{ inches}^4.$$

Hence the deflection is

$$d = \frac{5 \times 6{,}400 \times 144^3}{384 \times 1{,}800{,}000 \times 864} = 0.16 \text{ inch}.$$

EXAMPLES FOR PRACTICE.

1. Compute the deflection of a timber built-in cantilever 8 × 8 inches which projects 8 feet from the wall and bears an end load of 900 pounds. (This is the safe load for the cantilever, see example 1, Art. 65.)

Ans. 0.43 inch.

2. Compute the deflection caused by a uniform load of 40,000

pounds on a 42-pound 15-inch steel I-beam which is 16 feet long and rests on end supports.

Ans. 0.28 inch.

98. Twist of Shafts. Let Fig. 57 represent a portion of a shaft, and suppose that the part represented lies wholly between

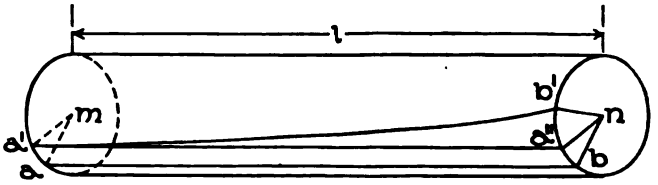


Fig. 57.

two adjacent pulleys on a shaft to which twisting forces are applied (see Fig. 54). Imagine two radii *ma* and *nb* in the ends of the portion, they being parallel as shown when the shaft is not twisted. After the shaft is twisted they will not be parallel, *ma* having moved to *ma'*, and *nb* to *nb'*. The angle between the two lines in their twisted positions (*ma'* and *nb'*) is called the **angle of twist,** or **angle of torsion,** for the length *l*. If *a'a''* is parallel to *ab*, then the angle *a''nb'* equals the angle of torsion.

If the stresses in the portion of the shaft considered do not exceed the elastic limit, and if the twisting moment is the same for all sections of the portion, then the angle of torsion a (in degrees) can be computed from the following:

For solid circular shafts,

$$a = \frac{584\ \mathrm{T}l}{\mathrm{E}^1\, d^4} = \frac{36{,}800{,}000\ \mathrm{H}l}{\mathrm{E}^1\, d^4 n}.$$

For hollow circular shafts,

$$a = \frac{584\ \mathrm{T}ld}{\mathrm{E}^1(d^4 - d_1^{\,4})} = \frac{36{,}800{,}000\ \mathrm{H}l}{\mathrm{E}^1\,(d^4 - d_1^{\,4})\, n}. \qquad (19)$$

Here T, l, d, d_1, H, and n have the same meanings as in Arts. 93 and 94, and should be expressed in the units there used. The letter E^1 stands for a quantity called **coefficient of elasticity for shear**; it is analogous to the coefficient of elasticity for tension and compression (E), Art. 95. The values of E^1 for a few materials average about as follows (roughly $\mathrm{E}^1 = \frac{2}{5}\,\mathrm{E}$):

For Steel,	11,000,000	pounds per square inch.
For Wrought iron,	10,000,000	" " " "
For Cast iron,	6,000,000	" " " "

Example. What is the value of the angle of torsion of a steel shaft 60 feet long when transmitting 6,000 horse-power at 50 revolutions per minute, if the shaft is hollow and its outer and inner diameters are 16 and 8 inches respectively?

Here $l = 720$ inches; hence, substituting in the appropriate formula (19), we find that

$$a = \frac{36{,}800{,}000 \times 6{,}000 \times 720}{11{,}000{,}000 \times (16^4 - 8^4)\ 50} = 4.7 \text{ degrees.}$$

EXAMPLE FOR PRACTICE.

Suppose that the first two pulleys in Fig. 54 are 12 feet apart; that the diameter of the shaft is 2 inches; and that $P_1 = 400$ pounds, and $a_1 = 15$ inches. If the shaft is of wrought iron, what is the value of the angle of torsion for the portion between the first two pulleys?

Ans. 3.15 degrees.

99. Non-elastic Deformation. The preceding formulas for elongation, deflection, and twist hold only so long as the greatest unit-stress does not exceed the elastic limit. There is no theory, and no formula, for non-elastic deformations, those corresponding to stresses which exceed the elastic limit. It is well known, however, that non-elastic deformations are not proportional to the forces producing them, but increase much faster than the loads. The value of the ultimate elongation of a rod or bar (that is, the amount of elongation at rupture), is quite well known for many materials. This elongation, for eight-inch specimens of various materials (see Art. 16), is :

For Cast iron,	about 1 per cent.	
For Wrought iron (plates),	12 - 15	per cent.
For " " (bars),	20 - 25	" " .
For Structural steel,	22 - 26	" " .

Specimens of ductile materials (such as wrought iron and structural steel), when pulled to destruction, **neck down**, that is, diminish very considerably in cross-section at some place along the length of the specimen. The decrease in cross-sectional area are the plates under compression at the sides of their holes. The "frictional strength" of a joint, however, is usually regarded as uncertain, and generally no allowance is made for friction in computations on the strength of riveted joints.

104. Tensile and Compressive Strength of Riveted Plates. The holes punched or drilled in a plate or bar weaken its tensile strength, and to compute that strength it is necessary to allow for the holes. By **net section**, in this connection, is meant the smallest cross-section of the plate or bar ; this is always a section along a line of rivet holes.

If, as in the foregoing article, t denotes the thickness of the plates joined ; d, the diameter of the holes; n_1, the number of rivets in a row ; and w, the width of the plate or bar; then the net section $= (w - n_1 d)\, t$.

Let S_t denote the tensile working strength of the plate ; then the strength of the unriveted plate is wtS_t, and the reduced tensile strength is $(w - n_1 d)\, t\, S_t$.

The compressive strength of a plate is also lessened by the presence of holes ; but when they are again filled up, as in a joint, the metal is replaced, as it were, and the compressive strength of the plate is restored. No allowance is therefore made for holes in figuring the compressive strength of a plate.

105. Computation of the Strength of a Joint. The strength of a joint is determined by either (1) the shearing value of the rivets ; (2) the bearing value of the plate ; or (3) the tensile strength of the riveted plate if the joint is in tension. Let P_s denote the strength of the joint as computed from the shearing values of the rivets ; P_c, that computed from the bearing value of the plates ; and P_t, the tensile strength of the riveted plates. Then, as before explained,

$$\left.\begin{aligned} P_t &= (w - n_1 d)\, tS_t; \\ P_s &= n_2\, 0.7854\, d^2 S_s; \text{ and} \\ P_c &= n_3 t d S_c\, ; \end{aligned}\right\} \qquad \textbf{(20)}$$

n_2 denoting the total number of rivets in the joint ; and n_3 denoting the total number of rivets in a lap joint, and one-half the number of rivets in a butt joint.

Examples. 1. Two half-inch plates $7\frac{1}{2}$ inches wide are connected by a single lap joint double-riveted, six rivets in two rows. If the diameter of the rivets is $\frac{3}{4}$ inch, and the working strengths are as follows : $S_t = 12,000$, $S_s = 7,500$, and $S_c = 15,000$ pounds per square inch, what is the safe tension which the joint can transmit ?

Here $n_1 = 3$, $n_2 = 6$, and $n_3 = 6$; hence

$$P_t = (7\frac{1}{2} - 3 \times \frac{3}{4}) \times \frac{1}{2} \times 12,000 = 31,500 \text{ pounds;}$$

$$P_s = 6 \times 0.7854 \times (\frac{3}{4})^2 \times 7,500 = 19,880 \text{ pounds ;}$$

$$P_c = 6 \times \frac{1}{2} \times \frac{3}{4} \times 15,000 = 33,750 \text{ pounds.}$$

Since P_s is the least of these three values, the strength of the joint depends on the shearing value of its rivets, and it equals 19,880 pounds.

2. Suppose that the plates described in the preceding example are joined by means of a butt joint (two cover-plates), and 12 rivets are used, being spaced as before. What is the safe tension which the joint can bear ?

Here $n_1 = 3$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_t = 31,500; \text{ and } P_c = 33,750 \text{ pounds; but}$$

$$P_s = 12 \times 0.7854 \times (\frac{3}{4})^2 \times 7,500 = 39,760 \text{ pounds.}$$

The strength equals 31,500 pounds, and the joint is stronger than the first.

3. Suppose that in the preceding example the rivets are arranged in rows of two. What is the tensile strength of the joint ?

Here $n_1 = 2$, $n_2 = 12$, and $n_3 = 6$; hence, as in the preceding example,

$$P_s = 39,760; \text{ and } P_c = 33,750 \text{ pounds; but}$$

$$P_t = (7\frac{1}{2} - 2 \times \frac{3}{4}) \frac{1}{2} \times 12,000 = 36,000 \text{ pounds.}$$

The strength equals 33,750 pounds, and this joint is stronger than either of the first two.

EXAMPLES FOR PRACTICE.

Note. Use working strengths as in example 1, above.

$S_t = 12{,}000$, $S_s = 7{,}500$, and $S_c = 15{,}000$ pounds per square inch.

1. Two half-inch plates 5 inches wide are connected by a lap joint, with two $\frac{3}{4}$-inch rivets in a row. What is the safe strength of the joint?

Ans. 6,625 pounds.

2. Solve the preceding example supposing that four $\frac{3}{4}$-inch rivets are used, in two rows.

Ans. 13,250 pounds.

3. Solve example 1 supposing that three 1-inch rivets are used, placed in a row lengthwise of the joint.

Ans. 17,670 pounds.

4. Two half-inch plates 5 inches wide are connected by a butt joint (two cover-plates), and four $\frac{3}{4}$-inch rivets are used, in two rows. What is the strength of the joint?

Ans. 11,250 pounds.

106. Efficiency of a Joint. The ratio of the strength of a joint to that of the solid plate is called the "efficiency of the joint." If ultimate strengths are used in computing the ratio, then the efficiency is called **ultimate efficiency;** and if working strengths are used, then it is called **working efficiency.** In the following, we refer to the latter. An efficiency is sometimes expressed as a per cent. To express it thus, multiply the ratio *strength of joint ÷ strength of solid plate*, by 100.

Example. It is required to compute the efficiencies of the joints described in the examples worked out in the preceding article.

In each case the plate is $\frac{1}{2}$ inch thick and $7\frac{1}{2}$ inches wide; hence the tensile working strength of the solid plate is

$$7\frac{1}{2} \times \frac{1}{2} \times 12{,}000 = 45{,}000 \text{ pounds.}$$

Therefore the efficiencies of the joints are:

$$(1)\quad \frac{19{,}880}{45{,}000} = 0.44, \text{ or } 44 \text{ per cent;}$$

$$(2)\quad \frac{31{,}500}{45{,}000} = 0.70, \text{ or } 70 \text{ per cent;}$$

$$(3)\quad \frac{33{,}750}{45{,}000} = 0.75, \text{ or } 75 \text{ per cent.}$$

applied. When the place of application is small so that it may be regarded as a point, it is called the "point of application." Thus the place of application of the pressure (push or force) which a locomotive wheel exerts on the rail is the part of the surface of the rail in contact with the wheel. For practically all purposes this pressure may be considered as applied at a point (the center of the surface of contact), and it is called the point of application of the force exerted by the wheel on the rail.

A force which has a point of application is said to have a **line of action**, and by this term is meant the line through the point of application of the force parallel to its direction. Thus, in the Fig. 1, the line of action of the force exerted on the body is the line representing the string. Notice clearly the distinction between the direction and line of action of the force; the direction of the force in the illustration could be represented by any horizontal line in the figure with an arrowhead upon it pointing toward the right, but the line of action can be represented only by the line representing the string, indefinite as to length, but definite in position.

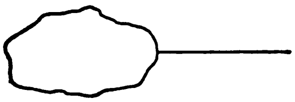


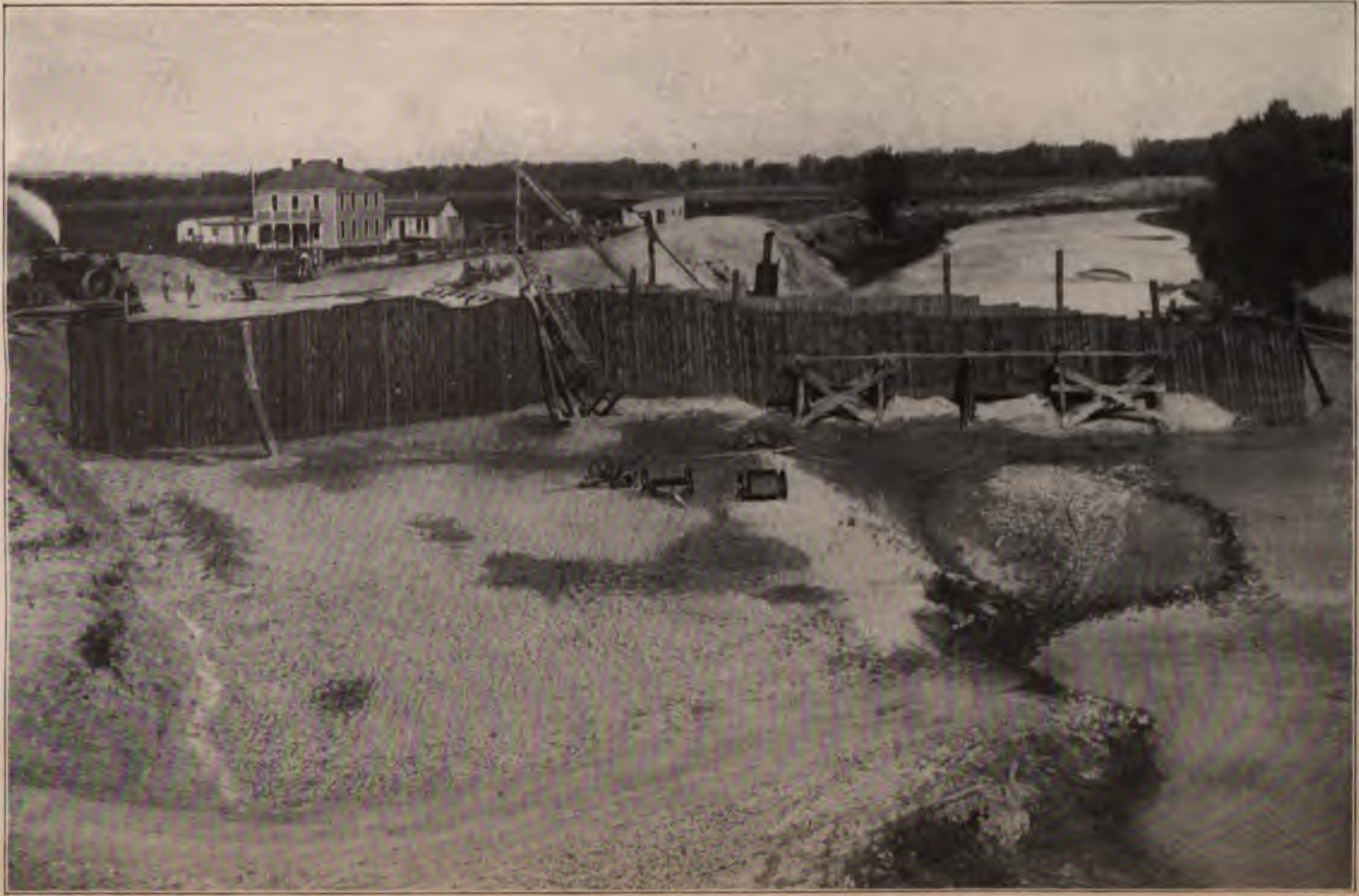
Fig. 1.

That part of the direction of a force which is indicated by means of the arrowhead on a line is called the **sense** of the force. Thus the sense of the force of the preceding illustration is toward the right and not toward the left.

2. Specification and Graphic Representation of a Force. For the purposes of statics, a force is completely specified or described if its

(1) magnitude, (2) line of action, and (3) sense are known or given.

These three elements of a force can be represented graphically, that is by a drawing. Thus, as already explained, the straight line (Fig. 1) represents the line of action of the force exerted upon the body; an arrowhead placed on the line pointing toward the right gives the sense of the force; and a definite length marked off on the line represents to some scale the magnitude of the force. For example, if the magnitude is 50 pounds, then to a scale of 100 pounds to the inch, one-half of an inch represents the magnitude of the force.



HEADWORKS OF MAIN INLET CANAL, BELLE FOURCHE PROJECT, SOUTH DAKOTA

It is often convenient, especially when many forces are concerned in a single problem, to use two lines instead of one to represent a force—one to represent the magnitude and one the line of action, the arrowhead being placed on either. Thus Fig. 2 also represents the force of the preceding example, AB (one-half inch long) representing the magnitude of the force and *ab* its line of action. The line AB might have been drawn anywhere in the figure, but its length is definite, being fixed by the scale.

The part of a drawing in which the body upon which forces act is represented, and in which the lines of action of the forces are drawn, is called the **space diagram** (Fig. 2*a*). If the body were drawn to scale, the scale would be a certain number of inches or feet to the inch. The part of a drawing in which the force magnitudes are laid off (Fig. 2*b*) is called by various names; let us call it the **force diagram.** The scale of a force diagram is always a certain number of pounds or tons to the inch.

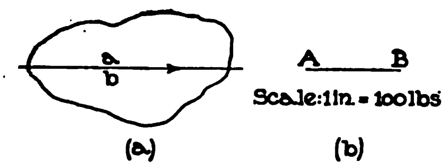


Fig. 2.

3. Notation. When forces are represented in two separate diagrams, it is convenient to use a special notation, namely: a capital letter at each end of the line representing the magnitude of the force, and the same small letters on opposite sides of the line representing the action line of the force (see Fig. 2). When we wish to refer to a force, we shall state the capital letters used in the notation of that force; thus "force AB" means the force whose magnitude, action line, and sense are represented by the lines AB and *ab*.

In the algebraic work we shall usually denote a force by the letter F.

4. Scales. In this subject, scales will always be expressed in feet or pounds to an inch, or thus, 1 inch = 10 feet, 1 inch = 100 pounds, etc. The number of feet or pounds represented by one inch on the drawing is called the *scale number.*

To find the length of the line to represent a certain distance or force, divide the distance or force by the scale number; the quotient is the length to be laid off in the drawing. To find the *magnitude of a distance or a force represented by a certain line in a drawing, multiply the length of the line by the scale number; the product is the magnitude of the distance or force, as the case may be.*

The scale to be used in making drawings depends, of course, upon how large the drawing is to be, and upon the size of the quantities which must be represented. In any case, it is convenient to select the scale number so that the quotients obtained by dividing the quantities to be represented may be easily laid off by means of the divided scale which is at hand.

Examples. 1. If one has a scale divided into 32nds, what is the convenient scale for representing 40 pounds, 32 pounds, 56 pounds, and 70 pounds?

According to the scale, 1 inch = 32 pounds, the lengths representing the forces are respectively :

$$\frac{40}{32}=1\tfrac{1}{4};\ \frac{32}{32}=1;\ \frac{56}{32}=1\tfrac{3}{4};\ \frac{70}{32}=2\tfrac{3}{16}\text{ inches.}$$

Since all of these distances can be easily laid off by means of the "sixteenths scale," 1 inch = 32 pounds is convenient.

2. What are the forces represented by three lines, 1.20, 2.11, and 0.75 inches long, the scale being 1 inch = 200 pounds?

According to the rule given in the foregoing, we multiply each of the lengths by 200, thus :

$$1.20 \times 200 = 240 \text{ pounds.}$$
$$2.11 \times 200 = 422 \text{ pounds.}$$
$$0.75 \times 200 = 150 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. To a scale of 1 inch = 500 pounds, how long are the lines to represent forces of 1,250, 675, and 900 pounds?

Ans. 2.5, 1.35, and 1.8 inches

2. To a scale of 1 inch = 80 pounds, how large are the forces represented by 1¼ and 1.6 inches?

Ans. 100 and 128 pounds.

5. Concurrent and Non-concurrent Forces. If the lines of action of several forces intersect in a point they are called **concurrent forces**, or a **concurrent system**, and the point of intersection is called the *point of concurrence* of the forces. If the lines of action of several forces do not intersect in the same point, they are called non-concurrent, or a non-concurrent system.

We shall deal only with forces whose lines of action lie in the same plane. It is true that one meets with problems in which there are forces whose lines of action do not lie in a plane, but such problems can usually be solved by means of the principles herein explained.

6. Equilibrium and Equilibrant. When a number of forces act upon a body which is at rest, each tends to move it; but the effects of all the forces acting upon that body may counteract or neutralize one another, and the forces are said to be *balanced* or *in equilibrium*. Any one of the forces of a system in equilibrium balances all the others. A single force which balances a number of forces is called the *equilibrant* of those forces.

7. Resultant and Composition. Any force which would produce the same effect (so far as balancing other forces is concerned) as that of any system, is called the *resultant* of that system. Evidently the resultant and the equilibrant of a system of forces must be equal in magnitude, opposite in sense, and act along the same line.

The process of determining the resultant of a system of forces is called composition.

8. Components and Resolution. Any number of forces whose combined effect is the same as that of a single force are called *components* of that force. The process of determining the components of a force is called *resolution*. The most important case of this is the resolution of a force into two components.

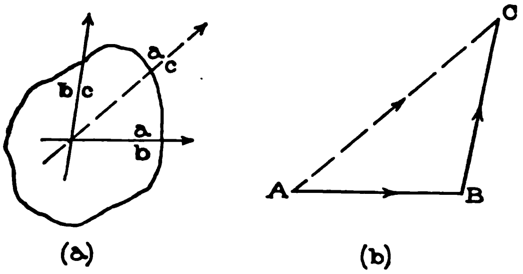
II. CONCURRENT FORCES; COMPOSITION AND RESOLUTION.

9. Graphical Composition of Two Concurrent Forces. *If two forces are represented in magnitude and direction by AB and BC (Fig. 3), the magnitude and direction of their resultant is represented by AC.* This is known as the "triangle law."

The line of action of the resultant is parallel to AC and passes through the point of concurrence of the two given forces; thus the line of action of the resultant is *ac*.

The law can be proved experimentally by means of two spring balances, a drawing board, and a few cords arranged as shown in

Fig. 4. The drawing board (not shown) is set up vertically, then from two nails in it the spring balances are hung, and these in turn support by means of two cords a small ring A from which a heavy body (not shown) is suspended. The ring A is in equilibrium under the action of three forces, a downward force equal to the weight of the suspended body, and two forces exerted by the upper cords whose values or magnitudes can be read from the spring balances. The first force is the equilibrant of the other two. Knowing the weight of the suspended body and the readings of the balances, lay off AB equal to the pull of the right-hand upper string according to some convenient scale, and BC parallel to the left-hand upper string and equal to the force exerted by it. It will then be found that the line joining A and C is vertical, and equals (by scale) the weight of the suspended body. Hence AC, with arrowhead pointing down, represents the equilibrant of the two upward pulls on the ring; and with arrowhead pointing up, it represents the resultant of those two forces.



(a) (b)

Fig. 3.

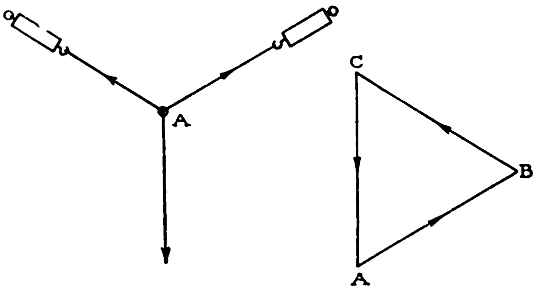


Fig. 4.

Notice especially how the arrowheads are related in the triangle (Fig. 3), and be certain that you understand this law before proceeding far, as it is the basis of most of this subject.

Examples. Fig. 5 represents a board 3 feet square to which forces are applied as shown. It is required to compound or find the resultant of the 100- and 80-pound forces.

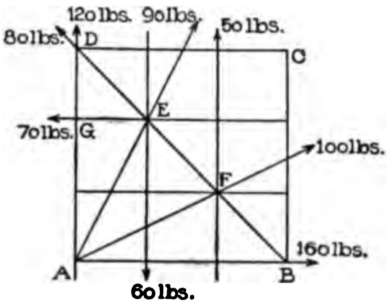
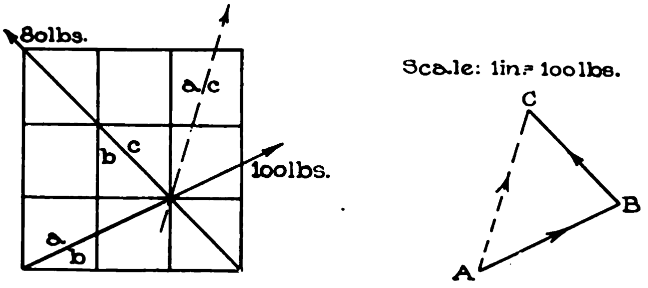


Fig. 5.

First we make a drawing of the board and mark upon it the lines of action of the two forces whose resultant is to be found, as in Fig. 6. Then by some convenient scale, as 100 pounds to the inch, lay off from any convenient point A, a line AB in the direction of the 100-pound force, and make AB one inch long, representing 100 pounds by the scale. Then from B lay off a line BC in the direction of the second force and make BC, 0.8 of an inch long, representing 80 pounds by the scale. Then the line AC, with the arrow pointing *from A to C*, represents the magnitude and direction of the resultant. Since AC equals 1.06 inch, the resultant equals



(a) (b)

Fig. 6.

$$1.06 \times 100 = 106 \text{ pounds}.$$

The line of action of the resultant is *ac*, parallel to AC and passing through the intersection of the lines of action (the point of concurrence) of the given forces. To complete the notation, we mark these lines of action *ab* and *bc* as in the figure.

EXAMPLES FOR PRACTICE.*

1. Determine the resultant of the 100- and the 120-pound forces represented in Fig. 5.

Ans. { The magnitude is 194 pounds; the force acts upward through A and a point 1.62 feet to the right of D.

2. Determine the resultant of the 120- and the 160-pound forces represented in Fig. 5.

Ans. { The magnitude is 200 pounds; the force acts upward through A and a point 9 inches below C.

10. Algebraic Composition of Two Concurrent Forces. If the angle between the lines of action of the two forces is not 90 degrees, the algebraic method is not simple, and the graphical is usually preferable. If the angle is 90 degrees, the algebraic method is usually the shorter, and this is the only case herein explained. Let F_1 and F_2 be two forces acting through some point of a body as represented in Fig. 7*a*. AB and BC represent the magnitudes and direction of F_1 and F_2 respectively; then, according to the triangle law (Art. 9), AC represents the magnitude and direction of the resultant of F_1 and F_2, and the line marked R (parallel to AC) is the line of action of that resultant. Since ABC is a right triangle,

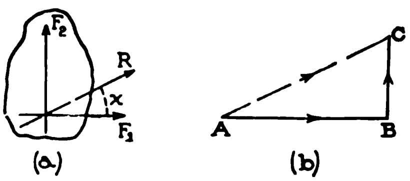


Fig. 7.

$$(AC)^2 = (AB)^2 + (BC)^2.$$

and,
$$\tan CAB = \frac{BC}{AB}.$$

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* Use sheets of paper not smaller than large letter size, and devote a full sheet to each example. In reading the answers to these examples, **remember that the board on which the forces act was stated to be 3 feet square.**

Now let R denote the resultant. Since AC, AB, and BC represent R, F_1, and F_2 respectively, and angle CAB = x,

$$R^2 = F_1^2 + F_2^2; \text{ or } R = \sqrt{F_1^2 + F_2^2};$$

and, $$\tan x = F_2 \div F_1.$$

By the help of these two equations we compute the magnitude of the resultant and inclination of its line of action to the force F_1.

Example. It is required to determine the resultant of the 120- and the 160-pound forces represented in Fig. 5.

Let us call the 160-pound force F_1; then,

$$R = \sqrt{160^2 + 120^2} = \sqrt{25,600 + 14,400}$$

$$= \sqrt{40,000} = 200 \text{ pounds};$$

and, $$\tan x = \frac{120}{160} = \tfrac{3}{4}; \text{ hence } x = 36° \, 52'.$$

The resultant therefore is 200 pounds in magnitude, acts through A (Fig. 5) upward and to the right, making an angle of 36° 52′ with the horizontal.

EXAMPLES FOR PRACTICE.

1. Determine the resultant of the 50- and 70-pound forces represented in Fig. 5.

Ans. { R = 86 pounds;
angle between R and 70-pound force = 35° 32′.

2. Determine the resultant of the 60- and 70-pound forces represented in Fig 5.

Ans. { R = 92.2 pounds;
angle between R and 70-pound force = 40° 36′.

11. Force Polygon. If lines representing the magnitudes and directions of any number of forces be drawn continuous and so that the arrowheads on the lines point the same way around on the series of lines, the figure so formed is called the *force polygon* for the forces. Thus ABCD (Fig. 8) is a force polygon for the 80-, 90-, and 100-pound forces of Fig. 5, for AB, BC, and CD represent the magnitudes and directions of those forces respectively, and the arrowheads point in the same way around, from A to D.

A number of force polygons can be drawn for any system of forces, no two alike. Thus A_1 B_1 C_1 D_1 and A_2 B_2 C_2 D_2 are other force polygons for the same three forces, 80, 90, and 100 pounds. Notice that A_3 B_3 C_3 D_3 is not a force polygon for the three forces although the lines represent the three forces in magnitude and direction. The reason why it is not a force polygon is that the arrowheads do not all point the same way around.

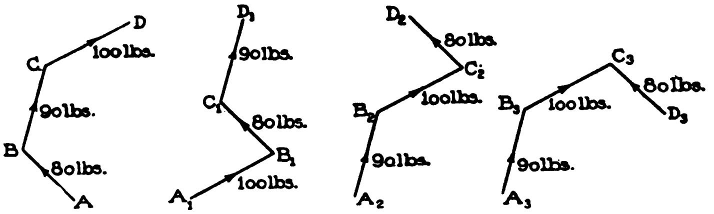


Fig. 8.

A force polygon is not necessarily a closed figure. If a force polygon closes for a system of concurrent forces, then evidently the resultant equals zero.

EXAMPLE FOR PRACTICE.

Draw to the same scale as many different force polygons as you can for the 100-, 120- and 160-pound forces of Fig. 5. Bear in mind that the arrowheads on a force polygon point the same way around.

12. Composition of More Than Two Concurrent Forces. The graphical is much the simpler method; therefore the algebraic one will not be explained. The following is a rule for performing the composition graphically:

(1). Draw a force polygon for the given forces.

(2). Join the two ends of the polygon and place an arrowhead on the joining line pointing from the beginning to the end of the polygon. That line then represents the magnitude and direction of the resultant.

(3). Draw a line through the point of concurrence of the given forces parallel to the line drawn as directed in (2). This line represents the action line of the resultant.

Example. It is required to determine the resultant of the four forces acting through the point E (Fig. 5).

First, make a drawing of the board and indicate the lines of action of the forces as shown in Fig. 9, but without lettering. Then to construct a force polygon, draw from any convenient point A, a line in the direction of one of the forces (the 70-pound force), and make AB equal to 70 pounds according to the scale (70 ÷ 100 = 0.7 inch). Then from B draw a line in the direction of the next force (80-pound), and make BC equal to 0.8 inch, representing 80 pounds. Next draw a line from C in the direction of the third force (90-pound), and make CD equal to 0.9 inch, representing 90 pounds. Finally draw a line from D in the direction of the last force, and make DE equal to 0.6 inch, representing 60 pounds. The force polygon is ABCDE, beginning at A and ending at E.

The second step is to connect A and E and place an arrowhead on the line pointing from A to E. This represents the magnitude and direction of the resultant. Since AE = 1.16 inches, the resultant is a force of

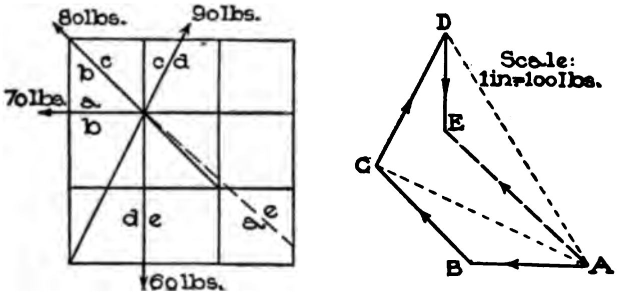


Fig. 9.

$$1.16 \times 100 = 116 \text{ pounds.}$$

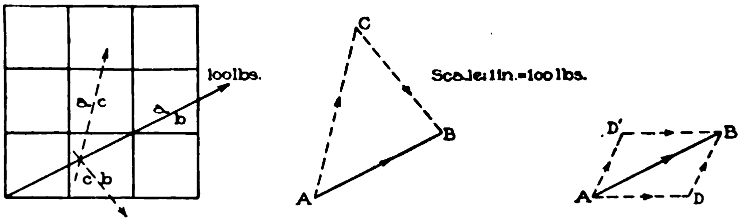
The third step is to draw a line *ae* through the point of concurrence and parallel to AE. This is the line of action of the resultant. (To complete the notation the lines of action of the 70-, 80-, 90- and 60-pound forces should be marked *ab*, *bc*, *cd*, and *de* respectively.)

That the rule for composition is correct can easily be proved. According to the triangle law, AC (Fig. 9), with arrowhead pointing from A to C, represents the magnitude and direction of the resultant of the 70- and 80-pound forces. According to the law, AD, with arrowhead pointing from A to D, represents the magnitude and direction of the resultant of AC and the 90-pound force, hence also of the 70-, 80-, and 90-pound forces. According to the law, AE with arrowhead pointing from A to E, represents the magnitude and direction of the resultant of AD and the 60-pound force. Thus we see that the foregoing rule and the triangle law lead to the same result, but the application of the rule is shorter as in it we do not need the lines AC and AD.

EXAMPLES FOR PRACTICE.

1. Determine the resultant of the four forces acting through the point A (Fig. 5).

Ans. { 380 pounds acting upward through A and a point 0.45 feet below C.



(a) (b) (c)

Fig. 10.

2. Determine the resultant of the three forces acting at the point F (Fig. 5).

Ans. { 155 pounds acting upward through F and a point 0.57 feet to left of C.

13. Graphical Resolution of Force into Two Concurrent Components. This is performed by applying the triangle law inversely. Thus, if it is required to resolve the 100-pound force of Fig. 5 into two components, we draw first Fig. 10 (a) to show the line of action of the force, and then AB, Fig. 10 (b), to represent the magnitude and direction. Then draw from A and B any two lines which intersect, mark their intersection C, and place arrowheads on AC and CB, pointing from A to C and from C to B. Also draw two lines in the space diagram parallel to AC and CB and so that they intersect on the line of action of the 100-pound force, *ab*.

The test of the correctness of a solution like this is to take the two components as found, and find their resultant; if the resultant thus found agrees in magnitude, direction, and sense with the given force (originally resolved), the solution is correct.

Notice that the solution above given is not definite, for the lines drawn from A and B were drawn at random. A force may therefore be resolved into two components in many ways. If, however, the components have to satisfy conditions, there may be but one solution. In the most important case of resolution, the lines of action of the components are given; this case is definite, there being but one solution, as is shown in the following example.

Example. It is required to resolve the 100-pound force (Fig. 5) into two components acting in the lines AE and AB.

Using the space diagram of Fig. 10, draw a line AB in Fig. 10 (c) to represent the magnitude and direction of the 100-pound force, and then a line from A parallel to the line of action of either of the components, and a line from B parallel to the other, thus locating D (or D′). Then AD and DB (or AD′ and D′B) represent the magnitudes and directions of the required components.

EXAMPLES FOR PRACTICE.

1. Resolve the 160-pound force of Fig. 5 into components which act in AF and AE.

Ans. { The first component equals 238½ pounds, and its sense is from A to F; the second component equals 119½ pounds, and its sense is from E to A.

2. Resolve the 50-pound force of Fig. 5 into two components, acting in FA and FB.

Ans. { The first component equals 37.3 pounds, and its sense is from A to F; the second component equals 47.0 pounds, and its sense is from B to F.

14. Algebraic Resolution of a Force Into Two Components. If the angle between the lines of action of the two components is not 90 degrees, the algebraic method is not simple and the graphical method is usually preferable. When the angle is 90 degrees, the algebraic method is usually the shorter, and this is the only case herein explained.

Let F (Fig. 11) be the force to be resolved into two components acting in the lines OX and OY. If AB is drawn to represent the magnitude and direction of F, and lines be drawn from A and B parallel to OX and OY, thus locating C, then AC and BC with arrowheads as shown represent the magnitudes and directions of the required components.

Now if F' and F'' represent the components acting in OX and OY, and x and y denote the angles between F and F', and F and F'' respectively, then AC and BC represent F' and F'', and the angles BAC and ABC equal x and y respectively. From the right triangle ABC it follows that

and, $F' = F \cos x$, and $F'' = F \cos y$.

If a force is resolved into two components whose lines of action are at right angles to each other, each is called a *rectangular component* of that force. Thus F' and F'' are rectangular components of F.

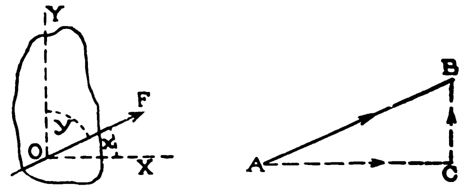


Fig. 11.

The foregoing equations show that *the rectangular component of a force along any line equals the product of the force and the cosine of the angle between the force and the line.* They show also that *the rectangular component of a force along its own line of action equals the force, and its rectangular component at right angles to the line of action equals zero.*

Examples. 1. A force of 120 pounds makes an angle of 22 degrees with the horizontal. What is the value of its component along the horizontal ?*

Since $\cos 22° = 0.927$, the value of the component equals $120 \times 0.927 = 111.24$ pounds.

2. What is the value of the component of the 90-pound force of Fig. 5 along the vertical ?

First we must find the value of the angle which the 90-pound force of Fig. 5 makes with the vertical.

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* When nothing is stated herein as to whether a component is rectangular or not, then rectangular component is meant.

Since $\tan \text{EAG} = \frac{\text{EG}}{\text{AG}} = \frac{1}{2}$, angle $\text{EAG} = 26° \, 34'$.

Hence the value of the desired component equals

$$90 \times \cos 26° \, 34' = 90 \times 0.8944 = 80.50 \text{ pounds.}$$

EXAMPLES FOR PRACTICE.

1. Compute the horizontal and vertical components of a force of 80 pounds whose angle with the horizontal is 60 degrees

$$\text{Ans.} \begin{cases} 40 \text{ pounds.} \\ 69.28 \text{ pounds.} \end{cases}$$

2. Compute the horizontal and vertical components of the 100-pound force in Fig. 5. What are their senses?

$$\text{Ans.} \begin{cases} 89.44 \text{ pounds to the right.} \\ 44.72 \text{ pounds upwards.} \end{cases}$$

3. Compute the component of the 70-pound force in Fig. 5 along the line EA. What is the sense of the component?

Ans. 31. 29 pounds ; E to A.

III. CONCURRENT FORCES IN EQUILIBRIUM.

15. Condition of Equilibrium Defined. By condition of equilibrium of a system of forces is meant a relation which they must fulfill in order that they may be in equilibrium or a relation which they fulfill when they are in equilibrium.

In order that any system may be in equilibrium, or be balanced, their equilibrant, and hence their resultant, must be zero, and this is a condition of equilibrium. If a system is known to be in equilibrium, then, since the forces balance among themselves, their equilibrant and hence their resultant also equals zero. This (the necessity of a zero resultant) is known as the general condition of equilibrium for it pertains to all kinds of force systems. For special kinds of systems there are special conditions, some of which are explained in the following.

16. Graphical Condition of Equilibrium. *The "graphical condition of equilibrium" for a system of concurrent forces is that the polygon for the forces must close.* For if the polygon closes, then the resultant equals zero as was pointed out in Art 11.

By means of this condition we can solve problems relating to

the algebraic sums of the components of the forces along each of two lines at right angles to each other must equal zero.

Examples. 1. It is required to determine the pull on the rope and the pressure on the plane in Example 1, Art. 16 (Fig. 12), it being given that the inclination of the plane to the horizontal is 30 degrees.

Let us denote the pull of the rope by F_1 and the pressure of the plane by F_2. The angles which these forces make the horizontal are 30° and 60° respectively; hence

the horizontal component of $F_1 = F_1 \times \cos 30° = 0.8660\ F_1$,
and " " " " $F_2 = F_2 \times \cos 60° = 0.5000\ F_2$;
also " " " " the weight $= 0$.

The angles which F_1 and F_2 make with the vertical are 60° and 30° respectively, hence

the vertical component of $F_1 = F_1 \times \cos 60° = 0.5000\ F_1$,
and the vertical component of $F_2 = F_2 \times \cos 30° = 0.8660\ F_2$;
also the vertical component of the weight $= 120$.

Since the three forces are in equilibrium, the horizontal and the vertical components are balanced, and hence

$$0.866\ F_1 = 0.5\ F_2$$

and

$$0.5\ F_1 + 0.866\ F_2 = 120.$$

From these two equations F_1 and F_2 may be determined; thus from the first,

$$F_2 = \frac{0.866}{0.5}\ F_1 = 1.732\ F_1.$$

Substituting this value of F_2 in the second equation we have

$$0.5\ F_1 + 0.866 \times 1.732\ F_1 = 120,$$

or

$$2\ F_1 = 120;$$

hence,

$$F_1 = \frac{120}{2} = 60 \text{ pounds},$$

and

$$F_2 = 1.732 \times 60 = 103.92 \text{ pounds}.$$

2. It is required to determine the pulls in the ropes of Fig. 13 by the algebraic method, it being given that the angles which the left- and right-hand ropes make with the ceiling are 30 and 70 degrees respectively and the body weighs 100 pounds.

Let us denote the pulls in the right- and left-hand ropes by F_1 and F_2 respectively. Then

the horizontal component of $F_1 = F_1 \times \cos 70^\circ = 0.342\ F_1$,
the horizontal component of $F_2 = F_2 \times \cos 30^\circ = 0.866\ F_2$,
the horizontal component of the weight $= 0$,
the vertical component of $F_1 = F_1 \times \cos 20^\circ = 0.9397\ F_1$,
the vertical component of $F_2 = F_2 \times \cos 60^\circ = 0.500\ F_2$,
and the vertical component of the weight $= 100$.

Now since these three forces are in equilibrium, the horizontal and the vertical components balance; hence

$$0.342 \cdot F_1 = 0.866\ F_2$$

and

$$0.9397\ F_1 + 0.5\ F_2 = 100.$$

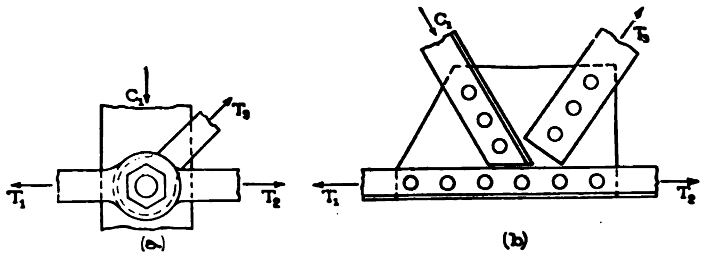

Fig. 15.

These equations may be solved for the unknown forces; thus from the first,

$$F_1 = \frac{0.866}{0.342} F_2 = 2.532\ F_2.$$

Substituting this value of F_1 in the second equation, we get

$$0.9397 \times 2.532\ F_2 + 0.5\ F_2 = 100,$$

or,

$$2.88\ F_2 = 100;$$

hence

$$F_2 = \frac{100}{2.88} = 34.72 \text{ pounds},$$

and

$$F_1 = 2.532 \times 34.72 = 87.91 \text{ pounds}.$$

EXAMPLES FOR PRACTICE.

1. Solve Ex. 1, Art. 16 algebraically. (First determine the angle which the inclined rope makes with the horizontal; you should find it to be 63° 26′.)

2. Solve Ex. 2, Art. 16 algebraically.
3. Solve Ex. 3, Art. 16 algebraically.

IV. ANALYSIS OF TRUSSES; "METHOD OF JOINTS."

18. **Trusses.** A truss is a frame work used principally to support loads as in roofs and bridges. Fig. 16, 25, 26 and 27 represent several forms of trusses. The separate bars or rods, $\overline{12}$, $\overline{23}$, etc. (Fig. 16) are called *members* of the truss and all the parts immediately concerned with the connection of a number of members at one place constitute a *joint*. A "pin joint" is shown in Fig. 15 (*a*) and a "riveted joint" in 15 (*b*).

19. **Truss Loads.** The loads which trusses sustain may be classified into fixed, or dead, and moving or live loads. A fixed, or dead load, is one whose place of application is fixed with reference to the truss, while a moving or live load is one whose place of application moves about on the truss.

Roof truss loads are usually fixed, and consist of the weight of the truss, roof covering, the snow, and the wind pressure, if any. Bridge truss loads are fixed and moving, the first consisting of the weights of the truss, the floor or track, the snow, and the wind pressure, and the second of the weight of the passing trains or wagons.

In this paper we shall deal only with trusses sustaining fixed loads, trusses sustaining moving loads being discussed later.

Weight of Roof Trusses. Before we can design a truss, it is necessary to make an estimate of its own weight; the actual weight can be determined only after the truss is designed. There are a number of formulas for computing the probable weight of a truss, all derived from the actual weights of existing trusses. If W denotes the weight of the truss, l the span or distance between supports in feet and a the distance between adjacent trusses in feet, then for steel trusses

$$W = al\left(\frac{l}{25} + 1\right);$$

and the weight of a wooden truss is somewhat less.

Roof Covering. The beams extending between adjacent trusses to support the roof are called *purlins*. On these there are sometimes placed lighter beams called *rafters* which in turn support *roof boards* or "*sheathing*" and the other covering. Sometimes the purlins are spaced closely, no rafters being used.

The following are weights of roof materials in *pounds per square foot* of roof surface:

Sheathing: Boards, 3 to 5.
Shingling: Tin, 1; wood shingles, 2 to 3; iron, 1 to 3; slate, 10; tiles, 12 to 25.
Rafters: 1.5 to 3.
Purlins: Wood, 1 to 3; iron, 2 to 4.

Snow Loads. The weight of the snow load that may have to be borne depends, of course, on location. It is usually taken from 10 to 30 pounds per square foot of area covered by the roof.

Wind Pressure. Wind pressure per square foot depends on the velocity of the wind and the inclination of the surface on which it blows to the direction of the wind. A horizontal wind blowing at 90 miles per hour produces a pressure of about 40 pounds per square foot on a surface perpendicular to the wind, while on surfaces inclined, the pressures are as follows:

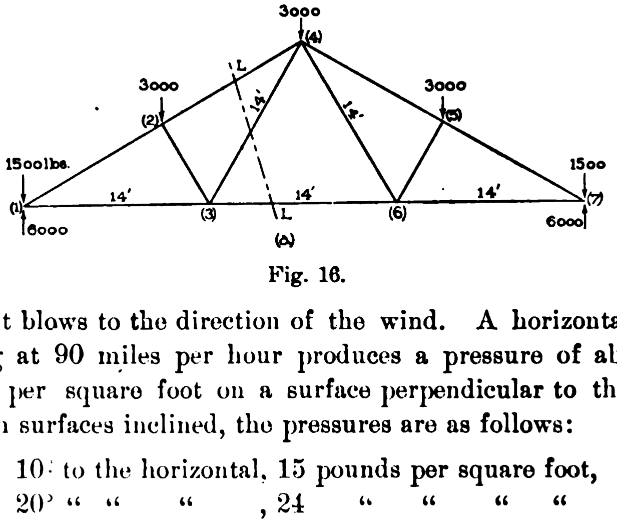

Fig. 16.

10° to the horizontal, 15 pounds per square foot,
20° " " " , 24 " " " "
30° " " " , 32 " " " "
40° " " " , 36 " " " "
50°-90° " " " , 40 " " " "

The wind pressure on an inclined surface is practically perpendicular to the surface.

20. **Computation of "Apex Loads."** The weight of the roof covering including rafters and purlins comes upon the trusses at the points where they support the purlins; likewise the

pressure due to wind and snow. Sometimes all the purlins are supported at joints; in such cases the loads mentioned act upon the truss at its joints. However, the roof, snow, and wind loads are always assumed to be applied to the truss at the upper joints of the trusses. This assumption is equivalent to neglecting the bending effect due to the pressure of those purlins which are not supported at joints. This bending effect can be computed separately.

The weight of the truss itself is assumed to come upon the truss at its upper joints; this, of course, is not exactly correct. Most of the weight does come upon the upper joints for the upper members are much heavier than the lower and the assumption is in most cases sufficiently correct.

Examples. 1. It is required to compute the apex loads for the truss represented in Fig. 16, it being of steel, the roof such that it weighs 15 pounds per square foot, and the distance between adjacent trusses 14 feet.

The span being 42 feet, the formula for weight of truss (Art. 19) becomes

$$14 \times 42\left(\frac{42}{25} + 1\right) = 1{,}575.84 \text{ pounds.}$$

The length $\overline{14}$ scales about 24¼ feet, hence the area of roofing sustained by one truss equals

$$48\tfrac{1}{2} \times 14 = 679 \text{ square feet,}$$

and the weight of the roofing equals

$$679 \times 15 = 10{,}185 \text{ pounds.}$$

The total load equals

$$1{,}575.84 + 10{,}185 = 11{,}760.84 \text{ pounds.}$$

Now this load is to be proportioned among the five upper joints, but joints numbered (1) and (7) sustain only one-half as much load as the others. Hence for joints (1) and (7) the loads equal

$$\frac{1}{8} \text{ of } 11{,}760 = 1{,}470,$$

and for (2), (4) and (5) they equal

$$\frac{1}{4} \text{ of } 11{,}760 = 2{,}940 \text{ pounds.}$$

and inclined members in Fig. 15 are in tension, they exert pulls on the joint, and if the vertical is a compression member, it exerts a push on the joint as indicated. *The forces acting at a joint are therefore concurrent and their lines of action are always known.*

23. **General Method of Procedure.** The forces acting at a joint constitute a system in equilibrium, and since the forces are concurrent and their lines of action are all known, we can determine the magnitude of two of the forces if the others are all known; for this is the important problem mentioned in Art. 16 which was illustrated there and in Art. 17.

Accordingly, after the loads and reactions on a truss, which is to be analyzed, have been ascertained*, we look for a joint at which only two members are connected (the end joints are usually such). Then we consider the forces at that joint and determine the two unknown forces which the two members exert upon it by methods explained in Arts. 16 or 17. The forces so ascertained are the direct stresses, or stresses, as we shall call them for short, and *they are the values of the pushes or pulls which those same members exert upon the joints at their other ends.*

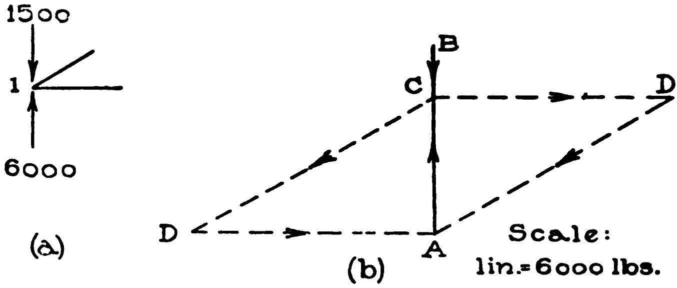

Fig. 17.

Next we look for another joint at which but two unknown forces act, then determine these forces, and continue this process until the stress in each member has been ascertained. We explain further by means of

Examples. 1. It is desired to determine the stresses in the members of the steel truss, represented in Fig. 16, due to its own weight and that of the roofing assumed to weigh 12 pounds per square foot. The distance between trusses is 14 feet.

---

* How to ascertain the values of the reactions is explained in Art. 37. For the present their values in any given case are merely stated.

The apex loads for this case were computed in Example 1, Art. 20, and are marked in Fig. 16. Without computation it is plain that each reaction equals one-half the total load, that is, $\frac{1}{2}$ of 12,000, or 6,000 pounds.

The forces at joint (1) are four in number, namely, the left reaction (6,000 pounds), the load applied there (1,500 pounds), and the forces exerted by members $\overline{12}$ and $\overline{13}$. For clearness, we represent these forces so far as known in Fig. 17 (*a*); we can determine the two unknown forces by merely constructing a closed force polygon for all of them. To construct the polygon, we first represent the known forces; thus AB (1 inch long with arrowhead pointing up) represents the reaction and BC ($\frac{1}{4}$ inch long with arrowhead pointing down) represents the load. Then from A and C we draw lines parallel to the two unknown forces and mark their intersection D (or D′). Then the polygon is ABCDA, and CD (1.5 inches = 9,000 pounds) represents the force exerted by the member $\overline{12}$ on the joint and DA (1.3 inches = 7,800 pounds) represents the force exerted by the member $\overline{13}$ on the joint. The arrowheads on BC and CD must point as shown, in order that all may point the same way around, and hence the force exerted by member $\overline{12}$ acts toward the joint and is a push, and that exerted by $\overline{13}$ acts away from the joint and is a pull. It follows that $\overline{12}$ is in compression and $\overline{13}$ in tension.

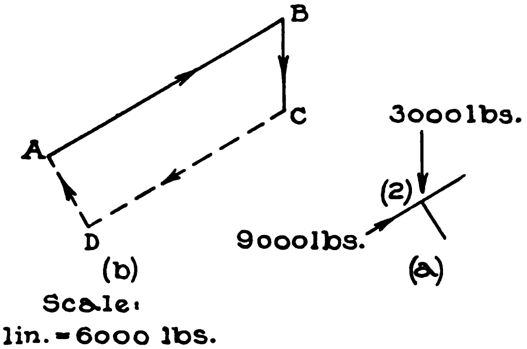

Fig. 18.

If D′ be used, the same results are reached, for the polygon is ABCD′A with arrowheads as shown, and it is plain that CD′ and DA also D′A and CD are equal and have the same sense. But one of these force polygons is preferable for reasons explained later.

Since $\overline{12}$ is in compression, it exerts a push (9,000 pounds) on joint (2) as represented in Fig. 18 (*a*), and since $\overline{13}$ is in tension it exerts a pull (7,800 pounds) on joint (3) as represented in Fig. 19 (*a*).

The forces at joint (2) are four in number, the load (3,000 pounds), the force 9,000 pounds, and the force exerted upon it by the members $\overline{24}$ and $\overline{23}$; they are represented as far as known in Fig. 18 (*a*). We determine the unknown forces by constructing a closed polygon for all of them. Representing the known forces first, draw AB (1.5 inches long with arrowhead pointing up) to represent the 9,000 pound force and BC (½ inch long with arrowhead pointing down) to represent the load of 3,000 pounds. Next from A and C draw lines parallel to the two unknown forces and mark their intersection D; then the force polygon is ABCDA and the arrowheads on CD and DA must point as shown. CD (1.25 inches = 7,500 pounds) represents the force exerted on joint (2) by $\overline{24}$; since it acts toward the joint the force is a push and member $\overline{24}$ is in compres ion. DA (0.43 inches = 2,580 pounds) represents the force exerted on the joint by member $\overline{23}$; since the force acts toward the joint it is a push and the member is in compression. Member 23 therefore exerts a push on joint (3) as shown in Fig. 19 (*a*).

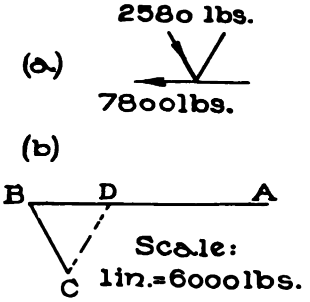

Fig. 19.

At joint (3) there are four forces, 7,800 pounds, 2,580 pounds, and the forces exerted on the joint by members $\overline{34}$ and $\overline{36}$. To determine these, construct the polygon for the four forces. Thus, AB (1.3 inches long with arrowhead pointing to the left) represents the 7,800-pound force and BC (0.43 inches long with arrowheads pointing down) represents the 2,580-pound force. Next draw from A and C two lines parallel to the unknown forces and mark their intersection D; then the force polygon is ABCDA and the arrowhead on CD and DA must point upward and to the right respectively. CD (0.43 inches = 2,580 pounds) represents the force exerted on the joint by member $\overline{34}$; since the force acts away from the joint it is a pull and the member is in tension. DA (0.87 inches = 5,220 pounds) represents the force exerted upon the joint by the member $\overline{36}$; since the force acts away from the joint, it is a pull and the member is in tension.

We have now determined the amount and kind of stress in members $\overline{12}$, $\overline{13}$, $\overline{23}$, $\overline{24}$, $\overline{34}$ and $\overline{36}$. It is evident that the stress in each of the members on the right-hand side is the same as the stress in the corresponding one on the left-hand side; hence further analysis is unnecessary.

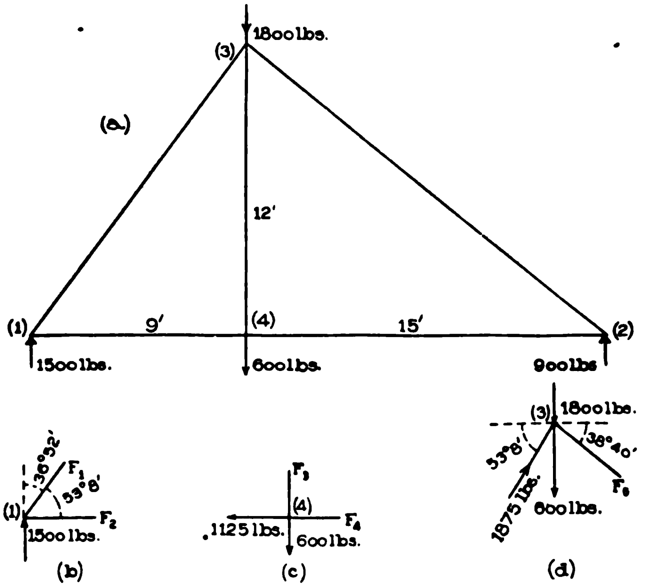

Fig. 20.

2. It is required to analyze the truss represented in Fig. 20 (*a*), the truss being supported at the ends and sustaining two loads, 1,800 and 600 pounds, as shown. (For simplicity we assumed values of the load; the lower one might be a load due to a suspended body. We shall solve algebraically.)

The right and left reactions equal 900 and 1,500 pounds as is shown in Example 1, Page 56. At joint (1) there are three forces, namely, the reaction 1,500 pounds and the forces exerted by members 13 and $\overline{14}$, which we will denote by F_1 and F_2 respect ively. The three forces are represented in Fig. 20 (*b*) as far as they are known. These three forces being in equilibrium, their horizontal and their vertical components balance. Since there are but two horizontal components and two vertical components it follows that (for balance of the components) F_1 must act downward and F_2 toward the right. Hence member $\overline{13}$ pushes on the joint and is under compression while member $\overline{14}$ pulls on the joint and is under tension. From the figure it is plain that

the horizontal component of $F_1 = F_1 \cos 53° \, 8' = 0.6 \, F_1$*,
the horizontal component of $F_2 = F_2$,
the vertical component of $F_1 = F_1 \cos 36° \, 52' = 0.8 \, F_1$,
and the vertical component of the reaction = 1,500.

Hence $\quad 0.6 \, F_1 = F_2$, and $0.8 \, F_1 = 1{,}500$;

or, $$F_1 = \frac{1{,}500}{0.8} = 1{,}875 \text{ pounds},$$

and $$F_2 = 0.6 \times 1{,}875 = 1{,}125 \text{ pounds}.$$

Since members $\overline{14}$ and $\overline{13}$ are in tension and compression respectively, 14 pulls on joint (4) as shown in Fig. 20 (*c*) and 13 pushes on joint (3) as shown in Fig. 20 (*d*).

The forces acting at joint (4) are the load 600 pounds, the pull 1,125 pounds, and the forces exerted by members 34 and 24; the last two we will call F_3 and F_4 respectively. The four forces being horizontal or vertical, it is plain without computation that for balance F_4 must be a pull of 1,125 pounds and F_3 one of 600 pounds. Since members 42 and 43 pull on the joint they are both in tension.

Member 43, being in tension, pulls down on joint (3) as shown in Fig. 20 (*d*). The other forces acting on that joint are the load 1,800 pounds, the push 1,875 pounds, the pull 600 pounds, and the force exerted by member $\overline{32}$ which we will call F_5. The only one of these forces having horizontal components are 1,875 and F_5; hence in order that these two components may balance, F_5 must act toward the left. F_5 is therefore a push and the member $\overline{32}$ is under compression.

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* The angles can be computed from the dimensions of the truss; often they can be ascertained easiest by scaling them with a protractor from a large size drawing of the truss.

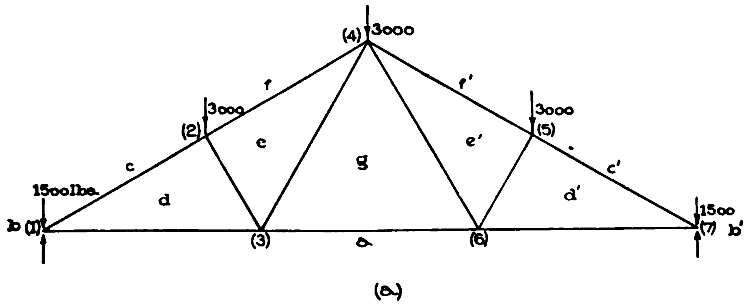
The horizontal component of 1,875 = 1,875 × cos 53° 8′ = 1,125; and the horizontal component of $F_5 = F_5 \times \cos 38° 40' = 0.7808\ F_5$.

Hence $$0.7808\ F_5 = 1,125,$$

or, $$F_5 = \frac{1,125}{.7808} = 1,440 \text{ pounds.}$$

(This same truss is analyzed graphically later.)

24. **Notation for Graphical Analysis of Trusses.** The notation described in Art. 3 can be advantageously systematized in this connection as follows: Each triangular space in the diagram of the truss and the spaces between consecutive lines of action of the loads and reactions should be marked by a small letter (see Fig. 21 *a*). Then the two letters on opposite sides of any line serve to denote that line and the same large letters are used to denote the force acting in that line. Thus *cd* (Fig. 21 *a*) refers to the member $\overline{12}$ and CD should be used to stand for the force or stress in that member.



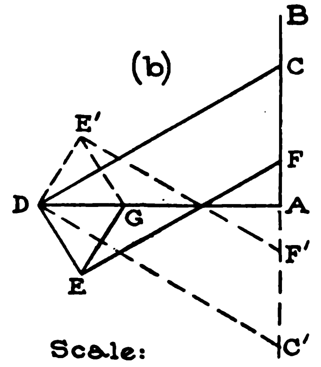
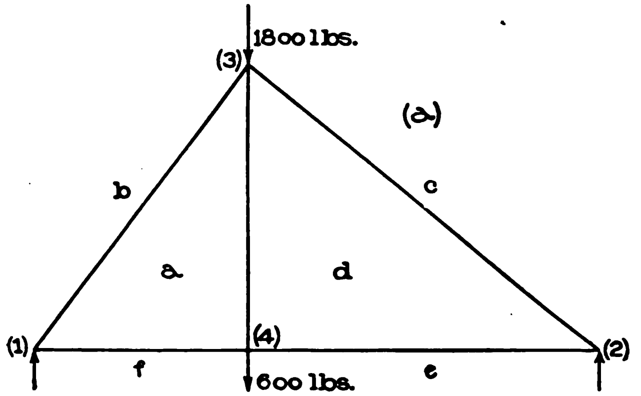


Fig. 21.

25. **Polygon for a Joint.** In drawing the polygon for all the forces at a joint, it is advantageous to represent the forces in the order in which they occur about the joint. Evidently there are always two possible orders thus (see Fig. 20 *d*) F_5, 600, 1,875, and 1,800 is one order around, and F_5, 1,800, 1,875, and 600 is another. The first is called a clockwise order and the second counter-clockwise.

A force polygon for the forces at a joint in which the forces are represented in either order in which they occur about the joint is called *a polygon for the joint*, and it will be called a clockwise or counter-clockwise polygon according as the order followed is clockwise or counter-clockwise. Thus in Fig. 17 (*a*), ABCDA is a clockwise polygon for joint (1). ABCD'A is a polygon for the *forces* at the joint; it is not a polygon for the *joint* because the order in which the forces are represented in that polygon is not the same as either order in which they occur about the joint.



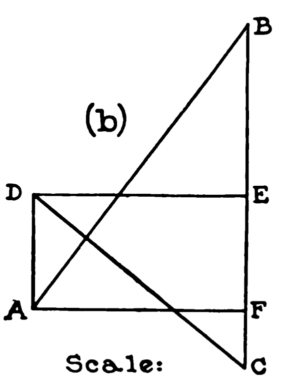


Fig. 22.

(Draw the counter-clockwise polygon for the joint and compare it with ABCDA and ABCD'A.)

26. **Stress Diagrams.** If the polygons for all the joints of a truss are drawn separately as in Example 1, Art. 23, the stress in each member will have been represented twice. It is possible to combine the polygons so that it will be unnecessary to represent the stress in any one member more than once, thus reducing the number of lines to be drawn. Such a combination of force polygons is called a stress diagram.

Fig 21 (*b*) is a stress diagram for the truss of Fig. 21 (*a*) same as the truss of Fig. 16. It will be seen that the part of the stress diagram consisting of solid lines is a combination of separate polygons previously drawn for the joints on the left half of the truss (Figs. 17, 18 and 19.) It will also be seen that the polygons are all clockwise, but counter-clockwise polygons could be combined into a stress diagram.

To Construct a Stress Diagram for a Truss Under Given Loads.

1. Determine the reactions*.
2. Letter the truss diagram as explained in Art. 24.
3. Construct a force polygon for all the forces applied to the truss (loads and reactions) representing them in the order in which they occur around the truss, clockwise or counter-clockwise. (The part of this polygon representing the loads is called a load line.)
4. On the sides of that polygon, construct the polygons for all the joints. They must be clockwise or counter-clockwise according as the polygon for the loads and reactions is clockwise or counter-clockwise. (The first polygon for a joint must be drawn for one at which but two members are connected—the joints at the supports are usually such. Then one can draw in succession the polygons for joints at which there are not more than two unknown forces until the stress diagram is completed.)

Example. It is desired to construct a stress diagram for the truss represented in Fig. 22 (*a*), it being supported at its ends and sustaining two loads of 1,800 and 600 pounds as shown.

The right and left reactions are 900 and 1,500 pounds as is shown in Example 1, Art. 37. Following the foregoing directions we first letter the truss, as shown. Then, where convenient, draw the polygon for all the loads and reactions, beginning with any force, but representing them in order as previously directed. Thus, beginning with the 1,800-pound load and following the clockwise order for example, lay off a line 1.8 inch in length representing 1,800 pounds (scale 1,000 pounds to an inch); since the line of action of the force is *bc*, the line is to be marked BC and B should be placed at the upper end of the line for a reason which will presently appear. The next force to be represented is the right reaction, 900 pounds; hence from C draw a line upward and 0.90 inch long. The line of action of this force being *ce*, the line just drawn should be marked CE and since C is already at the lower end, we mark the upper end E. (The reason for placing B at the upper end of the first line is now apparent.) The next force to be represented is the 600-pound load; therefore we draw from E a line downward and 0.6 inch long, and since the line of action of that force is *ef*, mark the lower end of the line F. The next force to be represented is the left reaction, 1,500 pounds, hence we draw a line 1.5 inches long and upward from F. If the lines have been carefully laid off, the end of the last line should fall at B, that is, the polygon should close.

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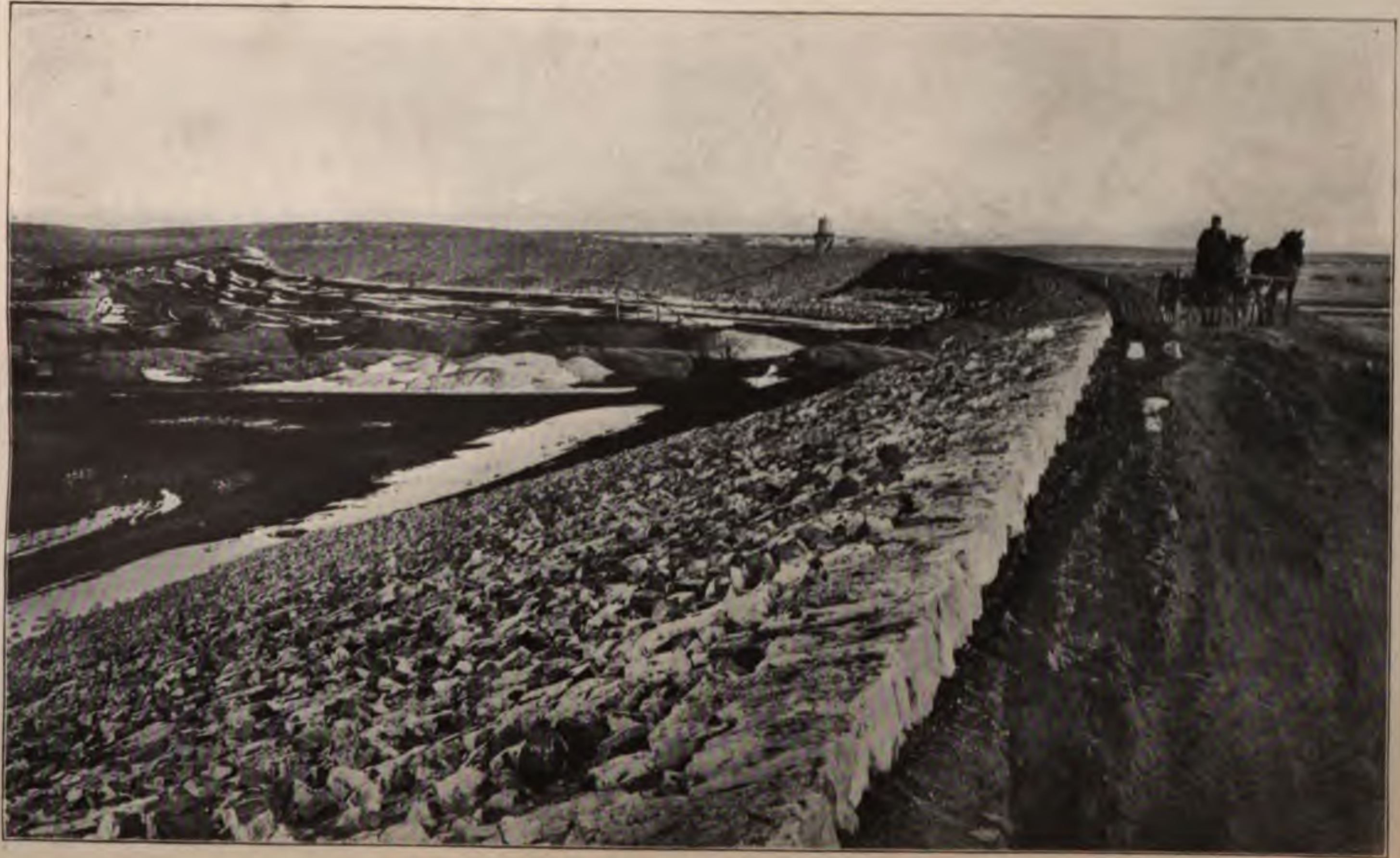
* As already stated, methods for determining reactions are explained in Art. 37; for the present the values of the reactions in any example will be given.

We are now ready to draw polygons for the joints; we may begin at the right or left end as we please but we should bear in mind that the polygons must be clockwise because the polygon for the loads and reactions (BCEFB) is such an one. Beginning at the right end for example, notice that there are three forces there, the right reaction, *de* and *dc*. The right reaction is represented by CE, hence from E draw a line parallel to *de* and from C one parallel to *dc* and mark their intersection D. Then CEDC is the clockwise polygon for the right-hand joint, and since CE acts up, the arrows on ED and DC would point to the left and down respectively. It is better to place the arrows near the joint to which they refer than in the stress diagram; this is left to the student. The force exerted by member *ed* on joint (2) being a pull, *ed* is under tension, and since ED measures 1.12 inches, the value of that tension is 1,120 pounds. The force exerted by member *dc* on joint (2) being a push, *dc* is under compression, and since DC measures 1.44 inches, the value of that compression is 1,440 pounds.

The member *dc* being in compression, exerts a push on the joint (3) and the member *de* being in tension, exerts a pull on the joint (4). Next indicate this push and pull by arrows.

We might now draw the polygon for any one of the remaining joints, for there are at each but two unknown forces. We choose to draw the polygon for the joint (3). There are four forces acting there, namely, the 1,800-pound load, the push (1,440 pounds) exerted by *cd*, and the forces exerted by members *ad* and



PAVING OF HONDO RESERVOIR, NEAR ROSWELL, NEW MEXICO

The early spring flood waters stored in this reservoir are diverted to the irrigation of 10,000 acres near Roswell. Owing to the fact that the bed of this reservoir is largely in gypsum, considerable trouble has been experienced through the characteristic tendency of this formation to develop leaks.

ab, unknown in amount and sense. Now the first two of these forces are already represented in the stress diagram by BC and CD, therefore we draw from D a line parallel to *da* and from B a line parallel to *ba* and mark their intersection A. Then BCDAB is the polygon for the joint, and since the arrowhead on BC and CD would point down and up respectively, DA acts down and AB up; hence place arrowheads in those directions on *da* and *ab* near the joint being considered. These arrows signify that member *da* pulls on the joint and *ba* pushes; hence *da* is in tension and *ba* in compression. Since DA and AB measure 0.6 and 1.88 inches respectively, the values of the tension and compression are 600 and 1,880 pounds.

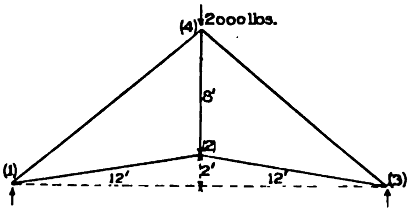


Fig. 23.

Next place arrowheads on *ab* and *ad* at joints (1) and (4) to represent a push and a pull respectively. There remains now but one stress undetermined, that in *af*. It can be ascertained by drawing the polygon for joint 1 or 4; let us draw the latter. There are four forces acting at that joint, namely, the 600-pound load, and the forces exerted by members *ed*, *da*, and *af*. The first three forces are already represented in the drawing by EF, DE and DA, and the polygon for those three forces (not closed) is ADEF. The fourth force must close the polygon, that is, a line from F parallel to *af* must pass through A, and if the drawing has been accurately done, it will pass through A. The polygon for the four forces then is ADEFA, and an arrowhead placed on FA ought to point to the left, but as before, place it in the truss diagram on *af* near joint (4). The force exerted by member *af* on joint (4) being a pull, *af* is under tension, and since AF measures 1.12 inches, the value of the tension is 1,120 pounds.

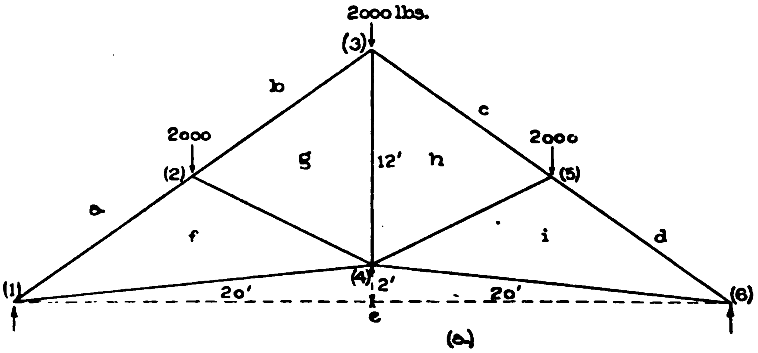
Since *af* is in tension it pulls on joint (1), hence we place an arrowhead on *af* near joint (1) to indicate that pull.

EXAMPLES FOR PRACTICE.

1. Construct a stress diagram for the truss of the preceding Example (Fig. 22*a*) making all the polygons counter-clockwise, and compare with the stress diagram in Fig. 22.

2. Determine the stresses in the members of the truss represented in Fig. 23 due to a single load of 2,000 pounds at the peak.

Ans. Stresses in $\overline{12}$ and $\overline{23}$ = 1,510 pounds,
Stresses in $\overline{14}$ and $\overline{43}$ = 1,930 pounds,
Stress in $\overline{24}$ = 490 pounds.



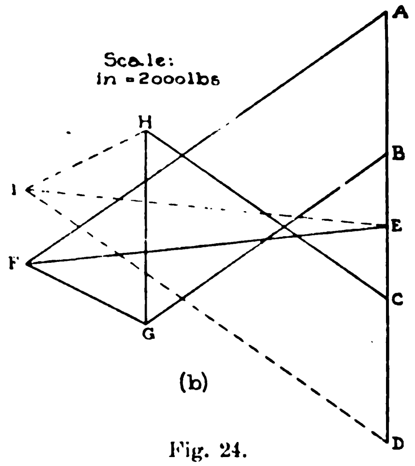


Fig. 24.

27. Stress Records. When making a record of the values of the stresses as determined in an analysis of a truss, it is convenient to distinguish between tension and compression by means of the signs plus and minus. Custom differs as to use of the signs for this purpose, but we shall use *plus for tension and minus for compression*. Thus + 4,560 means a tensile stress of 4,560 pounds, and - 7,500 means a compressive stress of 7,500 pounds.

The record of the stresses as obtained in an analysis can be conveniently made in the form of a table, as in Example 1 following, or in the truss diagram itself, as in Example 2 (Fig. 25).

As previously explained, the stress in a member is tensile or compressive according as the member pulls or pushes on the joints between which it extends. If the arrowheads are placed on the lines representing the members as was explained in Example 1 of Art. 26 (Fig. 22), the two arrowheads on any member

> point toward each other on tension members,
> and from each other on compression members.

If the system of lettering explained in Art. 24 is followed in the analysis of a truss, and if the first polygon (for the loads and reactions) is drawn according to directions (Art. 26), then the system of lettering will guide one in drawing the polygons for the joints as shown in the following illustrations. It must be remembered always that any two parallel lines, one in the truss and one in the stress diagram, must be designated by the same two letters, the first by small letters on opposite sides of it, and the second by the same capitals at its ends.

Examples. 1. It is required to construct a stress diagram for the truss represented in Fig. 24 supported at its ends and sustaining three loads of 2,000 pounds as shown. Evidently the reactions equal 3,000 pounds.

Following the directions of Art. 26, we letter the truss diagram, then draw the polygon for the loads and reactions. Thus, to the scale indicated in Fig. 24 (*b*), AB, BC, and CD represent the loads at joints (2), (3) and (5) respectively and DE and EA represent the right and the left reactions respectively. Notice that the polygon (ABCDEA) is a clockwise one.

At joint (1)there are three forces, the left reaction and the forces exerted by the members *af* and *fe*. Since the forces exerted by these two members must be marked AF and EF we draw from A a line parallel to *af* and from E one parallel to *ef* and mark their intersection F. Then EAFE is the polygon for joint (1), and since EA acts up (see the polygon), AF acts down and FE to the right. We, therefore, place the proper arrowheads on *af* and *fe* near (1) and record (see adjoining table) that the stresses in those members are compressive and tensile respectively. Measuring, we find that AF and FE equal 6,150 and 5,100 pounds respectively.

Member....	af	fe	bg	fg	gh
Stress.. ...	- 6.150	+ 5,100	- 4,100	- 1,875	+2,720

We may next draw the polygon for joint (6) or (2) since there are but two unknown forces at each. At joint (2) for instance, the unknown forces are those exerted by *fg* and *bg*, and the known are the load *ab* (2,000 pounds) and the force exerted by *af*. Since the unknown forces must be marked FG and GB, draw from F a line parallel to *fg*, from B a line parallel to *bg*, and mark their intersection G. Then the polygon for the joint is FABGF, and since AB acts down (see the polygon) (b) BG and GF act down and up respectively. Therefore, place the proper arrowheads on *bg* and *gf* near (2), and record that the stresses in those members are both compressive. Measuring, we find that BG and GF scale 4,100 and 1,875 pounds respectively.

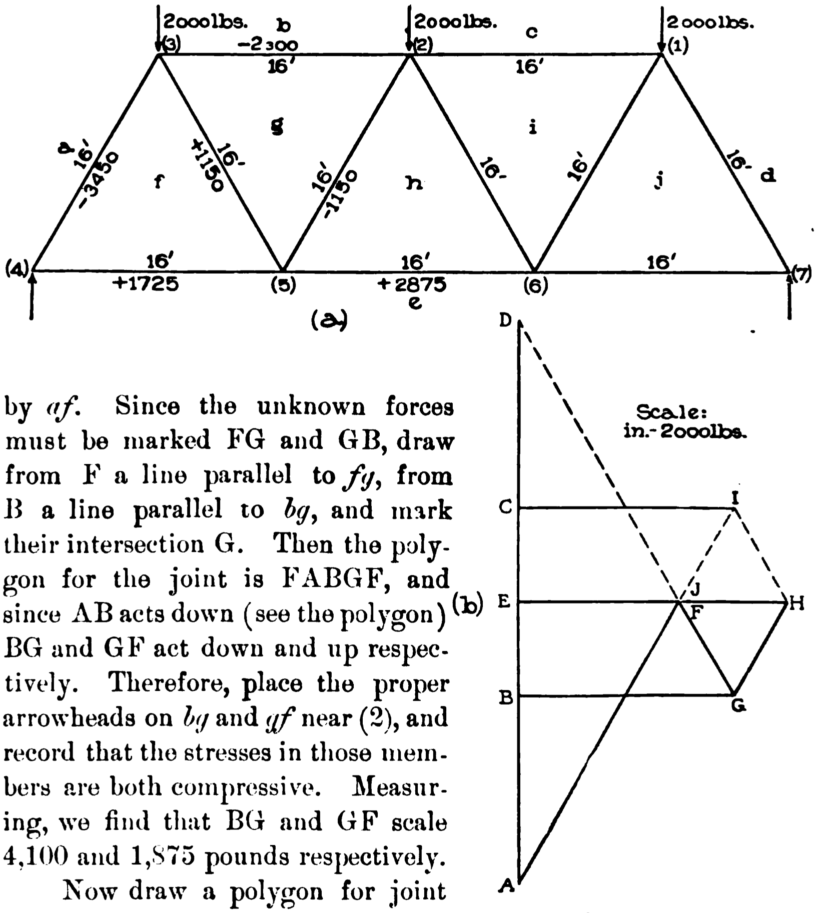


Fig. 25.

Now draw a polygon for joint (3) or (6) since there are but two unknown forces at each joint. At (3) for instance, the unknown forces are those exerted by *ch* and *gh*, the known forces being the load (2,000 pounds) and the force 4,100 pounds, exerted by *bg*. Since the unknown forces must be marked CH and GH, draw from C a line parallel to *ch*, from G one parallel to *gh*, and mark their intersection H. Then

GBCHG is the polygon for the joint, and since BC acts down (see the polygon) CH acts up and HG down. Therefore, place the proper arrowheads on *ch* and *hg* near (3), and record that the stresses in those members are compressive and tensile respectively. Measuring, we find that CH and HG scale 4,100 and 2,720 pounds respectively.

It is plain that the stress in any member on the right-hand side is the same as that in the corresponding member on the left, hence it is not necessary to construct the complete stress diagram.

2. It is required to analyze the truss of Fig. 25 which

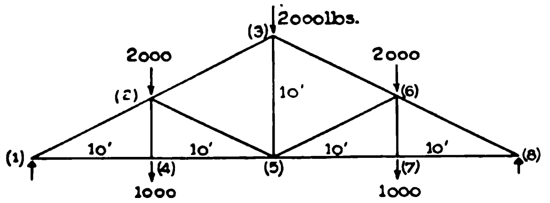


Fig. 26.

rests on end supports and sustains three loads each of 2,000 pounds as shown. Each member is 16 feet long.

Evidently, reactions are each 3,000 pounds. Following directions of Art. 26, first letter the truss diagram and then

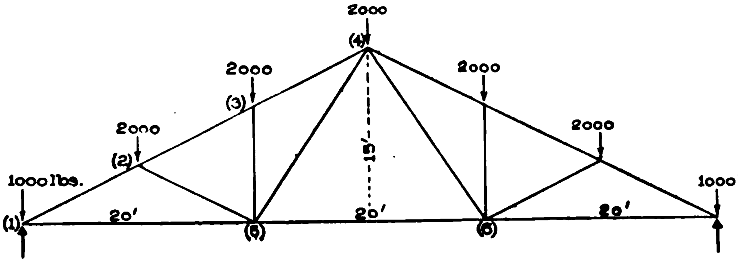


Fig. 27.

draw a polygon for the loads and reactions representing them in either order in which they occur about the truss. DCBAED is a counter-clockwise polygon, DC, CB, and BA representing the loads at joints (1), (2) and (3), AE the left reaction, ED the right reaction.

The construction of the polygons is carried out as in the preceding illustration, and little explanation is necessary. The polygon for joint (4) is AEFA, EF (1,725 pounds tension) representing the stress in *ef* and FA (3,450 pounds compression) that in *af*. The polygon for joint (3) is BAFGB, FG (1,150 pounds tension) representing the stress in *fg* and GB (2,000 pounds compression) that in *gb*. The polygon for joint (5) is GFEHG, EH (2,875 pounds tension) representing the stress in *eh* and HG (1,150 pounds compression) that in *hg*.

Evidently the stress in any member on the right side of the truss is like that in the corresponding member on the left, therefore it is not necessary to construct the remainder of the stress diagram.

EXAMPLES FOR PRACTICE.

1. Analyze the truss represented in Fig. 26, it being supported at its ends and sustaining three loads of 2,000 and two of 1,000 pounds as represented.

STRESS RECORD.

Member......	$\overline{12}$	$\overline{23}$	$\overline{14}$	$\overline{45}$	$\overline{24}$	$\overline{25}$	$\overline{35}$
Stress.........	-8,950	-5,600	+8,000	+8,000	+1,000	-3,350	+3,000

2. Analyze the truss represented in Fig. 27, it being supported at its ends and sustaining five 2,000-pound loads and two of 1,000 as shown.

STRESS RECORD.

Member	$\overline{12}$	$\overline{23}$	$\overline{34}$	$\overline{51}$	$\overline{52}$	$\overline{53}$	$\overline{54}$	$\overline{56}$
Stress...	-11,200	-8,900	-8,900	+10,000	-2,000	-2,000	+4,000	+6,000

28. **Analysis for Snow Loads.** In some cases the apex snow loads are a definite fractional part of the apex loads due to the weights of roof and truss. For instance, in Examples 1 and 2, Pages 25 and 26, it is shown that the apex loads are 1,500 and 3,000 pounds due to weight of roof and truss, and 735 and 1,470 due to snow; hence the snow loads are practically equal to one-half of the permanent dead loads. It follows that the stress in any member due to snow load equals practically one-half of the stress in that member due to the

permanent dead load. The snow load stresses in this case can therefore be obtained from the permanent load stresses and no stress diagram for snow load need be drawn.

In some cases, however, the apex loads due to snow at the various joints are not the *same* fractional part of the permanent load. This is the case if the roof is not all of the same slope, as for instance in Fig. 25 where a part of the roof is flat. In such a case the stresses due to the snow load cannot be determined from a stress diagram for the permanent dead load but a separate stress diagram for the snow load must be drawn. Such diagrams are drawn like those for permanent dead load.

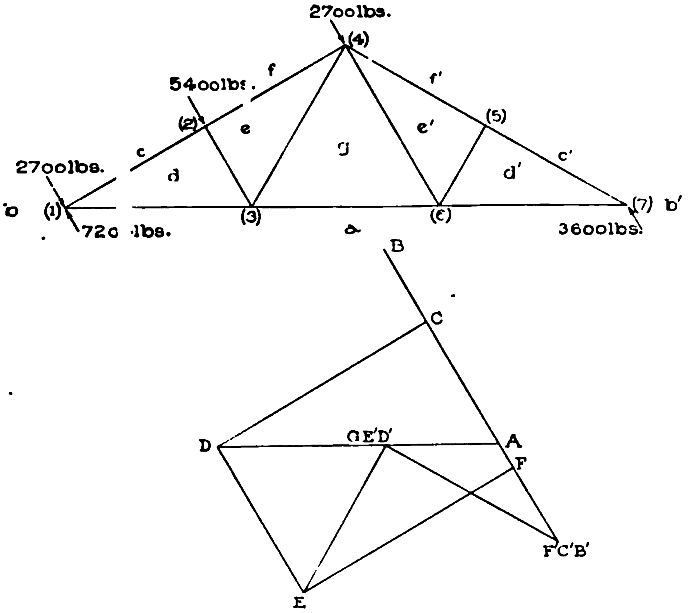


Fig. 28.

29. **Analysis for Wind Loads.** Stresses due to wind pressure cannot be computed from permanent load stresses; they can be most easily determined by means of a stress diagram. Since wind pressure exists only on one side of a truss at a time, the stresses in corresponding members on the right and left sides of a truss are unequal and the whole stress diagram must be drawn in analysis for "wind stresses." Moreover, where one end of the

truss rests on rollers, two stress diagrams must be drawn for a complete analysis, one for wind blowing on the right and one for wind blowing on the left (see Example 2 following).

Examples. 1. It is required to analyze the truss of Fig. 16 for wind pressure, the distance between trusses being 14 feet.

The apex loads for this case are computed in Example 3, Page 26, to be as represented in Fig. 28. Supposing both ends of the truss to be fastened to the supports, then the reactions (due to the wind alone) are parallel to the wind pressure and the right and left reactions equal 3,600 and 7,200 pounds as explained in Example 2, Page 57.

To draw a clockwise polygon for the loads and reactions, we lay off BC, CF, and FF′ to represent the loads at joints (1), (2), and (4) respectively; then since there are no loads at joints (5) and (7) we mark the point F′ by C′ and B′ also; then lay off B′A to represent the reaction at the right end. If the lengths are laid off carefully, AB will represent the reaction at the left end and the polygon is BCFF′C′B′AB.

At joint (1) there are four forces, the reaction, the load, and the two stresses. AB and BC represent the first two forces, hence from C draw a line parallel to *cd* and from A a line parallel to *ad* and mark their intersection D. Then ABCDA is the polygon for the joint and CD and DA represent the two stresses. The former is 7,750 pounds compression and the latter 9,000 pounds tension.

At joint (2) there are four forces, the stress in *cd* (7,750 pounds compression), the load, and the stresses in *fe* and *ed*. As DC and CF represent the stress 7,750 and the load, from F draw a line parallel to *fe* and from D a line parallel to *de*, and mark their intersection E. Then DCFED is the polygon for the joint and FE and ED represent the stresses in *fe* and *ed* respectively. The former is 7,750 pounds and the latter 5,400, both compressive.

At joint (3) there are four forces, the stresses in *ad* (9,000 pounds), *de* (5,400 pounds), *eg* and *ga*. AD and DE represent the first two stresses; hence from E draw a line parallel to *eg* and from A a line parallel to *ag* and mark their intersection G. Then ADEGA is the polygon for the joint and EG and GA represent the stresses in *eg* and *ga* respectively The former is 5,400 and the latter 3,600 pounds, both tensile.

At joint (4) there are five forces, the stresses in *eg* (5,400 pounds) and *ef* (7,750 pounds), the load, and the stresses in *f'e'* and *e'g*. GE, EF and FF' represent the first three forces; hence draw from F' a line parallel to *f'e'* and from G a line parallel to *e'g* and mark their intersection E'. (The first line passes through G, hence E' falls at G). Then the polygon for the joint is GEFF'E'G, and

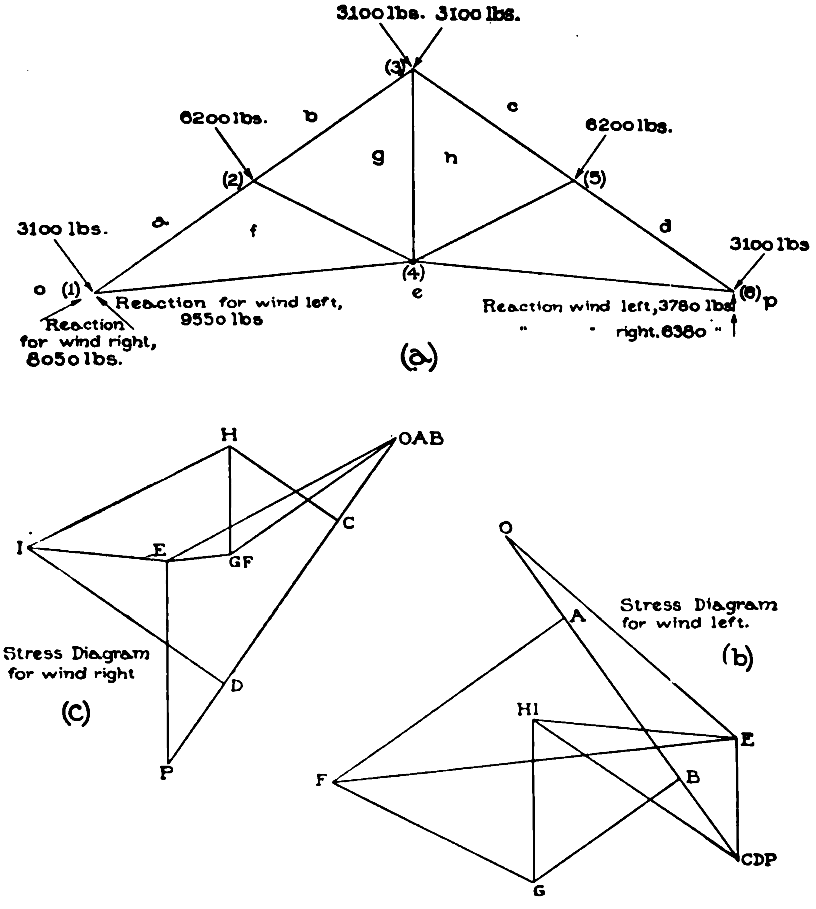

3100 lbs. 3100 lbs.
(3)
6200 lbs.
b
c
6200 lbs.
g
h
(2)
(5)
3100 lbs.
a
f
d
3100 lbs
o (1)
(4)
e
Reaction for wind left, 9550 lbs
Reaction for wind right, 8050 lbs.
Reaction wind left, 3780 lbs
" " right, 6380 "
(6) p
(a)
H
OAB
I
E
GF
C
D
P
Stress Diagram for wind right
(c)
O
A
Stress Diagram for wind left.
(b)
HI
E
F
B
G
CDP

Fig. 29.

F'E' (6,250 pounds compression) represents the stress in *f'e'*. Since E'G = 0, the wind produces no stress in member *ge'*.

At joint (5) three members are connected together and there is no load. The sides of the polygon for the joint must be parallel to the members joined there. Since two of those members are in the same straight line, two sides of the polygon will be parallel and it follows as a consequence that the third side must be zero. Hence the stress in the member $e'd'$ equals zero and the stresses in $f'e'$ and $d'c'$ are equal. This result may be explained slightly differently: Of the stresses in $e'f'$, $e'd'$, and $d'c'$ we know the first (6,250) and it is represented by E'F'. Hence we draw from F' a line parallel to $c'd'$ and one from E' parallel to $d'e$ and mark their intersection D'. Then the polygon for the joint is E'F'C'D'E'; C'D' (6,250 pounds compression) representing the stress in $c'd'$. Since E' and D' refer to the same point, E'D' scales zero and there is no stress in $e'd'$.

The stress in ad' can be determined in various ways. Since at joint (6) there are but two forces (the stresses in ge' and $e'd'$ being zero), the two forces must be equal and opposite to balance. Hence the stress in $d'a$ is a tension and its value is 3,600 pounds.

2. It is required to analyze the truss represented in Fig. 24 for wind pressure, the distance between trusses being 15 feet.

The length $\overline{13}$ equals $\sqrt{20^2 + 14^2}$ or

$$\sqrt{400 + 196} = 24.4 \text{ feet.}$$

Hence the area sustaining the wind pressure to be borne by one truss equals 24.4 × 15 = 366 square feet.

The tangent of the angle which the roof makes with the horizontal equals 14 ÷ 20 = 0.7; hence the angle is practically 35 degrees. According to Art. 19, the wind pressures for slopes of 30 and 40 degrees are 32 and 36 pounds per square foot; hence for 35 degrees it is 34 pounds per square foot. The total wind pressure equals, therefore, 366 × 34 = 12,444, or practically 12,400 pounds.

The apex load for

joint (2) is ½ of 12,400, or 6,200 pounds,
and for joints (1) and (3), ¼ of 12,400, or 3,100 pounds (see Fig. 29).

When the wind blows from the right the

load for joint (5) is 6,200 pounds, and
for joints (3) and (6) 3,100 pounds.

If the left end of the truss is fastened to its support and the right rests on rollers*, when the wind blows on the left side the right and left reactions equal 3,780 and 9,550 pounds respectively and act as shown. When the wind blows on the right side, the right and left reactions equal 6,380 and 8,050 pounds and act as shown. The computation of these reactions is shown in Example 1, Page 58.

For the wind on the left side, OA, AB, and BC (Fig. 29*b*) represent the apex loads at joints (1), (2) and (3) respectively and CE and EO represent the right and left reactions; then the polygon (clockwise) for the loads and reactions is OABCDPEO. The point C is also marked D and P because there are no loads at joints (5) and (6).

The polygon for joint (1) is EOAFE, AF and FE representing the stresses in *af* and *fe* respectively. The values are recorded in the adjoining table. The polygon for joint (2) is FABGF, BG and GF representing the stresses in *bg* and *fg*. The polygon for joint (3) is GBCHG, CH and HG representing the stresses in *ch* and *hg* respectively. At joint (5) there is no load and two of the members connected there are in the same line; hence there is no wind stress in the third member and the stresses in the other two members are equal. The point H is therefore also marked I to make HI equal to zero. The polygon for joint (5) is HCDIH.

STRESS RECORD.

Member.	Stress, Wind Left.	Stress, Wind Right.
af	- 8,850	- 6,300
fe	+12,700	- 2,000
bg	- 5,600	- 6,300
fg	- 7,000	0
hg	+ 5,100	+3,400
hi	0	- 7,000
ch	- 7,700	- 3,100
ie	+ 6,400	+4,400
di	- 7,700	- 7,500

At joint (4) there are four forces, all known except the one in *ie*. EF, FG, and GH represent the first three; hence the line

---

* Rollers to allow for free expansion and contraction of the truss would not be required for one as short as this. They are not used generally unless the truss is 55 feet or more in length.

It remains to compound AD and the remaining one of the given forces, hence we lay off DE to represent the magnitude and direction of the fourth force; then AE represents the magnitude and direction of the resultant of AD and the fourth force (also of the four given forces). The line of action of the resultant is parallel to AE and passes through the point of concurrence of the forces AD and DE. That line should be marked *ae* and the line of action of the fourth face *de*.

It is now plain that the magnitude and direction of the resultant is found exactly as in the case of concurrent forces, but finding the line of action requires an extra construction.

31. **When the Forces Are Parallel or Nearly So,** the method of composition explained must be modified slightly because there is no intersection from which to draw the line of action of the resultant of the first two forces. .

To make such an intersection available, resolve any one of the given forces into two components and imagine that force replaced by them; then find the resultant of those components and the other given forces by the methods explained in the preceding article. Evidently this resultant is the resultant of the given forces.

Example. It is required to find the resultant of the four parallel forces (50, 30, 40, and 60 pounds) represented in Fig. 31 (*a*).

Choosing the 30-pound force as the one to resolve, lay off AB to represent the magnitude and direct'on of that force and mark its line of action *ab*. Next draw lines from A and B intersecting at any convenient point O; then as explained in Art. 13, AO and OB (direction from A to O and O to B) represent the magnitudes and directions of two components of the 30-pound force, and the lines of action of those components are parallel to AO and OB and must intersect on the line of action of that force, as at 1. Draw next two such lines and mark them *ao* and *ob* respectively. Now imagine the 30-pound force replaced by its two components and then compound them with the 50-, 40- and 60-pound forces.

In the composition, the second component should be taken as the first force and the first component as the last. Choosing the 50-pound force as the second, lay off BC to represent the magnitude and direction of that force and mark the line of action *bc*. Then OC (direction O to C) represents the magnitude and direc-

Ans. Stress Record for Wind Left.

Member.	Stress.
12	– 8,600
14	+ 8,600
45	+ 8,600
24	0
23	– 5,000
25	– 6,080
35	+ 2,800
36	– 6,080
56	0
57	+ 4,200
68	– 6,200
67	0
78	+ 4,200

V. COMPOSITION OF NON-CONCURRENT FORCES.

30. Graphical Composition. As in composition of concurrent systems, we first compound any two of the forces by means of the Triangle Law (Art. 9), then compound the resultant of these two forces with the third, then compound the resultant of the first three with the fourth and so on until the resultant of all has been found. It will be seen in the illustration that the actual constructions are not quite so simple as for concurrent forces.

Example. It is required to determine the resultant of the four forces (100, 80, 120, and 60 pounds) represented in Fig. 30 (*a*).

If we take the 100- and 80-pound forces first, and from any convenient point A lay off AB and BC to represent the magnitudes and directions of those forces, then according to the triangle law AC represents the magnitude and direction of their resultant and its line of action is parallel to AC and passes through the point of concurrence of the two forces. This line of action should be marked *ac* and those of the 100- and 80-pound forces, *ab* and *bc* respectively.

If we take the 120-pound force as third, lay off CD to represent the magnitude and direction of that force; then AD represents the magnitude and direction of the resultant of AC and the third force, while the line of action of that resultant is parallel to AD and passes through the point of concurrence of the forces AC and CD. That line of action should be marked *ad* and that of the third force *cd*.

The method of construction may now be described as follows:

1. Draw a force polygon for the given forces. The line drawn from the beginning to the end of the polygon represents the magnitude and direction of the resultant.

2. Select a pole, draw the rays and then the string polygon. The line through the intersection of the first and last strings parallel to the direction of the resultant is the line of action of the resultant. (In constructing the string polygon, observe carefully that the two strings intersecting on the line of action of any one of the given forces are parallel to the two rays which are drawn to the ends of the line representing that force in the force polygon.)

EXAMPLES FOR PRACTICE.

1. Determine the resultant of the 50-, 70-, 80- and 120-pound forces of Fig. 5.

Ans. { 260 pounds acting upwards 1.8 and 0.1 feet
to the right of A and D respectively.

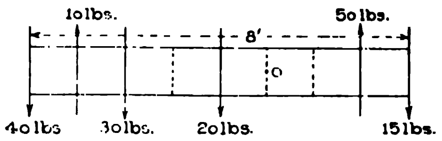


Fig. 32.

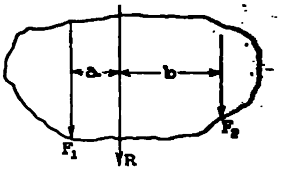


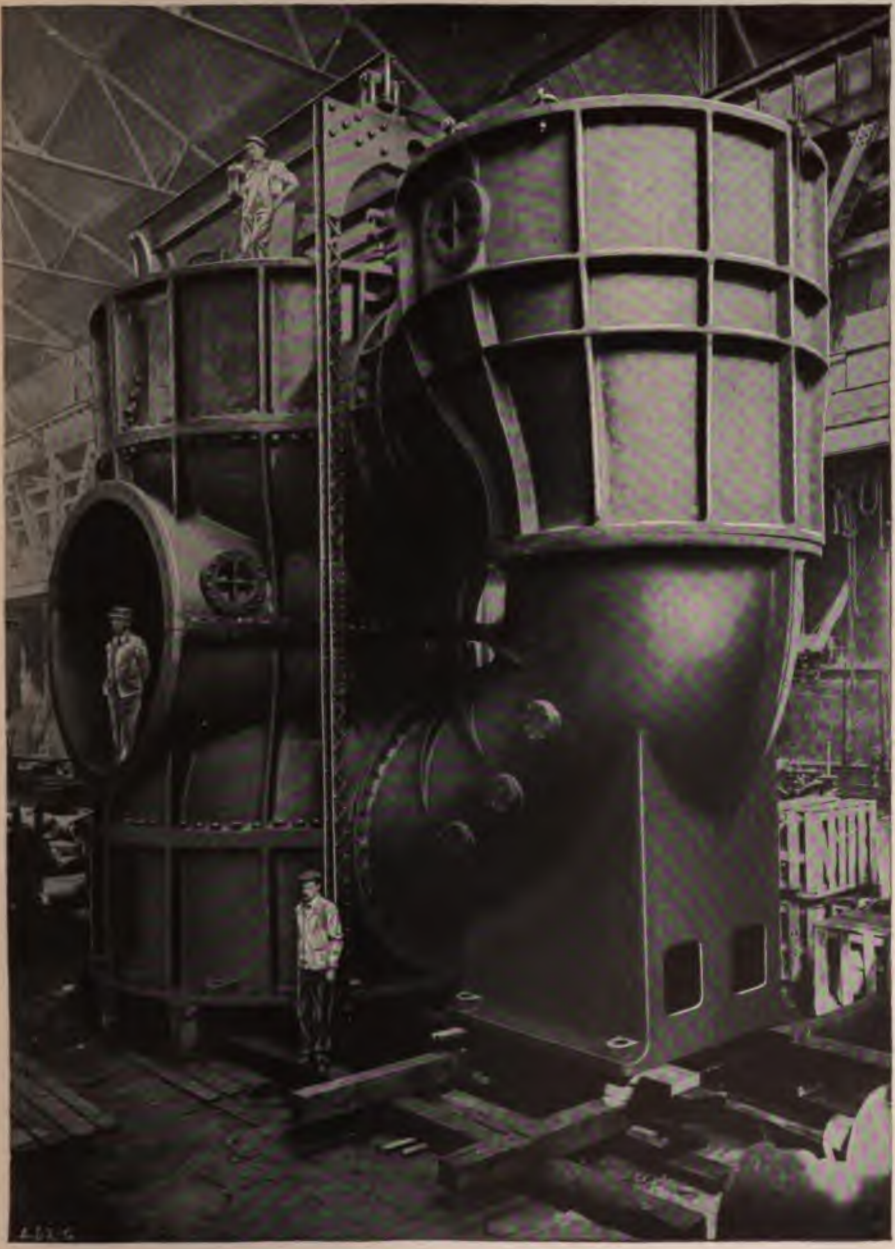
Fig. 33.

2. Determine the resultant of the 40-, 10-, 30- and 20-pound forces of Fig. 32.

Ans. { 80 pounds acting down $1\frac{5}{8}$ feet from left
end.

33. **Algebraic Composition.** The algebraic method of composition is best adapted to parallel forces and is herein explained only for that case.

If the plus sign is given to the forces acting in one direction, and the minus sign to those acting in the opposite direction, the magnitude and sense of the resultant is given by the algebraic sum of the forces; the magnitude of the resultant equals the value of the algebraic sum; the direction of the resultant is given by the sign of the sum, thus the resultant acts in the direction which has been called plus or minus according as the sign of the sum is plus or minus.



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If, for example, we call up plus and down minus, the algebraic sum of the forces represented in Fig. 32 is

$$-40 + 10 - 30 - 20 + 50 - 15 = -45;$$

hence the resultant equals 45 pounds and acts downward.

The line of action of the resultant is found by means of the principle of moments which is (as explained in "Strength of Materials") that *the moment of the resultant of any number of forces about any origin equals the algebraic sum of the moments of the forces.* It follows from the principle that the arm of the resultant with respect to any origin equals the quotient of the algebraic sum of the moments of the forces divided by the resultant; also the line of action of the resultant is on such a side of the origin that the sign of the moment of the resultant is the same as that of the algebraic sum of the moments of the given forces.

For example, choosing O as origin of moments in Fig. 32, the moments of the forces taking them in their order from left to right are

$$-40\times5 = -200,\ +10\times4 = +40,\ -30\times3 = -90,$$
$$-20\times1 = -20,\ -50\times2 = -100,\ +15\times3 = +45.^{*}$$

Hence the algebraic sum equals

$$-200 + 40 - 90 - 20 - 100 + 45 = -325 \text{ foot-pounds.}$$

The sign of the sum being negative, the moment of the resultant about O must also be negative, and since the resultant acts down, its line of action must be on the left side of O. Its actual distance from O equals

$$\frac{325}{45} = 7.22 \text{ feet.}$$

EXAMPLES FOR PRACTICE.

1. Make a sketch representing five parallel forces, 200, 150 100, 225, and 75 pounds, all acting in the same direction and 2 feet apart. Determine their resultant.

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* The student is reminded that when a force tends to turn the body on which it acts in the clockwise direction, about the selected origin, its moment is a given a plus sign, and when counter-clockwise, a minus sign.

Ans. { Resultant = 750 pounds, and acts in the same direction as the given forces and 4.47 feet to the left of the 75-pound force.

2. Solve the preceding example, supposing that the first three forces act in one direction and the last two in the opposite direction.

Ans. { Resultant = 150 pounds, and acts in the same direction with the first three forces and 16.3 feet to the left of the 75-pound force.

Two parallel forces acting in the same direction can be compounded by the methods explained in the foregoing, but it is sometimes convenient to remember that the resultant equals the sum of the forces, acts in the same direction as that of the two forces and between them so that the line of action of the resultant divides the distance between the forces inversely as their magnitudes. For example, let F_1 and F_2 (Fig. 33) be two parallel forces. Then if R denotes the resultant and a and b its distances to F_1 and F_2 as shown in the figure,

$$R = F_1 + F_2,$$

and

$$a : b :: F_2 : F_1.$$

34. Couples. Two parallel forces which are equal and act in opposite directions are called a couple. The forces of a couple cannot be compounded, that is, no single force can produce the same effect as a couple. The perpendicular distance between the lines of action of the two forces is called the *arm*, and the product of one of the forces and the arm is called the *moment of the couple*.

A plus or minus sign is given to the moment ot a couple according as the couple turns or tends to turn the body on which it acts in the clockwise or counter-clockwise direction.

VI. EQUILIBRIUM OF NON-CONCURRENT FORCES.

35. Conditions of Equilibrium of Non-Concurrent Forces Not Parallel may be stated in various ways; let us consider four. First:

1. The algebraic sums of the components of the forces along each of two lines at right angles to each other equal zero.

2. The algebraic sum of the moments of the forces about any origin equals zero.

Second:

1. The sum of the components of the forces along any line equals zero.
2. The sums of the moments of the forces with respect to each of two origins equal zero.

Third:

The sums of the moments of the forces with respect to each of three origins equals zero.

Fourth:

1. The algebraic sum of the moments of the forces with respect to some origin equals zero.
2. The force polygon for the forces closes.

It can be shown that if any one of the foregoing sets of conditions are fulfilled by a system, its resultant equals zero. Hence each is called a set of conditions of equilibrium for a non-concurrent system of forces which are not parallel.

The first three sets are "algebraic" and the last is "mixed," (1) of the fourth, being algebraic and (2) graphical. There is a set of graphical conditions also, but some one of those here given is usually preferable to a set of wholly graphical conditions.

Like the conditions of equilibrium for concurrent forces, they are used to answer questions arising in connection with concurrent systems known to be in equilibrium. Examples may be found in Art. 37.

36. Conditions of Equilibrium for Parallel Non-Concurrent Forces. Usually the most convenient set of conditions to use is one of the following:

First:

1. The algebraic sum of the forces equals zero, and
2. The algebraic sum of the moments of the forces about some origin equals zero.

Second:

The algebraic sums of the moments of the forces with respect to each of two origins equal zero.

37. Determination of Reactions. The weight of a truss, its loads and the supporting forces or reactions are balanced and constitute a system in equilibrium. After the loads and weight are ascertained, the reactions can be determined by means of conditions of equilibrium stated in Arts. 35 and 36.

The only cases which can be taken up here are the following common ones: (1) The truss is fastened to two supports and (2) The truss is fastened to one support and simply rests on rollers at the other.

Case (1) Truss Fastened to Both Its Supports. If the loads are all vertical, the reactions also are vertical. If the loads are not vertical, we assume that the reactions are parallel to the resultant of the loads.

The algebraic is usually the simplest method for determining the reactions in this case, and two moment equations should be used. Just as when finding reactions on beams we first take moments about the right support to find the left reaction and then about the left support to find the right reaction. As a check we add the reactions to see if their sum equals the resultant load as it should.

Examples. 1. It is required to determine the reactions on the truss represented in Fig. 20, it being supported at its ends and sustaining two vertical loads of 1,800 and 600 pounds as shown.

The two reactions are vertical; hence the system in equilibrium consists of parallel forces. Since the algebraic sum of the moments of all the forces about any point equals zero, to find the left reaction we take moments about the right end, and to find the right reaction we take moments about the left end. Thus, if R_1 and R_2 denote the left- and right-reactions respectively, then taking moments about the right end,

$$(R_1 \times 24) - (1800 \times 15) - (600 \times 15) = 0,$$

or $$24R_1 = 27{,}000 + 9{,}000 = 36{,}000;$$

hence $$R_1 = \frac{36{,}000}{24} = 1{,}500 \text{ pounds.}$$

Taking moment about the left end,

$$-R_2 \times 24 + 1{,}800 \times 9 + 600 \times 9 = 0,$$

or $$24R_2 = 16{,}200 + 5{,}400 = 21{,}600;$$

hence $$R_2 = 900 \text{ pounds.}$$

As a check, add the reactions to see if the sum equals that of the loads as should be the case. (It will be noticed that reactions on trusses and beams under vertical loads are determined in the same manner.)

2. It is required to determine the reactions on the truss represented in Fig. 28 due to the wind pressures shown (2,700, 5,400 and 2,700 pounds), the truss being fastened to both its supports.

The resultant of the three loads is evidently a single force of 10,800 pounds, acting as shown in Fig. 34. The reactions are parallel to this resultant; let R_1 and R_2 denote the left and right reactions respectively.

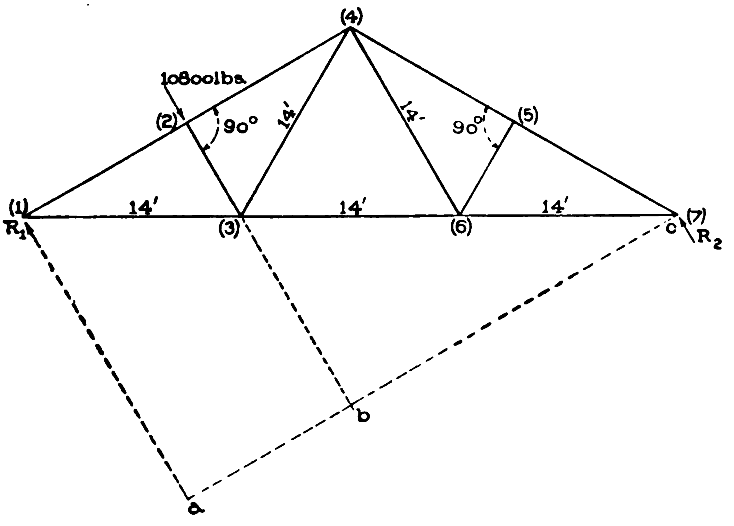

Fig. 34.

The line abc is drawn through the point 7 and perpendicular to the direction of the wind pressure; hence with respect to the right support the arms of R_1 and resultant wind pressure are ac and bc, and with respect to the left support, the arms of R_2 and the resultant wind pressure are ac and ab. These different arms can be measured from a scale drawing of the truss or be computed as follows: The angle $\overline{17a}$ equals the angle $\overline{417}$, and $\overline{417}$ was shown to be 30 degrees in Example 3, Page 26. Hence

$$ab = 14 \cos 30^\circ,\ bc = 28 \cos 30^\circ,\ ac = 42 \cos 30^\circ.$$

Since the algebraic sums of the moments of all the forces acting on the truss about the right and left supports equal zero,

$$R_1 \times 42 \cos 30^\circ = 10{,}800 \times 28 \cos 30^\circ,$$

and $$R_2 \times 42 \cos 30^\circ = 10{,}800 \times 14 \cos 30^\circ.$$

From the first equation,

$$R_1 = \frac{10{,}800 \times 28}{42} = 7{,}200 \text{ pounds},$$

and from the second,

$$R_2 = \frac{10{,}800 \times 14}{42} = 3{,}600 \text{ pounds}.$$

Adding the two reactions we find that their sum equals the load as it should.

Case (2) One end of the truss rests on rollers and the other is fixed to its support. The reaction at the roller end is always vertical, but the direction of the other is not known at the outset unless the loads are all vertical, in which case both reactions are vertical.

When the loads are not all vertical, the loads and the reactions constitute a non-concurrent non-parallel system and any one of the sets of conditions of equilibrium stated in Art. 35 may be used for determining the reactions. In general the fourth set is probably the simplest. In the first illustration we apply the four different sets for comparison.

Examples. 1. It is required to compute the reactions on the truss represented in Fig. 29 due to the wind pressures shown on the left side (3,100, 6,200 and 3,100 pounds), the truss resting on rollers at the right end and being fastened to its support at the left.

(*a*) Let R_1 and R_2 denote the left and right reactions. The direction of R_2 (at the roller end) is vertical, but the direction of R_1 is unknown. Imagine R_1 resolved into and replaced by its horizontal and vertical components and call them R_1' and R_1'' respectively (see Fig. 35.) The six forces, R_1', R_1'', R_2 and the three wind pressures are in equilibrium, and we may apply any one of the sets of statements of equilibrium for this kind of a system (see Art. 35) to determine the reactions. If we choose to use the first set we find, resolving forces along a horizontal line,

$$-R_1' + 3{,}100 \cos 55^\circ + 6{,}200 \cos 55^\circ + 3{,}100 \cos 55^\circ = 0;$$

resolving forces along a vertical line,

$$+R_1'' + R_2 - 3{,}100 \cos 35^\circ - 6{,}200 \cos 35^\circ - 3{,}100 \cos 35^\circ = 0;$$

taking moments about the left end,

$$+ 6{,}200 \times 12.2 + 3{,}100 \times 24.4 - R_2 \times 40 = 0.$$

From the first equation,

$$R_1' = 3{,}100 \cos 55^\circ + 6{,}200 \cos 55^\circ + 3{,}100 \cos 55^\circ = 7{,}113 \text{ pounds},$$

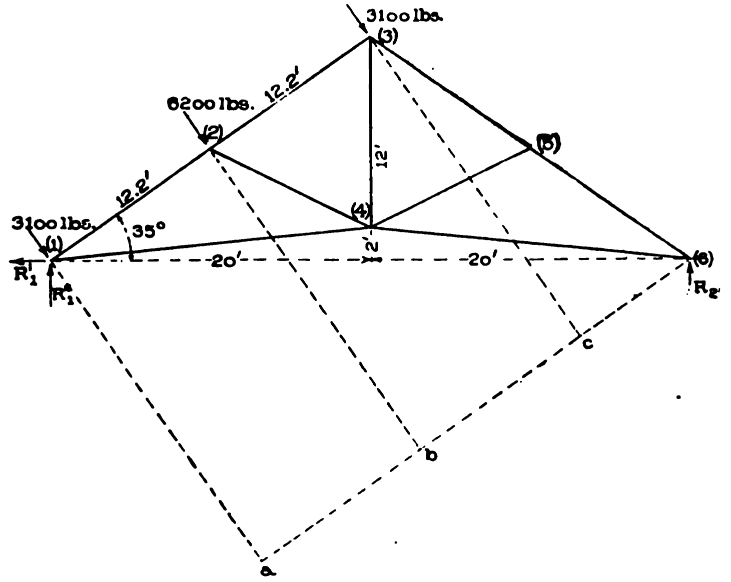

Fig. 35.

and from the third,

$$R_2 = \frac{6{,}200 \times 12.2 + 3{,}100 \times 24.4}{40} = 3{,}782 \text{ pounds}.$$

Substituting this value of R_2 in the second equation we find that

$$R_1'' = 3{,}100 \cos 35^\circ + 6{,}200 \cos 35^\circ + 3{,}100 \cos 35^\circ - 3{,}782$$

$$= 10{,}156 - 3{,}782 = 6{,}374 \text{ pounds}.$$

If desired, the reaction R_1 can now be found by compounding its two components R_1' and R_1''.

(*b*) Using the second set of conditions of equilibrium stated in Art. 35 we obtain the following three "equilibrium equations":
As in (1), resolving forces along the horizontal gives

$$-R_1' + 3{,}100 \cos 55^\circ + 6{,}200 \cos 55^\circ + 3{,}100 \cos 55^\circ = 0,$$

and taking moments about the left end,

$$6{,}200 \times 12.2 + 3{,}100 \times 24.4 - R_2 \times 40 = 0.$$

Taking moments about the right end gives

$$R_1'' \times 40 - 3{,}100 \times \overline{a6} - 6{,}200 \times \overline{b6} - 3{,}100 \times \overline{c6} = 0$$

Just as in (*a*), we find from the first and second equations the values of R_1' and R_2. To find R_1'' we need values of the arms $\overline{a6}$, $\overline{b6}$, and $\overline{c6}$. By measurement from a drawing we find that

$$\overline{a6} = 32.7, \overline{b6} = 20.5, \text{ and } \overline{c6} = 8.3 \text{ feet.}$$

Substituting these values in the third equation and solving for R_1'' we find that

$$R_1'' = \frac{3{,}100 \times 32.7 + 6{,}200 \times 20.5 + 3{,}100 \times 8.3}{40} = 6{,}355 \text{ pounds.}$$

(*c*) Using the third set of conditions of equilibrium stated in Art. 35 we obtain the following three equilibrium equations:
As in (*b*), taking moments about the right and left ends we get

$$R_1'' \times 40 - 3{,}100 \times 32.7 - 6{,}200 \times 20.5 - 3{,}100 \times 8.3 = 0,$$

and $\qquad -R_2 \times 40 + 6{,}200 \times 12.2 + 3{,}100 \times 24.4 = 0.$

Choosing the peak of the truss as the origin of moments for the third equation we find that

$$R_1' \times 14 + R_1'' \times 20 - 3{,}100 \times 24.4 - 6{,}200 \times 12.2 - R_2 \times 20 = 0.$$

As in (*b*) we find from the first two equations the values of R_1'' and R_2. These values substituted in the third equation change it to

$$R_1' \times 14 + 6{,}373 \times 20 - 3{,}100 \times 24.4 - 6{,}200 \times 12.2 - 3{,}782 \times 20 = 0$$

or

$$R_1' = \frac{-6,373 \times 20 + 3,100 \times 24.4 + 6,200 \times 12.2 + 3,782 \times 20}{14}$$

$= 7,104.$*

(*d*) When using the fourth set of conditions we always determine the reaction at the roller end from the moment equation. Then, knowing the value of this reaction, draw the force polygon for all the loads and reactions and thus determine the magnitude and direction of the other reaction.

Taking moments about the left end, we find as in (*a*), (*b*), and (*c*) that $R_2 = 3,782$. Then draw AB, BC and CD (Fig. 36) to represent the wind loads, and DE to represent R_2. Since the force polygon for all the forces must close, EA represents the magnitude and direction of the left reaction; it scales 9,550 pounds.

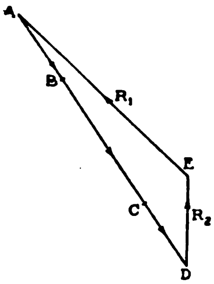

Fig. 36.

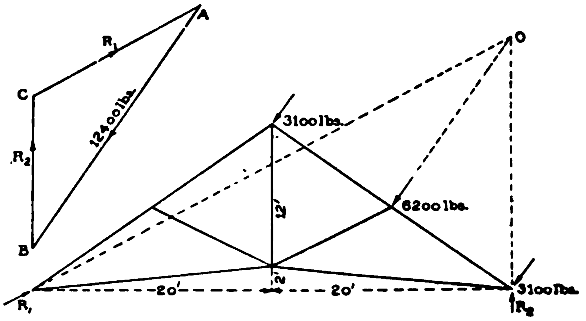

Fig. 37.

2. It is required to determine the reactions on the truss of the preceding illustration when the wind blows from the right.

The methods employed in the preceding illustration might be used here, but we explain another which is very simple. The truss and its loads are represented in Fig. 37. Evidently the resultant of the three wind loads equals 12,400 pounds and acts in the same line with the 6,200-pound load. If we imagine this resultant to replace the three loads we may regard the truss acted upon by three forces, the 12,400-pound force and the reactions, and these three forces as in equilibrium. Now when three forces

---

[*] The slight differences in the answers obtained from the different sets of equilibrium equations are due to inaccuracies in the measured arms of some of the forces.

are in equilibrium they must be concurrent or parallel, and since the resultant load (12,400 pounds) and R_2 intersect at O, the line of action of R_1 must also pass through O. Hence the left reaction acts through the left support and O as shown. We are now ready to determine the values of R_1 and R_2. Lay off AB to represent the resultant load, then from A and B draw lines parallel to R_1 and R_2, and mark their intersection C. Then BC and CA represent the magnitude and directions of R_2 and R_1 respectively; they scale 6,380 and 8,050 pounds.

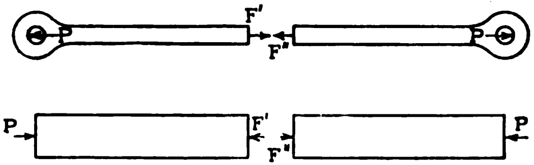

Fig. 38.

EXAMPLE FOR PRACTICE.

1. Determine the reactions on the truss represented in Fig. 26 due to wind pressure, the distance between trusses being 15 feet, supposing that both ends of the truss are fastened to the supports.

Ans. { Reaction at windward end is 6,682½ pounds.
Reaction at leeward end is 3,037½ pounds.

VII. ANALYSIS OF TRUSSES (CONTINUED); METHOD OF SECTIONS.

38. Forces in Tension and Compression Members. As explained in "Strength of Materials" if a member is subjected to forces, any two adjacent parts of it exert forces upon each other which hold the parts together. Figs. 38 (*a*) and 38 (*b*) show how these forces act in a tension and in a compression member. F' is the force exerted on the left part by the right, and F'' that exerted on the right part by the left. The two forces F' and F'' are equal, and in a tension member are pulls while in a compression member they are pushes.

39. Method of Sections. To determine the stress in a member of a truss by the method explained in the foregoing (the "method of joints"), we begin at one end of the truss and draw polygons for joints from that end until we reach one of the joints to which that member is connected. If the member is near the middle of a long truss, such a method of determining the stress in it requires the construction of several polygons. It is sometimes desirable to determine the stress in a member as directly as possible without having first determined stresses in other members. A method for doing this will now be explained; it is called the method of sections.

Fig. 39 (*a*) is a partial copy of Fig. 16. The line LL is intended to indicate a "section" of the truss "cutting" members 24, 34 and 36. Fig. 39 (*b*) and (*c*) represents the parts of the truss to the left and right of the section. By "part of a truss" we mean either of the two portions into which a section separates it when it cuts it completely.

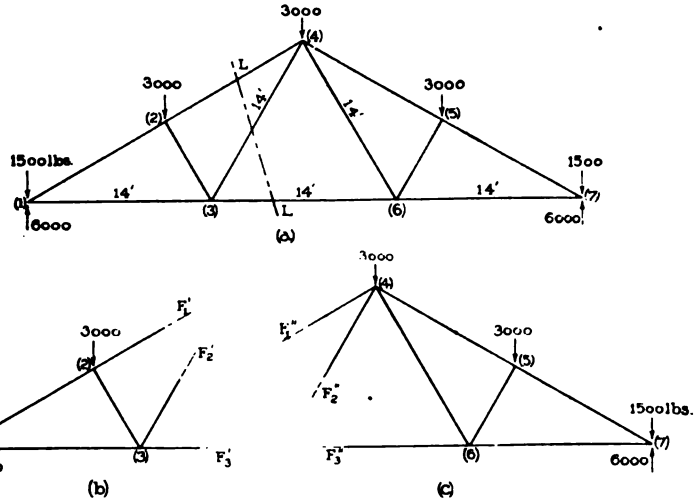

Fig. 39.

Since each part of the truss is at rest, all the forces acting on each part are balanced, or in equilibrium. The forces acting on each part consist of the loads and reactions applied to that part together with the forces exerted upon it by the other part. Thus the forces which hold the part in Fig. 39 (*b*) at rest are the 1,500- and 3,000-pound loads, the reaction 6,000 and the forces which the right part of the cut members exert upon the left parts. These latter forces are marked F_1', F_2' and F_3'; their senses are unknown.

but each acts along the axis of the corresponding member. The forces which hold the part in Fig. 39 (c) at rest are the two 3,000-pound loads, the 1,500-pound load, the right reaction 6,000 pounds and the forces which the left parts of the cut members exert upon the right parts. These are marked F_1'', F_2'' and F_3''; their senses are also unknown but each acts along the axis of the corresponding member. The forces F_1' and F_1'', F_2' and F_2'', and F_3' and F_3'' are equal and opposite; they are designated differently only for convenience.

If, in the system of forces acting on either part of the truss, there are not more than three unknown forces, then those three can be computed by "applying" one of the sets of the conditions of equilibrium stated in Art. 35.* In writing the equations of equilibrium for the system it is practially necessary to assume a sense for one or more of the unknown forces. *We shall always assume that such forces are pulls that is, act away from the part of the truss upon which they are exerted. Then if the computed value of a force is plus, the force is really a pull and the member is in tension and if the value is minus, then the force is really a push and the member is in compression.*

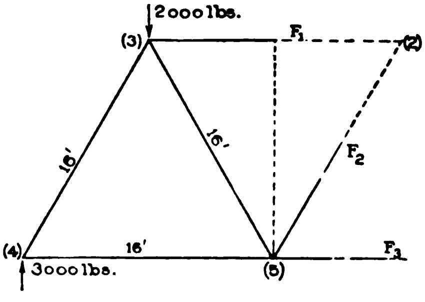

Fig. 40.

To determine the stress in any particular member of a truss in accordance with the foregoing, "pass a section" through the truss cutting the member under consideration, and then apply as many conditions of equilibrium to all the forces acting on either part of the truss as may be necessary to determine the desired force. In passing the section, care should be taken to cut as few members as possible, and never should a section be passed so as to cut more than three, the stresses in which are unknown; neither should the three be such that they intersect in a point.

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*If, however, the lines of action of the three forces intersect in a point then the statement is not true.

Examples. 1. It is required to determine the amount and kind of stress in the member $\overline{24}$ of the truss represented in Fig. 39 (*a*) when loaded as shown.

Having determined the reactions (6,000 pounds each) we pass a section through the entire truss and cutting $\overline{24}$; LL is such a section. Considering the part of the truss to the left of the section, the forces acting upon that part are the two loads, the left reaction and the forces on the cut ends of members $\overline{24}$, $\overline{34}$ and $\overline{36}$ (F_1', F_2', and F_3'). F_1' can be determined most simply by writing a moment equation using (3) as the origin, for with respect to that origin the moments of F_2' and F_3' are zero, and hence those forces will not appear in the equation. Measuring from a large scale drawing, we find that the arm of F_1' is 7 feet and that of the 3,000-pound load is 3.5 feet. Hence

$$(F_1' \times 7) + (6{,}000 \times 14) - (1{,}500 \times 14) - (3{,}000 \times 3.5) = 0$$

$$\text{or } F_1' = \frac{-(6{,}000 \times 14) + (1{,}500 \times 14) + (3{,}000 \times 3.5)}{7} = -7{,}500$$

The minus sign means that F_1' is a push and not a pull, hence the member $\overline{24}$ is under 7,500 pounds compression.

The stress in the member 24 may be computed from the part of the truss to the right of the section. Fig. 39 (*c*) represents that part and all the forces applied to it. To determine F_1'' we take moments about the intersection of F_2'' and F_3''. Measuring from a drawing we find that the arm of F_1'' is 7 feet,

that of the load at joint (4) is 7 feet,
that of the load at joint (5) is 17.5 feet,
that of the load at joint (7) is 28 feet,
and that of the reaction is 28 feet.

Hence, assuming F_1'' to be a pull,

$$-(F_1'' \times 7) + (3{,}000 \times 7) + (3{,}000 \times 17.5) + (1{,}500 \times 28) - (6{,}000 \times 28) = 0,$$

$$\text{or } F_1'' = \frac{(3{,}000 \times 7) + (3{,}000 \times 17.5) + (1{,}500 \times 28) - (6{,}000 \times 28)}{7} = -7{,}500$$

The minus sign means that F_1'' is a push, hence the member $\overline{24}$ is under compression of 7,500 pounds, a result agreeing with that previously found.

2. It is required to find the stress in the member *gh* of the truss represented in Fig. 25, due to the loads shown.

If we pass a section cutting *bg*, *gh* and *hc*, and consider the left part, we get Fig. 40, the forces on that part being the 2,000-pound load, the left reaction, and the forces F_1, F_2 and F_3 exerted by the right part on the left. To compute F_2 it is simplest to use the condition that the algebraic sum of all the vertical components equals zero. Thus, assuming that F_2 is a pull, and since its angle with the vertical is 30°,

$$F_2 \cos 30^\circ - 2{,}000 + 3{,}000 = 0; \text{ or,}$$

$$F_2 = \frac{2{,}000 - 3{,}000}{\cos 30^\circ} = \frac{-1{,}000}{0.866} = -1{,}154.$$

The minus sign means that F_2 is a push, hence the member is under a compression of 1,154 pounds.

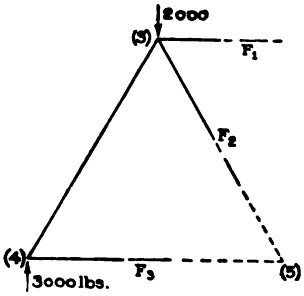

Fig. 41.

3. It is required to determine the stress in the member *bg* of the truss represented in Fig. 25, due to the loads shown.

If we pass a section cutting *bg* as in the preceding illustration, and consider the left part, we get Fig. 40. To compute F_1 it is simplest to write the moment equation for all the forces using joint 5 as origin. From a large scale drawing, we measure the arm of F_1 to be 13.86 feet hence, assuming F_1 to be a pull,

$$F_1 \times 13.86 - 2{,}000 \times 8 + 3{,}000 \times 16 = 0;$$

or,

$$F_1 = \frac{2{,}000 \times 8 - 3{,}000 \times 16}{13.86} = \frac{-32{,}000}{13.86} = -2{,}309.$$

The minus sign means that F_1 is a push; hence the member is under a compression of 2,308 pounds.

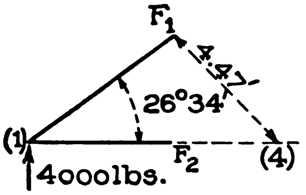

Fig. 42.

The section might have been passed so as to cut members *bg*, *fg*, and *fe*, giving Fig. 41 as the left part, and the desired force might be obtained from the system of forces acting on that part (3,000, 2,000, F_1, F_2, and F_3.) To compute F_1 we take moments about the intersection of F_2 and F_3, thus

$$F_1 \times 13.86 - 2{,}000 \times 8 + 3{,}000 \times 16 = 0.$$

This is the same equation as was obtained in the first solution, and hence leads to the same result.

4. It is required to determine the stress in the member $\overline{12}$ of the truss represented in Fig. 26, due to the loads shown.

Passing a section cutting members $\overline{12}$ and $\overline{14}$, and considering the left part, we get Fig. 42. To determine F_1 we may write a moment equation preferably with origin at joint 4, thus:

$$F_1 \times 4.47 + 4{,}000 \times 10 = 0^*;$$

or,
$$F_1 = \frac{-4{,}000 \times 10}{4.47} = -8{,}948 \text{ pounds},$$

the minus sign meaning that the stress is compressive.

F_1 might be determined also by writing the algebraic sum of the vertical components of all the forces on the left part equal to zero, thus:

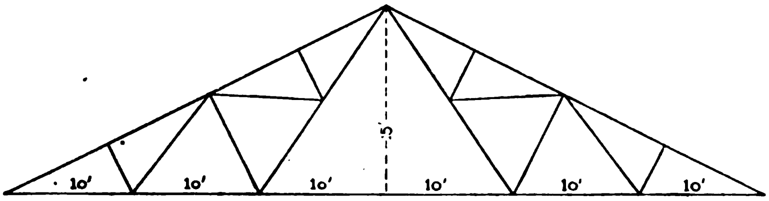

Fig. 43.

$$F_1 \sin 26^\circ\, 34' + 4{,}000 = 0;$$

or,
$$F_1 = \frac{-4{,}000}{\sin 26^\circ\, 34'} = \frac{-4{,}000}{0.447} = -8{,}948,$$

agreeing with the first result.

EXAMPLES FOR PRACTICE.

1. Determine by the method of sections the stresses in members $\overline{23}$, $\overline{25}$, and $\overline{45}$ of the truss represented in Fig. 26, due to the loads shown.

Ans. $\begin{cases} \text{Stress in } \overline{23} = -5{,}600 \text{ pounds}, \\ \text{Stress in } \overline{25} = -3{,}350 \text{ pounds}, \\ \text{Stress in } \overline{45} = +8{,}000 \text{ pounds}. \end{cases}$

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[*] The arm of F_1 with respect to (4) is 4.47 feet.

2. Determine the stresses in the members $\overline{12}$, $\overline{15}$, $\overline{34}$, and $\overline{56}$ of the truss represented in Fig. 27, due to the loads shown.

$$\text{Ans.}\begin{cases}\text{Stress in } \overline{12} = -\,11{,}170 \text{ pounds,}\\ \text{Stress in } \overline{15} = +\,10{,}000 \text{ pounds,}\\ \text{Stress in } \overline{34} = -\,8{,}940 \text{ pounds,}\\ \text{Stress in } \overline{56} = +\,6{,}000 \text{ pounds.}\end{cases}$$

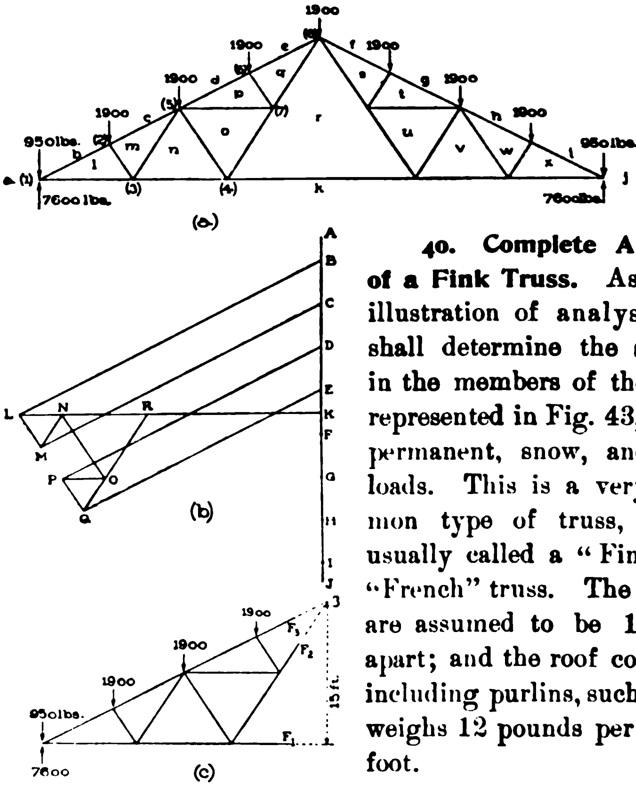

Fig. 44.

40. Complete Analysis of a Fink Truss. As a final illustration of analysis, we shall determine the stresses in the members of the truss represented in Fig. 43, due to permanent, snow, and wind loads. This is a very common type of truss, and is usually called a "Fink" or "French" truss. The trusses are assumed to be 15 feet apart; and the roof covering, including purlins, such that it weighs 12 pounds per square foot.

The length from one end to the peak of the truss equals

$$\sqrt{15^2 + 30^2} = \sqrt{1{,}125} = 33.54 \text{ feet,}$$

hence the area of the roofing sustained by one truss equals

$$(33.54 \times 15)\ 2 = 1{,}006.2 \text{ square feet,}$$

and the weight of that portion of the roof equals

$$1{,}006.2 \times 12 = 12{,}074 \text{ pounds.}$$

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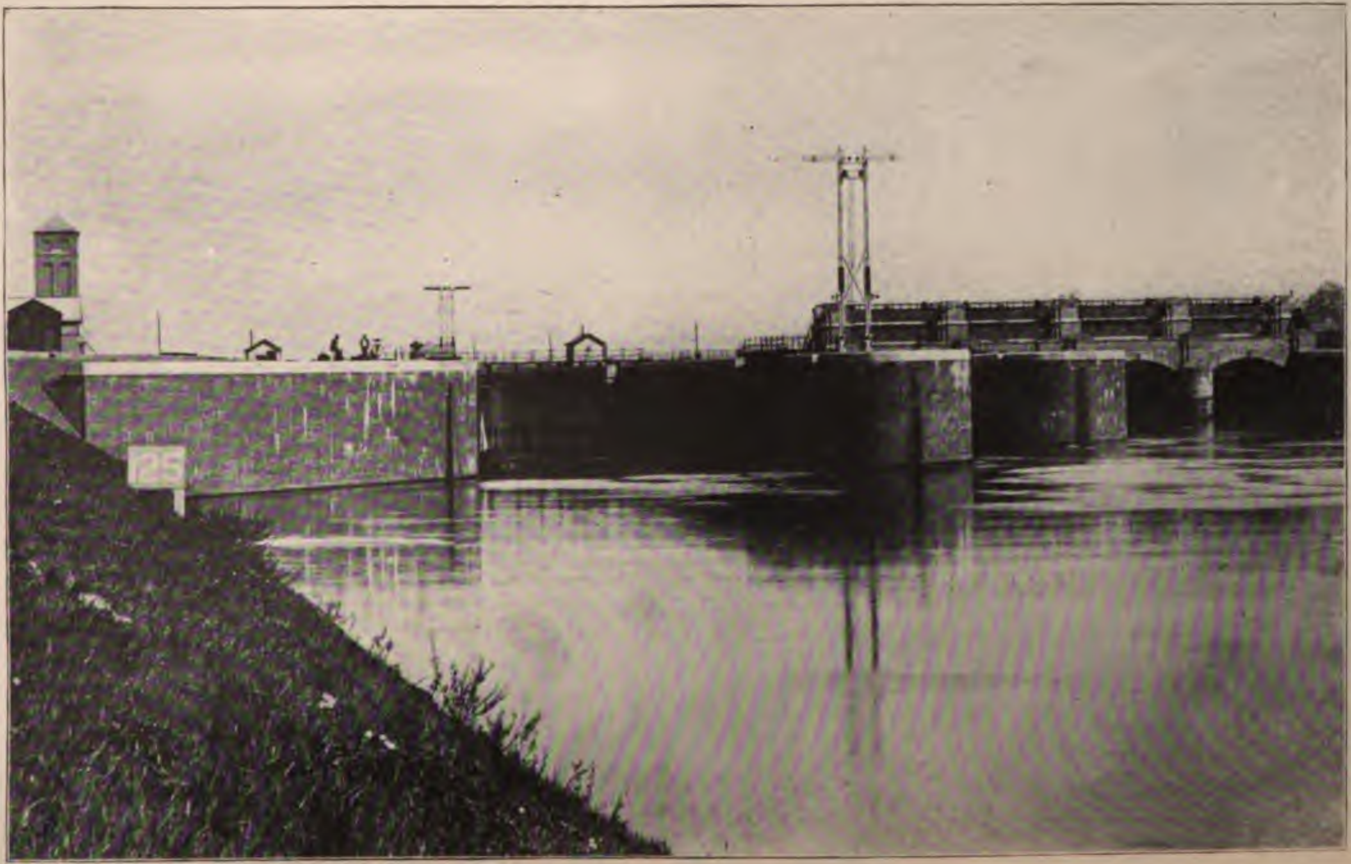


BARTON LOCKS ON MANCHESTER SHIP CANAL

This canal, opened May 21, 1894, made Manchester, the chief manufacturing city of England, a seaport, giving it direct connection with the River Mersey and the port of Liverpool.

The probable weight of the truss (steel), according to the formula of Art. 19, is

$$15 \times 60 \left(\frac{60}{25} + 1\right) = 3{,}060 \text{ pounds.}$$

The total permanent load, therefore, equals

$$12{,}074 + 3{,}060 = 15{,}134 \text{ pounds;}$$

the end loads equal $\frac{1}{16}$ of the total, or 950 pounds, and the other apex loads equal $\frac{1}{8}$ of the total, or 1,900 pounds.

Dead Load Stress. To determine the dead load stresses, construct a stress diagram. Evidently each reaction equals one-half the total load, that is 7,600 pounds; therefore ABCDEFGHIJKA (Fig. 44*b*) is a polygon for all the loads and reactions. First, we draw the polygon for joint 1; it is KABLK, BL and LK representing the stress in *bl* and *lk* (see record Page 72 for values). Next draw the polygon for joint 2; it is LBCML, CM and ML representing the stresses in *cm* and *ml*. Next draw the polygon for joint 3; it is KLMNK, MN and NK representing the stresses in *mn* and *nk*.

At each of the next joints (4 and 5), there are three unknown forces, and the polygon for neither joint can be drawn. We might draw the polygons for the joints on the right side corresponding to 1, 2, and 3, but no more until the stress in one of certain members is first determined otherwise. If, for instance, we determine by other methods the stress in *rk*, then we may construct the polygon for joint 4; then for 5, etc., without further difficulty.

To determine the stress in *rk*, we pass a section cutting *rk*, *qr*, and *eq*, and consider the left part (see Fig. 44*c*). The arms of the loads with respect to joint 8 are 7.5, 15, 22.5, and 30 feet; and hence, assuming F_1 to be a pull,

$$-F_1 \times 15 - 1{,}900 \times 7.5 - 1{,}900 \times 15 - 1{,}900 \times 22.5 - 950 \times 30 + 7{,}600 \times 30 = 0; \text{ or,}$$

$$F_1 = \frac{-1{,}900 \times 7.5 - 1{,}900 \times 15 - 1{,}900 \times 22.5 - 950 \times 30 + 7{,}600 \times 30}{15}$$

$$= 7{,}600 \text{ pounds}$$

Since the sign of F_1 is plus, the stress in *rk* is tensile.

Now lay off KR to represent the value of the stress in *kr* just found, and then construct the polygon for joint 4. The polygon is KNORK, NO and OR representing the stresses in *no* and *or*. Next draw the polygon for joint 5; it is ONMCDPO, DP and PO representing the stresses in *dp* and *po*. Next draw the

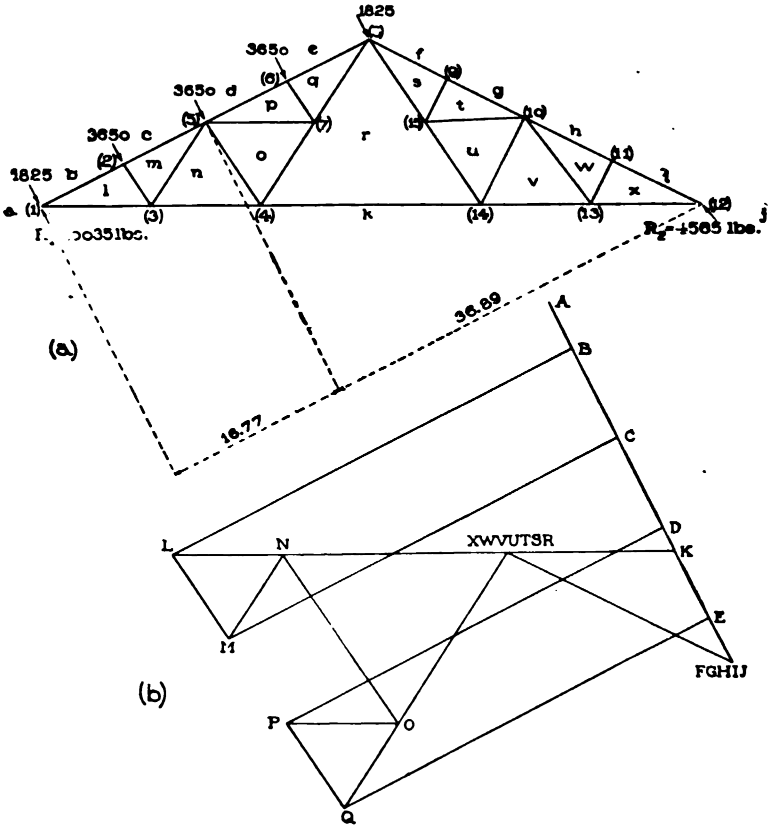


Fig. 45.

polygon for joint 6 or joint 7; for 6 it is PDEQP, EQ and QP representing the stresses in *eq* and *qp*. At joint 7 there is now but one unknown force, namely, that in *qr*. The polygon for the three others at that joint is ROPQ; and since the unknown force must close the polygon, QR must represent that force, and must be parallel to *qr*.

On account of the symmetry of loading, the stress in any member on the right side is just like that in the corresponding member on the left; hence, it is not necessary to draw the diagram for the right half of the truss.

Snow-Load Stress. The area of horizontal projection of the roof which is supported by one truss is 60 × 15 = 900 square feet; hence the snow load borne by one truss is 900 × 20 = 18,000 pounds, assuming a snow load of 20 pounds per horizontal square foot. This load is nearly 1.2 times the dead load, and is applied similarly to the latter; hence the snow load stress in any member equals 1.2 times the dead load stress in it. We record, therefore, in the third column of the stress record, numbers equal to 1.2 times those in the second as the snow-load stresses.

Wind Load Stress. The tangent of the angle which the roof makes with the horizontal equals $\frac{15}{30}$ or $\frac{1}{2}$; hence the angle is 26° 34′, and the value of wind pressure for the roof equals practically 29 pounds per square foot, according to Art. 19. As previously explained, the area of the roof sustained by one truss equals 1,006.2 square feet; and since but one-half of this receives wind pressure at one time, the wind pressure borne by one truss equals

$$503.1 \times 29 = 14{,}589.9, \text{ or practically } 14{,}600 \text{ pounds.}$$

When the wind blows from the left, the apex loads are as represented in Fig. 45*a*, and the resultant wind pressure acts through joint 5. To compute the reactions, we may imagine the separate wind pressures replaced by their resultant. We shall suppose that both ends of the truss are fixed; then the reactions will be parallel to the wind pressure. Let R_1 and R_2 denote the left and right reactions respectively; then, with respect to the right end, the arms of R_1 and the resultant wind pressure (as may be scaled from a drawing) are 16.77 + 36.89 and 36.89 feet respectively; and with respect to the left end, the arms of R_2 and the resultant wind pressure are 16.77 + 36.89 and 16.77 feet respectively.

Taking moments about the right end we find that

$$-14{,}600 \times 36.89 + R_1 \times (16.77 + 36.89) = 0;$$

or,

$$R_1 = \frac{14{,}600 \times 36.89}{16.77 + 36.89} = 10{,}035 \text{ pounds.}$$

Taking moments about the left end, we find that

$$14{,}600 \times 16.77 - R_2 \times (16.77 + 36.89) = 0;$$

or,
$$R_2 = \frac{14{,}600 \times 16.77}{16.77 + 36.89} = 4{,}565 \text{ pounds.}$$

To determine the stresses in the members, we construct a stress diagram. In Fig. 45*b*, AB, BC, CD, DE, and EF represent the wind loads at the successive joints, beginning with joint 1. The point F is also marked G, H, I, and J, to indicate the fact that there are no loads at joints 9, 10, 11, and 12. JK represents the right reaction, and KA the left reaction.

We may draw the polygon for joint 1 or 12; for 1 it is KABLK, BL and LK representing the stresses in *bl* and *lk*. We may next draw the polygon for joint 2; it is LBCML, CM and ML representing the stresses in *cm* and *ml*.

Stress Record.

MEMBER.	STRESSES.					
	Dead Load.	Snow Load.	Wind Left.	Wind Right.	Resultant.	Resultant.
bl	- 14,700	- 17,600	- 16,400	0	- 48,700	- 32,300
cm	- 13,700	- 16,400	- 15,900	0	- 46,000	- 30,100
dp	- 12,600	- 15,100	- 15,400	0	- 43,100	- 28,000
eq	- 11,600	- 13,900	- 14,900	0	- 40,400	- 26,500
lm	- 1,650	- 2,000	- 3,700	0	- 7,350	- 5,350
mn	+ 1,650	+ 2,000	+ 3,700	0	+ 7,350	+ 5,350
no	- 3,300	- 4,000	- 7,400	0	- 14,700	- 10,700
op	+ 1,850	+ 2,200	+ 4,100	0	+ 8,150	+ 5,950
pq	- 1,650	- 2,000	- 3,700	0	- 7,350	- 5,350
rq	+ 5,000	+ 6,000	+ 11,000	0	+ 22,000	+ 16,000
ro	+ 3,400	+ 4,100	+ 7,400	0	+ 14,900	+ 10,800
kl	+ 13,300	+ 16,000	+ 18,300	+ 6,100	+ 47,600	+ 31,600
kn	+ 11,300	+ 13,600	+ 14,200	+ 6,100	+ 39,100	+ 25,500
kr	+ 8,000	+ 9,600	+ 6,100	+ 6,100	+ 23,700	+ 17,600
kv	+ 11,300	+ 13,600	+ 6,100	+ 14,200	+ 39,100	+ 25,500
kx	+ 13,300	+ 16,000	+ 6,100	+ 18,300	+ 47,600	+ 31,600
ru	+ 3,400	+ 4,100	0	+ 7,400	+ 14,900	+ 10,800
rs	+ 5,000	+ 6,000	0	+ 11,000	+ 22,000	+ 16,000
st	- 1,650	- 2,000	0	+ 3,700	- 7,350	- 5,350
tu	+ 1,850	+ 2,200	0	+ 4,100	+ 8,150	+ 5,950
uv	- 3,300	- 4,000	0	- 7,400	- 14,700	- 10,700
vw	+ 1,650	+ 2,000	0	- 3,700	+ 7,350	+ 5,350
wx	- 1,650	- 2,000	0	- 3,700	- 7,350	- 5,350
fs	- 11,600	- 13,900	0	- 14,900	- 40,400	- 26,500
gt	- 12,600	- 15,100	0	- 15.400	- 43,100	- 28,000
hw	- 13,700	- 16,400	0	- 15,900	- 46,000	- 30,100
ix	- 14,700	- 17,600	0	- 16,400	- 48,700	- 32,300

We may draw next the polygon for joint 3; it is KLMNK, MN and NK representing the stresses in *mn* and *nk*. No polygon for a joint on the left side can now be drawn, but we may begin at the right end. For joint 12 the polygon is JKXIJ, KX and XI representing the stresses in *kx* and *xi*.

At joint 11 there are three forces; and since they are balanced, and two act along the same line, those two must be equal and opposite, and the third must equal zero. Hence the point X is also marked W to indicate the fact that XW, or the stress in *xw*, is zero. Then, too, the diagram shows that WH equals XI. Having just shown that there is no stress in *xw*, there are but three forces at joint 13. Since two of these act along the same line, they must be equal and opposite, and the third zero. Therefore the point W is also marked V to indicate the fact that WV, or the stress in *wv*, equals zero. The diagram shows also that VK equals XK. This same argument applied to joints 9, 15, 10, and 14 successively, shows that the stresses in *st*, *tu*, *uv*, *ur*, and *sr* respectively equal zero. For this reason the point X is also marked UTS and R. It is plain, also, that the stresses in *sf* and *tg* equal those in *wh* and *xi*, and that the stress in *kr* equals that in *kv* or *kx*. Remembering that we are discussing stress due to wind pressure only, it is plain, so far as wind pressure goes, that the intermediate members on the right side are superfluous.

We may now resume the construction of the polygons for the joints on the left side. At joint 4, we know the forces in the members *kn* and *kr*; hence there are only two unknown forces there. The polygon for the joint is KNORK, NO and OR representing the stresses in *no* and *or*. The polygon for joint 5 may be drawn next; it is ONMCDPO, DP and PO representing the stresses in *dp* and *po*. The polygon for joint 6 or joint 7 may be drawn next; for 6 it is PDEQP, EQ and QP representing the stresses in *eq* and *qp*. At joint 7 there is but one unknown force, and it must close the polygon for the known forces there. That polygon is ROPQ; hence QR represents the unknown force. (If the work has been correctly and accurately done, QR will be parallel to *qr*.)

When the wind blows upon the right side, the values of the reactions, and the stresses in any two corresponding members, are reversed. Thus, when the wind blows upon the left side, the stresses in kl and kx equal 18,300 and 6,100 pounds respectively; and when it blows upon the right they are respectively 6,100 and 18,300 pounds. It is not necessary, therefore, to construct a stress diagram for the wind pressure on the right. The numbers in the fifth column (see table, Page 72) relate to wind right, and were obtained from those in the fourth.

41. Combination of Dead, Snow, and Wind-Load Stresses. After having found the stress in any member due to the separate loads (dead, snow, and wind), we can then find the stress in that member due to any combination of loads, by adding algebraically the stresses due to loads separately. Thus, in a given member, suppose:

Dead-load stress = + 10,000 pounds,
Snow-load " = + 15,000 "
Wind-load " (right) = − 12,000 "
" " (left) = + 4,000 "

Since the dead load is permanent (and hence the dead-load stress also) the resultant stress in the member when there is a snow load and no wind pressure, is

+ 10,000 + 15,000 = + 25,000 pounds (tension);

when there is wind pressure on the right, the resultant stress equals

+ 10,000 − 12,000 = − 2,000 pounds (compression);

when there is wind pressure on the left, the resultant stress is

+ 10,000 + 4,000 = + 14,000 pounds (tension);

and when there is a snow load and wind pressure on the left, the resultant stress is

+ 10,000 + 15,000 + 4,000 = + 29,000 pounds (tension).

If all possible combinations of stress for the preceding case be made, it will be seen that the greatest tension which can come upon the member is 29,000 pounds, and the greatest compression is 2,000 pounds.

In roof trusses it is not often that the wind load produces a "reversal of stress" (that is, changes a tension to compression, or *vice versa*); but in bridge trusses the rolling loads often produce reversals in some of the members. In a record of stresses the reversals of stress should always be noted, and also the value of the greatest tension and compression for each one.

The numbers in the sixth column of the record (Page 72) are the values of the greatest resultant stress for each member. It is sometimes assumed that the greatest snow and wind loads will not come upon the truss at the same time. On this assumption the resultant stresses are those given in the seventh column.

EXAMPLE FOR PRACTICE.

1. Compile a complete record for the stresses in the truss of Fig. 24, for dead, snow, and wind loads. See Example 1, Article 27, for values of dead-load stresses, and Example 2, Article 29, for values of the wind-load stresses. Assume the snow load to equal 1.2 times the dead load.

After the record is made, compute the greatest possible stress in each member, assuming that the wind load and snow load will not both come upon the truss at the same time.

The greatest resultant stresses are as follows:

Member.	*af*	*fe*	*bg*	*fg*	*gh*	*hi*	*hc*	*ie*	*id*
Resultant....	-14,950	+17,800	-10,400	-8,900	+7,800	-8,900	-11,800	+11,500	-13,800

42. Truss Sustaining a Roof of Changing Slope. Fig 46 represents such a truss. The weight of the truss itself can be estimated by means of the formula of Art. 19. Thus if the distance between trusses equals 12 feet, the weight of the truss equals

$$W = 12 \times 32 \left(\frac{32}{25} + 1\right) = 875 \text{ pounds.}$$

The **weight of the roofing** equals the product of the area of the roofing and the weight per unit area. The area equals 12 times the sum of the lengths of the members $\overline{12}$, $\overline{23}$, $\overline{34}$, and $\overline{45}$, that is, $12 \times 36\frac{1}{2} = 438$ square feet. If the roofing weighs 10 pounds per square foot, then the weight of roofing sustained by one truss equals $438 \times 10 = 4,380$ pounds. The total dead load then equals

$$875 + 4,380 = 5,255 \text{ pounds;}$$

and the apex dead loads for joints 2, 3, and 4 equal:

$$\frac{1}{4} \times 5{,}255 = 1{,}314 \text{ (or approximately 1,300) pounds;}$$

while the loads for joints 1 and 5 equal

$$\frac{1}{8} \times 5{,}255 = 657 \text{ (or approximately 650) pounds.}$$

The **snow loads** for the joints are found by computing the snow load on each separate slope of the roof. Thus, if the snow

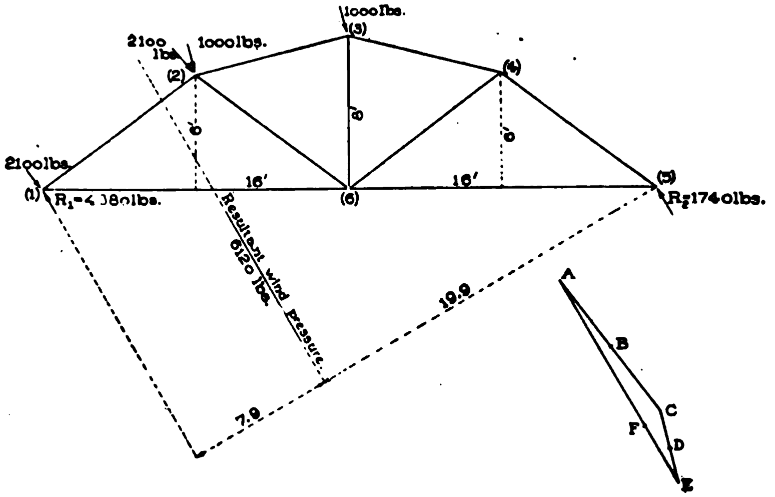


Fig. 46.

weighs 20 pounds per square foot (horizontal), the load on the portion 12 equals 20 times the area of the horizontal projection of the portion of the roof represented by $\overline{12}$. This horizontal projection equals 8×12 (= 96) square feet; snow load equals 96×20 (= 1,920) pounds. This load is to be equally divided between joints 1 and 2.

In a similar way the snow load borne by $\overline{23}$ equals 20 times the area of the horizontal projection of the roof represented by $\overline{23}$; this horizontal projection equals 8×12 (= 96) square feet as before, and the snow load hence equals 1,920 pounds also. This load is to be equally divided between joints 2 and 3.

Evidently the loads on parts $\overline{34}$ and $\overline{45}$ also equal 1,920 pounds each; hence the apex loads at joints 1 and 5 equal 960 pounds and at joints 2, 3 and 4, 1,920 pounds.

The wind load must be computed for each slope of the roof separately. The angles which $\overline{12}$ and $\overline{23}$ make with the horizontal, scale practically 37 and 15 degrees. According to the table of wind pressures (Art. 19), the pressures for these slopes equal about 35 and 20 pounds per square foot respectively. Since member $\overline{12}$ is 10 feet long, the wind pressure on the 37-degree slope equals $10 \times 12 \times 35 = 4{,}200$ pounds.

This force acts perpendicularly to the member $\overline{12}$, and is to be equally divided between joints 1 and 2 as represented in the figure. Since the member $\overline{23}$ is $8\frac{1}{4}$ feet long, the wind pressure on the 15-degree slope equals

$$8\tfrac{1}{4} \times 12 \times 20 = 1{,}980 \text{ or approximately } 2{,}000 \text{ pounds.}$$

This pressure acts perpendicularly to member $\overline{23}$, and is to be equally divided between joints 2 and 3 as represented.

The stress diagram for dead, snow, or wind load for a truss like that represented in Fig. 46, is constructed like those previously explained; but there are a few points of difference in the analysis for wind stress, and these will be explained in what follows.

Example. Let it be required to determine the stresses in the truss of Fig. 46, due to wind loads on the left as represented.

It is necessary to ascertain the reactions due to the wind loads; therefore, find the resultant of the wind pressures, see Art. 32; it equals 6,120 pounds and acts as shown. Now, if both ends of the truss are fastened to the supports, then the reactions are parallel to the resultant wind pressure, and their values can be readily found from moment equations. Let R_1 and R_2 denote the left and right reactions respectively; then, since the arms of R_1 and the resultant wind pressure with respect to the right end equal 27.8 and 19.9 feet respectively,

$$R_1 \times 27.8 = 6{,}120 \times 19.9 = 121{,}788\ ;$$

hence,

$$R_1 = \frac{121{,}788}{27.8} = 4{,}380 \text{ pounds approximately.}$$

Since the arms of R_2 and the resultant wind pressure with respect to the left support are 27.8 and 7.9 feet respectively,

$$R_2 \times 27.8 = 6{,}120 \times 7.9 = 48{,}348\ ;$$

hence, $R_2 = \frac{48,348}{27.8} = 1,740$ pounds approximately.

The next step is to draw the polygon for the loads and reactions; so we draw lines AB, BC, CD, and DE to represent the loads at joint 1, the two at joint 2, and that at joint 3, respectively; and then EF to represent the right reaction. (If the reactions have been correctly determined and the drawing accurately done, then FA will represent the left reaction.)

The truss diagram should now be lettered (agreeing with the letters on the polygon just drawn), and then the construction of the stress diagram may be begun. Since this construction presents no points not already explained, it will not be here carried out.

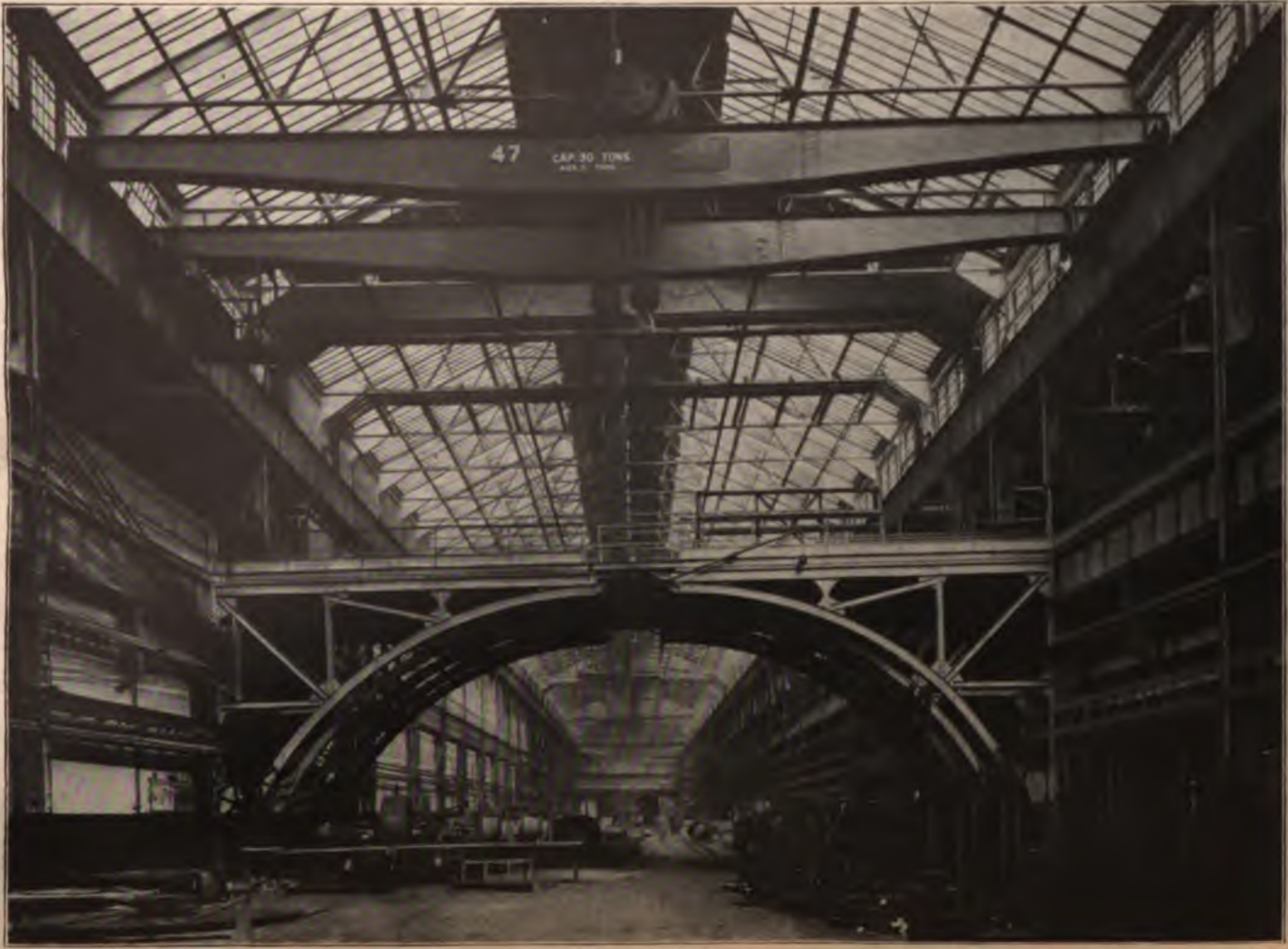
EXAMPLE FOR PRACTICE.

Analyze the truss of Fig. 46 for dead, snow, and wind loads as computed in the foregoing, and compute the greatest resultant stress in each member due to combined loads, assuming that the snow and wind do not act at the same time.

Stress Record.

Ans.

Member.	Dead.	Snow.	Wind Left.	Wind Right.	Resultant.
12	−3,250	−4,800	−3,450	−2,500	−8,050
23	−2,700	−4,000	−2,850	−3,100	−6,700
16	+2,600	+3,850	+3,750	+1,150	+6,450
26	0	0	−2,000	+1,250	−2,000 +1,250
36	0	0	+ 450	+ 450	+ 450
46	0	0	+1,250	−2,000	+1,250 −2,000
56	+2,600	+3,850	+1,150	+3,750	+6,450
43	−2,700	−4,000	−3,100	−2,850	−6,700
54	−3,250	−4,800	−2,500	−3,450	−8,050



HALF VIEW DOWN CENTER AISLE OF MACHINE SHOP OF WESTINGHOUSE ELECTRIC & MFG. CO., EAST PITTSBURG, PA.

ROOF TRUSSES

1. Classes of Roof Trusses. Roof trusses may be divided into three classes according to the shape of their upper chord. These three classes are:

(1) Triangular roof trusses;
(2) Crescent roof trusses;
(3) Roof trusses other than these.

Fig. 1 shows various forms of triangular roof trusses. The ***Pratt*** and ***Howe*** trusses are shown respectively by *a* and *b*. These trusses obtain their name on account of their web bracing being of the Pratt or Howe type. The triangular truss in most common use is the ***Fink***, next to which is the *Saw-tooth*. The Fink truss is built in a variety of forms, as shown in Fig. 1 (*c*, *d*, *e*, and *f*), *c* being for spans up to 60 feet; *e* for spans up to 70 feet, and *d* and *f* for spans up to 80 feet and over. The great advantage of this style of truss is that many of its members have the same stress, and therefore it can be constructed more cheaply on account of the fact that a large amount of the same sized material can be purchased at once.

When the top chord of a roof truss becomes bent as shown in Fig. 2, the truss is called a *crescent* roof truss. The bracing in the crescent roof trusses is not of any particular form, being as a usual thing built of members which can take either tension or compression. This is made necessary by the fact that the curved upper chord may cause either tension or compression in the webbing, according to the angle of its inclination with the horizontal.

Roof trusses which do not come under either of the above classes may be regarded in a class of their own. To this class belong those trusses which are somewhat like a bridge truss in that the two chords are horizontal or nearly so. The ends of these trusses may be rectangular or not. For various types of this class of truss, see Fig. 3.

In addition to the above classification, which is based on the form of the chords, roof trusses may be divided according to the manner in which their members are connected. This classification

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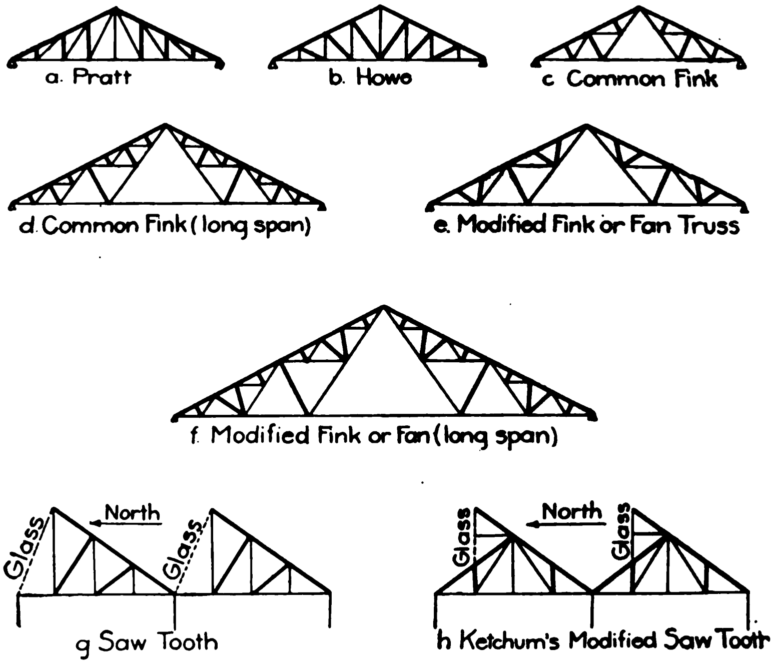

a. Pratt
b. Howe
c. Common Fink
d. Common Fink (long span)
e. Modified Fink or Fan Truss
f. Modified Fink or Fan (long span)
North
Glass
Glass
g Saw Tooth
North
Glass
Glass
h Ketchum's Modified Saw Tooth

Fig. 1. Triangular Roof Trusses.

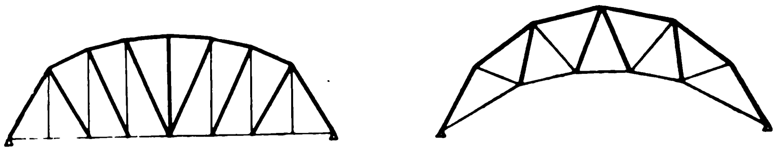


Fig. 2. Crescent Roof Trusses.

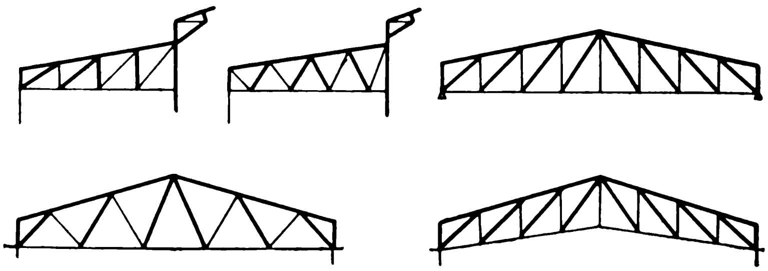


Fig. 3. Trusses with Chords Almost Parallel.

is that of *pin-connected* and *riveted*. For a definition of this, and for figures showing such joints, see "Statics," pp. 22 and 23.

Trusses are seldom built as pin-connected unless they are of long span, since roof trusses are comparatively light, and pin-connected trusses, unless of considerable weight, do not give very great stiffness.

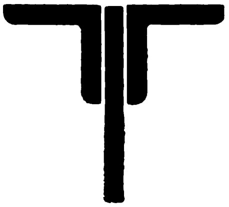


Fig. 4. Chord Section for Heavy Stresses.

Riveted roof trusses are used for nearly all practical purposes, since they give great rigidity under the action of wind and of moving loads, such as cranes, which may be attached to them.

2. Physical Analysis of Roof Trusses. In pin-connected roof trusses, the tension members consist of I-bars or rods; and the compression members are made of channels or angles and plates, either plain or latticed. In riveted trusses, both tension and compression members are made up of angles and plates or a combination of the two. The top chords of roof trusses of medium span usually consist of two angles placed back to back. If the stress becomes too great to be taken up by two angles larger than 5 by 3½ inches, then two angles and a plate are used (see Fig. 4). In case the roof truss is of great size and the stresses are exceedingly large, the chord member may be built up in a manner somewhat similar to a bridge truss, being constructed of two channels and a plate, or four angles and three plates. Figs. 5 and 6 show cross-sections of chords for long-span riveted trusses. These cross-sections may also be used for pin-connected trusses.

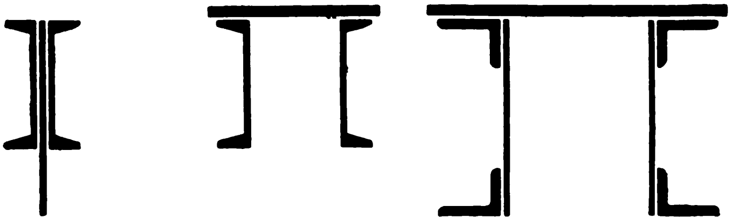


Fig. 5. Fig. 6

Chord Sections for Trusses of Long Span.

The web members of a truss usually consist of one angle; and if this is insufficient, two angles back to back are used. Fig. 80, page 65, gives a diagram of a roof, and shows not only the roof trusses but also various other parts which will be referred to in the succeeding articles.

3. **Wind Pressure and Snow Loads.** The wind pressure on a flat surface varies, of course, with the velocity of the wind, and is very closely given by the formula:

$$P = 0.004V^2$$

By substituting in this formula, the values shown in Table I are determined for given velocities in miles per hour.

TABLE I

Wind Pressure at Various Velocities

VELOCITY (Miles per hour)	PRESSURE (Lbs per square foot)	REMARKS
10	0.4	**Breeze**
20	1.6	**Strong breeze**
30	3.6	**Strong wind**
40	6.4	**High wind**
50	10.0	**Storm**
60	14.4	**Heavy storm**
70	19.6	**Hurricane**
80	25.6	"
100	40.0	"

The pressures indicated in Table I are perpendicular to the direction of the wind. When the wind blows on an inclined surface, the wind is assumed to be acting horizontally, and the normal component on the inclined surface is determined. This component is not equal to the horizontal pressure times the sine of the angle of inclination, as one would suppose (see Fig. 7), but is greater by a small amount. Roofs are usually figured on a basis of 40 pounds pressure on a vertical surface. The value of the normal component for a horizontal wind pressure of 40 pounds per square foot, is given on page 24 of "Statics," and is here, for convenience, reduced to the normal pressure for any given pitch.

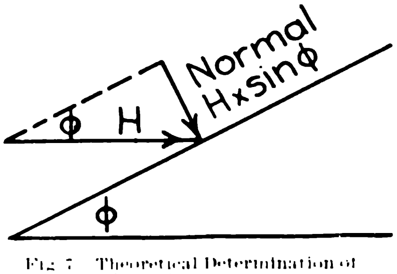
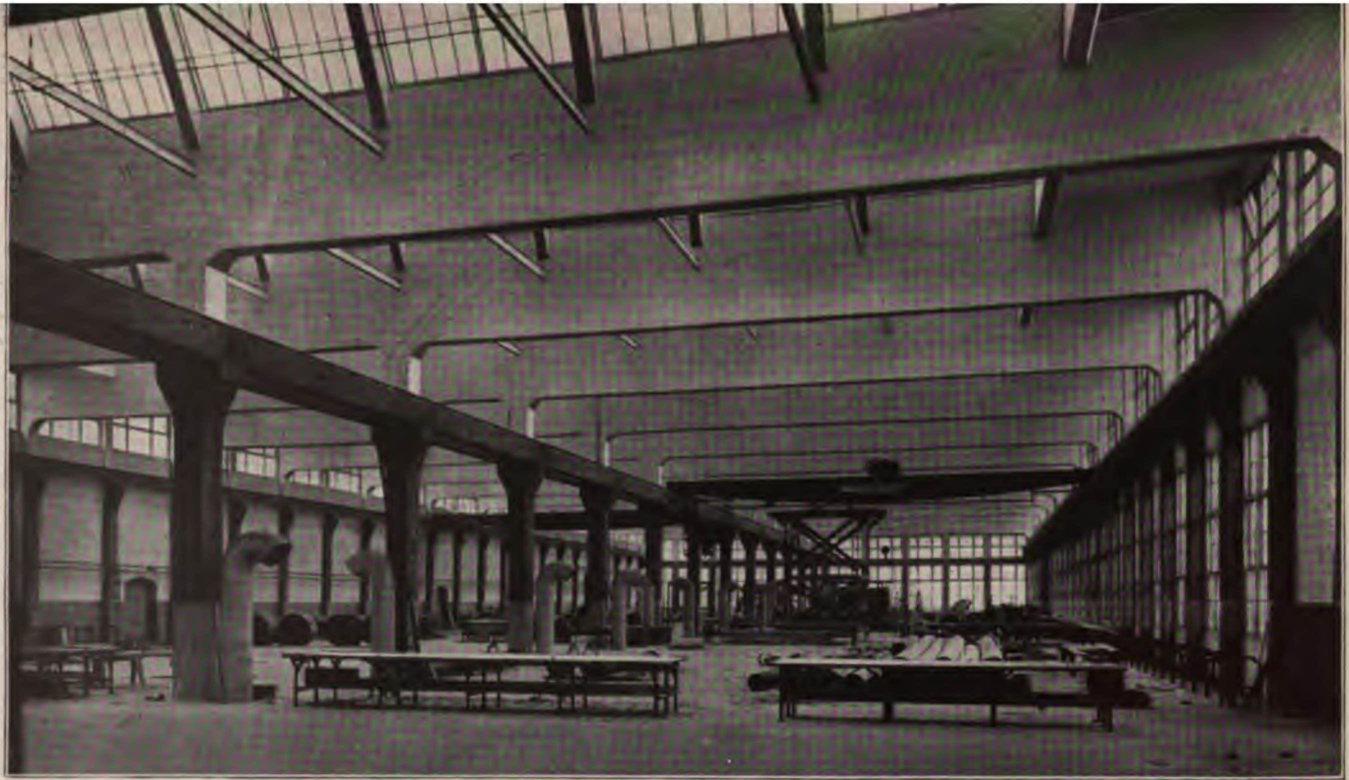


Fig. 7. Theoretical Determination of Normal Component.



INTERIOR OF ASSEMBLY BUILDING OF THE GEORGE N. PIERCE COMPANY, BUFFALO, N. Y.

Spans under the sawtooth roof are 61-ft., giving the building two unbroken bays, each 61 ft. by 401 ft. Kahn System of Reinforced Concrete.
Courtesy of Trussed Concrete Steel Company, Detroit, Mich.

Pitch	Normal Wind Pressure
1/3	34 pounds per square foot.
30°	32 " " " "
1/4	30 " " " "
1/5	26 " " " "
1/6	22 " " " "

If the normal pressure on a roof making any other angle with the horizontal is desired, see "Statics," p. 24.

The determination of these values is based for the most part on data obtained by experiment. In the computations relative to the design of buildings, the wind is usually assumed to exert a pressure on the walls of 30 pounds per square foot.

The snowfall varies with the locality. The heaviest snow loads which come upon a roof are not always in the locality of the heaviest snowfall, since a comparatively light snowfall may occur, and if this is followed by wind and sleet, the result will be a load greatly in excess of the snowfall itself. The snow load per square foot of roof surface varies with the pitch of the roof, and will be greater the smaller the pitch. The ice and sleet will be comparatively constant. Fig. 8* gives values of snow and sleet loads which are recommended for use. It is customary to figure the snow load by taking it as so much per square foot of horizontal projection.

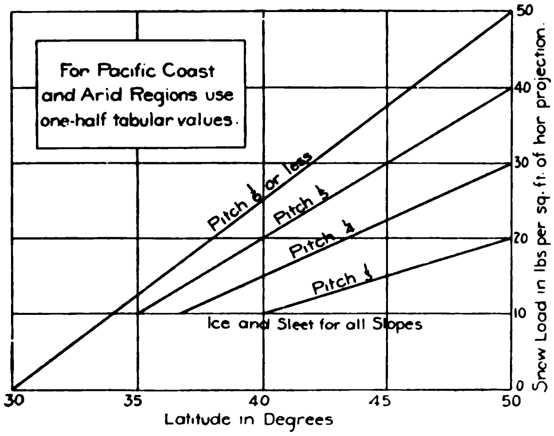


Fig. 8. Unit Snow Loads.

EXAMPLES FOR PRACTICE

1. Compute the wind panel load on a roof whose pitch is ¼, and whose panel length is 15 feet, the distance between trusses being 16 feet.

2. Compute the snow panel load for the truss of Problem 1, above.

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*Ketchum's "Steel Mill Buildings," p. 11.

4. Weights of Roof Trusses. The weight of a roof truss varies with the material of which it is constructed, the span, the distance between trusses, the pitch, and the capacity of the truss. The actual weight, of course, cannot be determined until after the truss is designed; but an approximate weight may be obtained from any of the empirical formulæ which are now in use. Table II gives the most common and best of the empirical formulæ, together with the names of their authors.

TABLE II

Formulæ Giving Weights of Roof Trusses

FORMULA	AUTHOR
$W = \frac{3}{4}\, al \left(1 + \frac{l}{10}\right)$	Mansfield Merriman
$W = al \left(1 + \frac{l}{25}\right)$	E. R. Maurer, (p. 23, "Statics")
$W = al^2 \left(\frac{1}{25} + \frac{l}{6\,000}\right)$, wooden trusses.	N. C. Ricker
$W = \frac{Pal}{45} \left(1 + \frac{l}{5\sqrt{a}}\right)$	Milo S. Ketchum*
$W = 2a\left(4 + \frac{l}{25}\right) \sqrt{l^2 + r^2}$	C. W. Bryan
$W = al\,(0.06\,l + 0.6)$ for heavy loads $W = al\,(0.04\,l + 0.4)$ " light "	C. E. Fowler

In the above formulæ,

W ≐ Weight of steel in truss, in pounds;
P = Capacity of truss in pounds per square foot of horizontal projection of roof;
r = Rise of peak, in feet;
a = Distance center to center of trusses, in feet;
l = Span of truss, in feet.

ROOF COVERINGS

5. The roof is covered with some material which will protect the interior of the building from the action of the elements. This covering may consist of any one or more of the materials which, together with their weights per square foot, are indicated in Table III. The weights here given for materials which must be laid upon sheathing, do not include the weight of the sheathing, which is given separately. A short description, together with necessary information for use in estimates, will now be given.

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* "Steel Mill Buildings," p. 5

TABLE III

Approximate Weights of Roof Coverings

MATERIAL	WEIGHT PER SQUARE FOOT
White pine sheathing 1 inch thick	3 lbs.
Yellow pine sheathing 1 inch thick	4 "
Batten sheathing, 4-in. by 1-in.	2½ "
Slate	10 "
Skylight glass, including frame	10 "
Tin	1 "
Shingles	3 "
Corrugated steel	2 "
Flat tiles	12 to 25 "
Corrugated tiles	10 "
Concrete slabs	35 to 50 "
Felt, asphalt and gravel	10 "
Felt and gravel	10 "
Patent roofings	½ to 1¼ "
Sheet steel	1½ "
Non-condensing base	1 "

Sheathing. Sheathing is generally laid directly upon the purlins (see Article 6); and upon this are laid the shingles, slate, tin, or tile. Sheathing is usually made of a single thickness of planks, 1 to 2½ inches thick, laid close together. In some cases, however, when batten sheathing is used, it is spaced from 2 to 4 inches apart. This has the advantage of being cheap and at the same time allowing good circulation of air beneath the roof covering, and consequently dampness due to any cause will soon dry out. Batten sheathing is much used where the roof covering consists of slate, shingles, or tile.

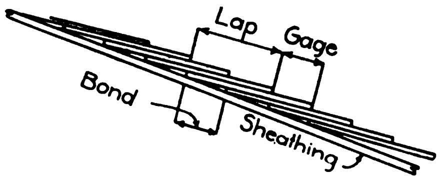


Fig. 9. Method of Laying Slate.

Slate. Roofing slate is generally of the characteristic slaty color, but may be obtained in nature in greens, purple, reds, and other colors. It is made in thicknesses of from ⅛ to ¼ inch, in widths from 6 to 24 inches, and in lengths from 12 to 44 inches. The 12 by 18 by $\frac{3}{16}$-in. slate is probably the most commonly used. Slate should be laid as shown in Fig. 9, and the pitch of the roof should not be less than ¼. If the pitch of the roof is less than this, the lap should be made greater than 3 inches, as is shown in Fig. 9. The lap should be increased at least ½ inch for every $\frac{1}{100}$ in pitch; and the minimum pitch should never be less than $\frac{1}{8}$, since it is practically impossible to prevent roofs with a smaller pitch than this from leaking, especially if a strong wind is blowing.

The number of different sizes of slate required to lay 100 square feet of surface, and also their weight, are given in the handbooks of the various slate companies. With a 3-inch lap, it takes 160 of the size and thickness mentioned above to lay 100 square feet, and the total weight of this square is 650 pounds. Slate is one of the most durable of roofing materials. Its first cost is high, being from 5 to 8 dollars per hundred square feet; but the cost of maintenance is almost nothing, since it is affected neither by the elements nor by the action of gases or acids. In case the roof would be subjected to the action of gases or acids, it is advisable to use copper slating nails.

Skylight Glass. Skylights usually consist of glass about $\frac{3}{16}$ to $\frac{3}{8}$ inch in thickness, supported on light members of iron or galvanized iron which act as a framework. The actual weight of glass of different kinds can be accurately obtained from manufacturers' catalogues, and the student is referred to these; they may be had by addressing the manufacturers (see Figs. 73 and 74).

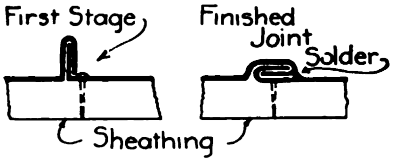


Fig. 10. Tin Laid with Flat Seam.

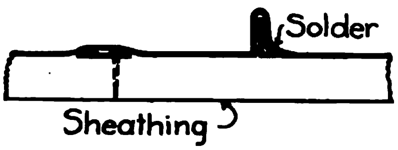
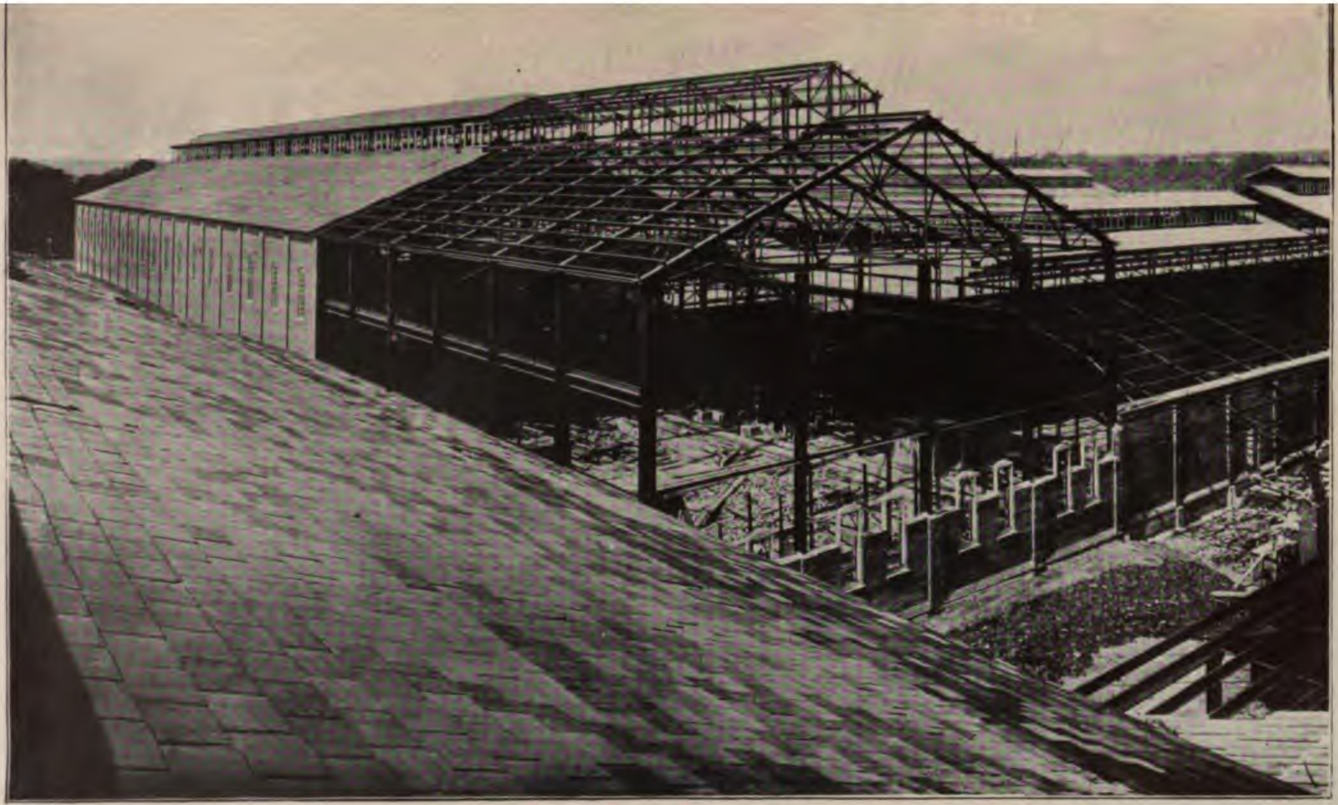


Fig. 11. Tin Laid with Standing Seam.

Tin. This is made by coating thin, flat sheets of iron or steel, either with tin alone or with a mixture of tin and lead. In the first case the product is called *bright tinplate*, and in the second case *terne plate*. Terne plate must not be used where it will be subjected to the action of acids or corrosive gases, since the lead coating is rapidly destroyed, and then of course the iron also.

Tin plates come in various sizes and thicknesses; but usually 112 come in one box. The most commonly used is a sheet 20 by 28 inches, and of sheet iron of No. 27 gauge, which weighs 10 ounces to the square foot. This is marked "IX." If the box were marked "IC," it would indicate that the sheets were of No. 29 gauge metal, which weighs 8 ounces to the square foot. The value of the roofing



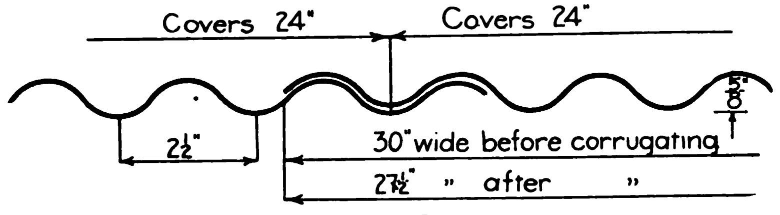
BUILDINGS FOR THE PITTSBURG PLATE GLASS COMPANY, CRYSTAL CITY, MISSOURI

Grinder and Polisher building, 408 ft. by 532 ft.; Laying Yard building, 100 ft. by 578 ft.; Lehr building, 68 ft. by 475 ft. and 114 ft. by 171 ft.; Stripper building, 90 ft. by 408 ft.; Warehouse, 153 ft. by 304 ft. and 90 ft. by 173 ft. Total weight of steel work, 6,280,000 lbs.

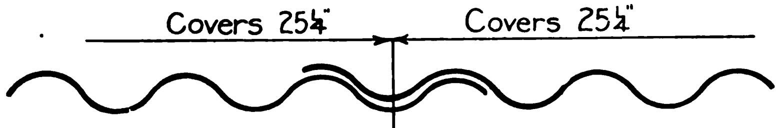
Courtesy of American Bridge Company.

depends to a great extent upon the amount of tin used in the coating. This will vary from 8 to 50 pounds for a box of the 20 by 28-inch sheets.

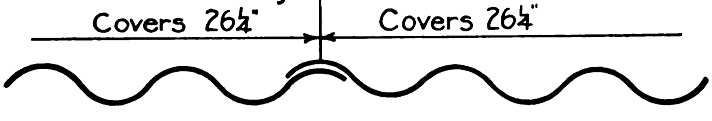
A tin roof is formed by fixing together a number of these sheets. The sheets may be connected as shown in Fig. 10, or as shown in Fig. 11. In the first case, they are said to be laid with a *flat seam,* and in the second case they are said to be laid with a *standing seam.* Tin roofs rot out very readily unless they are kept painted. If a new coat is given them every couple of years, they will last from twenty to thirty years.



2 Corrugation Side Lap



1½ Corrugation Side Lap



1 Corrugation Side Lap

Fig. 12. Lapping of Corrugated Steel.

Tin can be laid on roofs whose pitch is very small, say $\frac{1}{10}$. The first cost is about as much as that of slate, but the cost of maintenance is very high.

Shingles. Shingle roofs are very seldom used on buildings for manufacturing purposes, for the reason that they take fire quite readily, leak quite easily, and require renewal quite often. Shingles are from 18 to 24 inches long, and usually run from 2 to 8 inches in width, although they can be obtained of a uniform width of from 4 to 6 inches. They are laid like slate, the lap being made 4 inches or more. They should never be laid on roofs whose pitch is less than $\frac{1}{3}$. It takes about from 800 to 1,000 shingles to lay 100 square feet of roof. The cost is about $5.00 per 100 square feet; but under the best conditions, the life of shingles is only about ten years.

Corrugated Steel. Corrugated steel is made from flat sheets of standard gauges, and may be either galvanized or left as it comes from the rolls. The corrugations are of different sizes and widths; the total width of the plates runs from 24 to 28 inches, and their length from 5 to 10 feet, varying by $\frac{1}{2}$ foot. The sheet most used for roof construction has $2\frac{1}{2}$-inch corrugations; is 26 inches wide after corrugating; and will cover a width of 24 inches with one lap and $21\frac{1}{2}$ inches with two laps. This roofing should be laid with a pitch of not less than $\frac{1}{4}$, and should have from 6 to 8 inches lap at the ends. For further information regarding the method of lapping and the width covered, see Fig. 12.

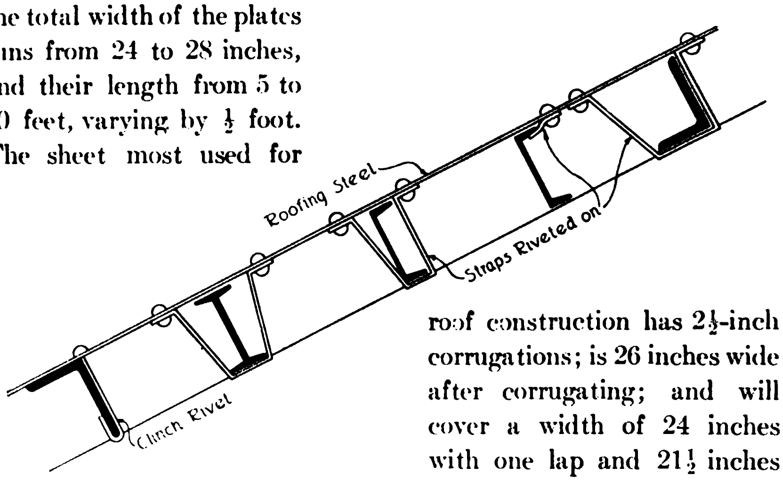


Fig. 13. Showing How Steel Roofing is Fastened to Purlins.

Corrugated steel is fastened either directly to wooden purlins by means of nails, or directly to iron purlins either by means of a bolt and clip or by a clinch nail (see Fig. 13).

It is often advisable to know the strength of corrugated steel when supported at certain distances apart by supports perpendicular to the corrugations. This unsupported length determines in many cases the spacing of the purlins. The load in pounds per square foot which can be carried by a plate of span l, parallel to the corrugation, is given by the formula:

$$W = \frac{330\,S\,d\,t}{l^2},$$

in which,

l = Unsupported length of sheet, in inches;
t = Thickness of sheet, in inches;
S = Allowable unit-stress;
d = Depth of corrugation, in inches.

Table IV, giving data relative to corrugated sheets, is taken from page 172 of the Pocket Companion of the Carnegie Steel Company (edition of 1902), where also other valuable information is given.

TABLE IV

Corrugated Steel Data

No. by Birmingham Gauge	Thickness (Inches)	Weight in lbs. per 100 Sq. Ft. of Roof when Laid with 6-in. End Lap and One Corrugation, 2½-in., Side Lap, and Length of:					
		5 ft.	6 ft.	7 ft.	8 ft.	9 ft.	10 ft.
16	0.065 in.	365	358	353	350	348	346
18	0.049 "	275	270	267	264	262	261
20	0.035 "	196	192	190	188	186	185
22	0.028 "	156	154	152	150	149	148
24	0.022 "	123	121	119	118	117	117
26	0.018 "	101	99	97	97	96	95

Tiles. One of the most common sizes of plain roofing tile is 10½ inches long by 6¼ inches wide and ⅝ inch thick. Tile of this size weigh about 2¼ pounds each. They are laid with a lap equal to one-half their length. They may be laid directly upon plank sheathing in a manner similar to shingles or slate, or they may be laid directly upon purlins (see Figs. 14, 15, and 16). In the first case they are nailed directly to the sheathing, and in the second case they are connected with the purlins either with copper wire or clinch nails. Flat tiles are usually laid in cement; corrugated tiles are made so as to interlock, and consequently in most cases require no cement. One convenience of the tile roof is that the skylights may be formed by laying glass tile in place of the other.

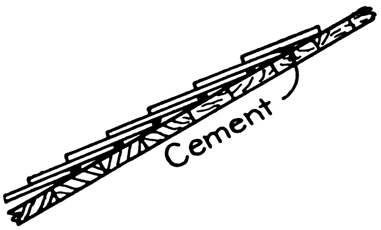


Fig. 14. Method of Laying Flat Tile on Plank Sheathing.

Tile roofs are very substantial; but are very costly, in regard not only to the tiles themselves, but also in regard to the additional weight required in the trusses by reason of the great weight to be supported. Tile weigh from 700 to 1,000 pounds per 100 square feet of roof surface. They cost from $12.00 to $40.00 per 100 square feet on the roof.

Concrete Slabs. These are usually moulded directly in place by suspending forms from the roof trusses. They may or may not be reinforced, and in any case are usually not over 4 inches in thickness. Their weight is about 50 pounds per square foot. Their cost is from $16.00 to $30.00 per hundred square feet of roof surface. They are expensive, not only on their own account, but also from the fact that the weight of the roof trusses must be increased in order to carry the weight of the slabs. Concrete roofing may be used on roofs which are practically flat, ½ inch to 1 foot being sufficient pitch.

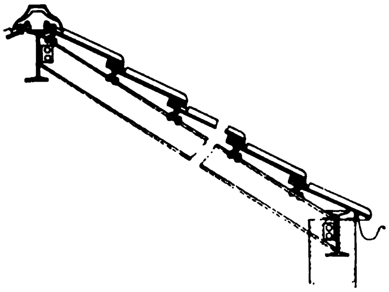


Fig. 15. Ludowici Tile on Steel Purlins.

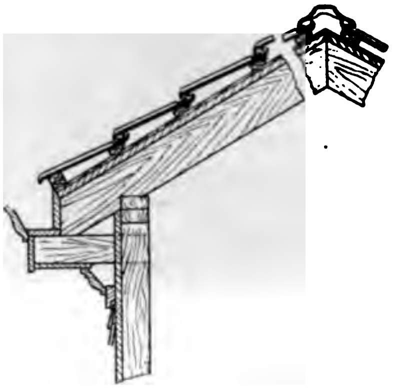


Fig. 16. Ludowici Tile on Sheathing.

Felt and Asphalt. This roofing is laid upon shingles, and consists of one thickness of dry felt, three or four thicknesses of roofing felt well cemented together with asphalt cement, and laid with good laps where they join, and a coating of from 100 to 200 pounds of asphalt per 100 square feet of roof surface. Upon this asphalt, while hot, gravel screened through a ⅜-inch mesh is spread in the quantity of about ⅙ of a cubic yard per 100 square feet. This class of roofing should never be laid on roofs whose pitch is greater than ⅕, since, when heated by the rays of the sun, the asphalt will run and destroy the surface. It gives good satisfaction on roofs whose pitch is $\frac{1}{12}$. This class of roofing can be bought in rolls, and in this case the gravel is exceedingly fine, being screened through a ⅛-inch mesh.

Felt and Gravel Roofing. This roofing is similar to the above; only, in this case, tar instead of asphalt is used for the cementing constituent. This roofing does well on roofs of flat pitch, and should never be used on roofs whose pitch exceeds $\frac{1}{8}$. It can also be bought in rolls ready for laying, in which case the gravel is screened through a $\frac{1}{8}$-inch mesh. The prepared roofings are cheaper than those laid by hand; but they do not give good service unless great care is taken to fasten them down securely. In economy of first cost and maintenance, they are equal to or better than tin.

Sheet Steel. This should not be laid on a pitch less than $\frac{1}{4}$, unless the ends are cemented together where they lap. It comes in sheets 28 inches wide and from 4 to 12 feet long, or it may be purchased in rolls 26 inches wide and about 50 feet long. When used in sheets, it may be had with standing crimped edges, in which case it is laid as shown in Fig. 17. In case it comes in rolls, it may be laid in the same manner as tin, with either standing seams or horizontal flat ones as shown in Figs. 10 and 11. Like corrugated steel, it comes in different gauges, No. 28 being that most commonly used. It can be laid cheaper than tin, on account of the long lengths obtainable.

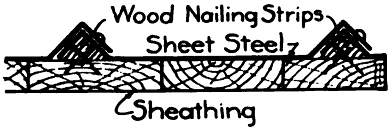


Fig. 17. Method of Laying Sheet Steel with Crimped Edges.

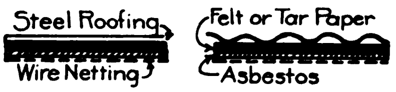


Fig. 18. Method of Laying Roofing on an Anti-Condensing Base.

Patent roofings of many kinds are on the market. These come in rolls usually from 2 feet to 3 feet wide, and cover about 200 square feet of roof surface. The basis of most of these covers is asbestos, felt, magnesia, or rubber; and this is treated with either asphalt, tar, or some other preparation, and in some cases is covered with fine gravel.

Non-Condensing Roofing. In cases where a metal, slate, or tile roof is used without sheathing, moisture is liable to condense upon the under side and drip on the floor beneath. This can be prevented by laying the material upon an anti-condensing base consisting of a layer of wire netting on top of which are placed one or more sheets of asbestos paper about $\frac{1}{16}$ inch thick (see Fig. 18).

Snow load = $10 \times 16 \times 12$	1 920	pounds
Roof covering = $10 \times 16 \times 14$	2 240	"
16 rafters 6 by 2-in., 10 feet long, at 3 lbs. per 144 cu. in.	480	"
Total	4 640	pounds

The moment caused by this weight is:

$$(4\,640 \times 16 \times 12) \div 8 = 111\,360 \text{ pound-inches.}$$

The determination of the beam which will be used to withstand this bending moment is made by means of its section modulus. The formula $\frac{M}{S} = \frac{I}{c}$ is used in the design of beams. The values of I and c are constant for any given beam, and therefore the value of $I \div c$ for any particular beam is a constant, and this constant is called the *section modulus*. It is therefore evident that if we have a certain bending moment and a certain allowable unit-stress, we can obtain the value of the section modulus by dividing the moment by the allowable unit-stress. Then, looking into one of the steel handbooks, the beam can be determined which will have a section modulus equal to or slightly in excess of the value that has been obtained by dividing the bending moment by the unit-stress. This beam will be the beam which, with a unit-stress equal to the one assumed, will withstand the bending moment under consideration.

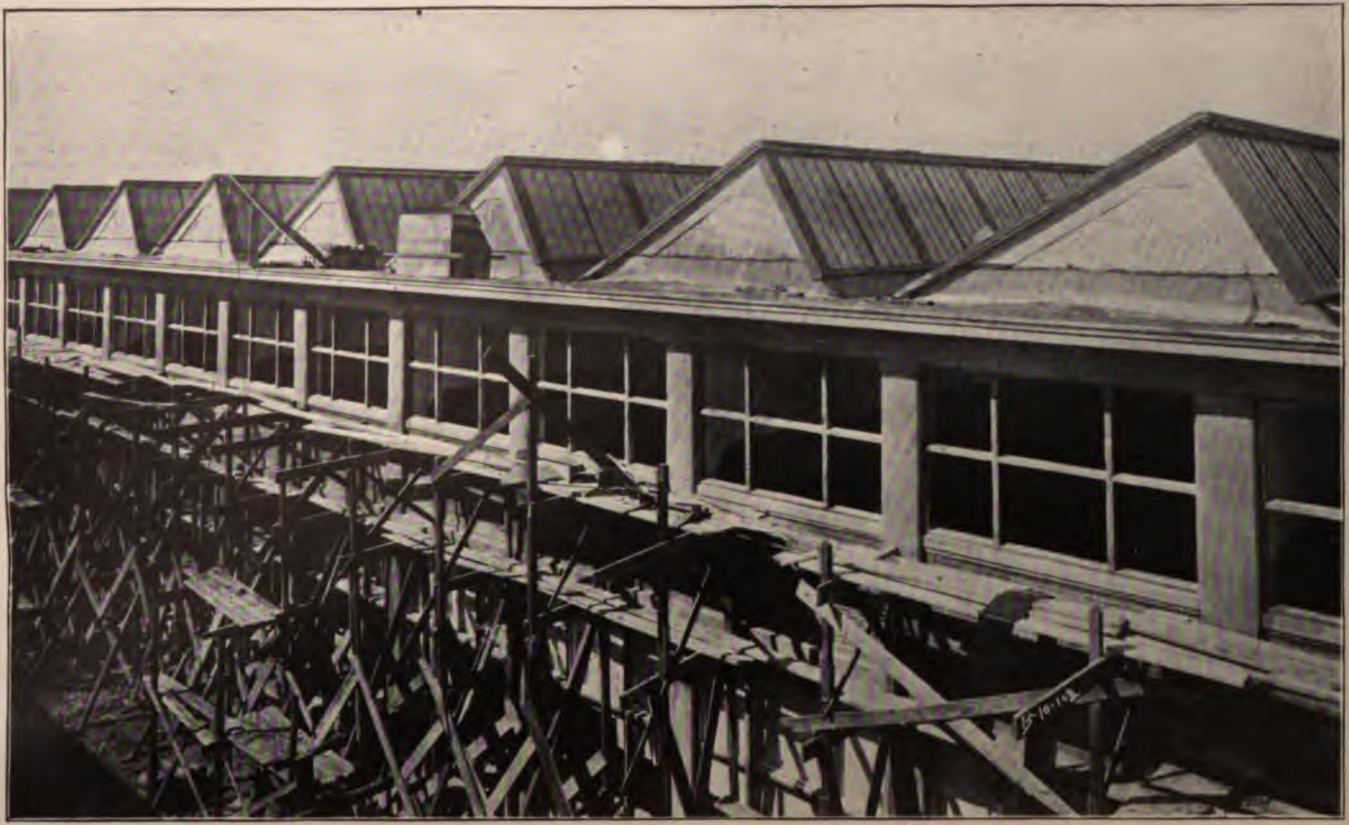
The handbooks issued by many of the steel companies are indispensable to the intelligent design of structural steel. That issued by the Carnegie Steel Company (edition of 1903) is one of the most convenient; and since it will be frequently referred to in this text, its purchase by the student is desired. This book may be obtained by addressing the Carnegie Steel Company at its offices in any of the larger cities. The cost to students has usually been 50 cents; to others, $2.00.

Assuming an allowable unit-stress of 18 000 pounds per square inch on the extreme fibre, the section modulus required to withstand the bending moment of 111 360 pound-inches is:

$$\frac{111\,360}{18\,000} = 6.19.$$

Looking in the Carnegie Handbook at column 11 on page 100, column 11 on page 102, and column 9 on page 104, it will be seen that any one of the following shapes will be sufficient:

One 5-inch 14.75-pound I-beam;
One 7-inch 9.75 " channel;
One $4\frac{1}{8}$ by $3\frac{3}{16}$ by $\frac{9}{16}$-inch Z-bar weighing 17.9 pounds per linear foot.



PART OF SAWTOOTH ROOF OF ASSEMBLY BUILDING OF THE GEORGE N. PIERCE COMPANY, BUFFALO, N. Y.

Giving a partial view of the 61-foot girders. Kahn System of Reinforced Concrete.

Courtesy of Trussed Concrete Steel Co., Detroit, Mich.

Instead of the 5-inch I-beam as given above, a 6-inch 12.25-pound I-beam with a section modulus of 7.3 could be used, and would be more economical, since it is less in weight; and it would also be stiffer, since its depth is greater and its section modulus is greater. A comparison of the above weights shows the channel to be the most economical, since its weight is considerably less than either of the other two shapes. Channels usually make the most economical purlins; and for this reason no other shapes are usually inspected, the channels being used in the first case without being compared with other sections. Inspection of column 11, page 110, Carnegie Handbook, shows that a 6 by 4 by $\frac{3}{4}$-inch angle could have been used for the purlin, since it gives a section modulus of 6.25. The weight of this angle, 23.6 pounds per linear foot, shows it to be far too uneconomical to employ.

EXAMPLES FOR PRACTICE

1. Design the rafters when the total weight of the snow and roof covering is 30 pounds per square foot, and the purlins are spaced 15 feet apart. Use 1 000 pounds per square inch as the allowable unit-stress.

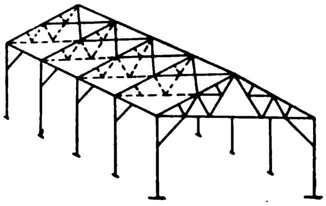
2. Design the purlins if the trusses are 12 feet center to center; the purlins are spaced 8 feet apart; the roof covering, which weighs 6 pounds per square foot, is laid upon 1-inch yellow pine sheathing resting directly upon the purlins; and the snow load is 10 pounds to the square foot of roof surface. Use 18 000 pounds per square inch as the allowable unit-stress, and use a channel for the purlin section.

7. Bracing. In order to keep the roof trusses erect, bracing is employed to join together their top chords and also their bottom chords. This bracing may consist either of small round or square rods, or it may consist of angles. The latter is the best practice, since it gives great rigidity to the structure; and in fact it should be used in all cases where machinery of any kind is attached to the trusses. One disadvantage of the rod bracing is that good connections with the trusses are usually difficult. The bracing between the lower chords is lighter than that between the top chords, since its office is merely to prevent vibration, while that between the upper chords must take up the stresses caused by the wind blowing upon the ends of the building. The stresses in each of these classes of bracing can only be approximately determined; and for that reason it has become customary to determine their section by judgment rather than by computation. For lower chord bracing, single angles 3 by

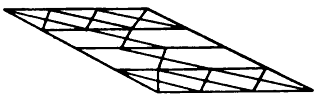
2 by $\frac{5}{16}$-inch are recommended; and for upper chord bracing, 3 by 3 by $\frac{5}{16}$-inch angles should be used.

It is not customary to place bracing between each pair of trusses, but to place them between each alternate pair or between every third pair of trusses. Fig. 23 shows several ways in which the bracing may be inserted.

8. **Economical Spacing and Pitch of Trusses.** The term pitch which has been used in the preceding pages is the fraction obtained by dividing the span into the height of the truss at the center of the span. For example, if a truss has a span of 60 feet, and a rise of 12 feet at the center, it would be said to have a pitch of $\frac{1}{5}$; if the rise were 15 feet, the pitch would be $\frac{1}{4}$; and if the rise were 20 feet, the pitch would be $\frac{1}{3}$. The pitch of a truss is seldom expressed in degrees by giving the angle that the top chord makes with the horizontal. One exception is very common. It is to use the 30° pitch. This has the advantage of making the height of the center equal to one-half the length of one side of the top chord—a fact which lends itself to ease in making the shop drawings.



Upper Chord Bracing



Looking Upward - The Lower Chord Bracing

Fig. 23. Methods of Inserting Bracing between Trusses.

The maximum or minimum allowable pitch for any given roof depends to a great extent upon the class of roof covering employed. For pitches required for any given class of roof covering, see Article 5, p. 6. It might be noted that most of the patent roofings, or any roofing in which tar or asphalt is an ingredient, should not be laid upon roofs with a pitch greater than $\frac{1}{5}$ or $\frac{1}{6}$; while most of the coverings which consist of steel or clay products require pitches of $\frac{1}{5}$ or over.

Pitches varying from $\frac{1}{6}$ to $\frac{1}{3}$ have very little effect upon the weight of the trusses. This is true only for trusses with horizontal lower chords. If the lower chord is *cambered*—that is, raised above the horizontal position—it greatly increases the stresses in the truss, and consequently the weight of the truss. The greater the camber, the greater the weight of the truss, the pitch remaining the same. If the camber is constant, then the greater the stresses (and consequently the weight of the truss), the smaller the pitch. It is advisable not to camber the lower chord unless it is positively necessary. A camber of 5 per cent of the span will increase the weight of the truss from 10 to 40 per cent, according to the pitch.

Taking all things into consideration, a pitch of $\frac{1}{3}$ or $\frac{1}{4}$ is to be preferred over that of $\frac{1}{5}$ or less, since, after the pitch becomes less than $\frac{1}{5}$, the weight of the truss increases quite rapidly, the span being constant.

For any given roof, there is an economical spacing of the trusses. As the spacing of the trusses increases, the weight of the purlins and bracing per square foot of area increases, while the weights of the trusses, the columns that support them, and the girts, or members which run from one column to the other and on which the siding of the building is placed, decrease. The most economical spacing of the trusses is such as will make the *cost* of the above quantities a minimum. It is evident that this spacing for trusses which rest upon masonry supports will be different from the spacing in case they rest upon steel columns. Attention is called to the statement that the sum of the costs, instead of the sum of the weights of the above-mentioned quantities, should be a minimum. This is due to the fact that the unit-cost of the purlins is considerably less than that of the trusses, it being in some cases only about one-half.

The spacing of trusses is sometimes governed by local conditions, such as the placing of the machinery in the building and the probable position of future additions. Considering the spacing from a purely economical standpoint, it is probably well to space trusses about as indicated in Table V.

TABLE V

Spacing of Trusses

SPAN, IN FEET	SPACING, IN FEET
10 to 30	12
30 to 60	15
60 to 75	20
75 to 150	21 to 25

The spacing indicated in Table V is for triangular roof trusses of equal size and span. For other conditions—such as when the main roof consists of one span, and the side roofs consist of different spans and different classes of trusses—the economical spacing may be somewhat different, and is usually less.

The best method of determining the economical spacing is either to make a comparative design or to consult the back volumes of *The Engineering Record*, *Engineering News*, or some other good engineering periodicals. Designs of buildings which have been constructed are frequently given in these periodicals; and from these the student may, in addition to the spacing of the trusses, obtain much other valuable information regarding roof construction.

Bulletin No. 16 of the University of Illinois Experiment Station gives a systematic study of roof trusses, and shows the effect on the variation in the weights of rafters and purlins due to a variation in the length of span. This bulletin, which can be had free for the asking, should be in the hands of the student. It may be had by addressing "The Director," Engineering Experiment Station, University of Illinois, Urbana, Illinois. A most valuable book giving a systematic and extensive study of roof trusses and mill buildings, is "Steel Mill Buildings," by M. S. Ketchum, Engineering News Publishing Company, New York, N. Y.

9. **Stresses in Roof Trusses, and Sizes of Members.** Stresses in roof trusses of any form can be computed by the methods of "Statics" (pp. 23 to 73). On account of the ease and economy of manufacture, some form of truss is usually used in which there are many members with equal stresses. The Fink truss, or some modification of it, is almost universally used (see Fig. 1, *c*, *d*, *e*, *f*). On pages 21 and 22 are shown some forms of trusses, together with the pitches which are commonly used.

The stresses in the various members due to a vertical panel load of one pound are given. To obtain the stress in that member due to any other vertical panel load, multiply the stress here given by the vertical panel load.

For example, if the stresses in U_2L_2 (Fig. 24) or L_0L_1 (Fig. 31) due to a panel load of 3 000 pounds, were required, they would be determined as follows:

$U_2\,L_2$ (Fig. 24) $3\,000 \times -1.73 = -5\,190$ pounds.
$L_0\,L_1$ (Fig 31) $3\,000 \times +5.00 = +15\,000$ pounds

These diagrams are especially useful, since it is the custom of many engineers not to compute the stresses due to wind, snow, and

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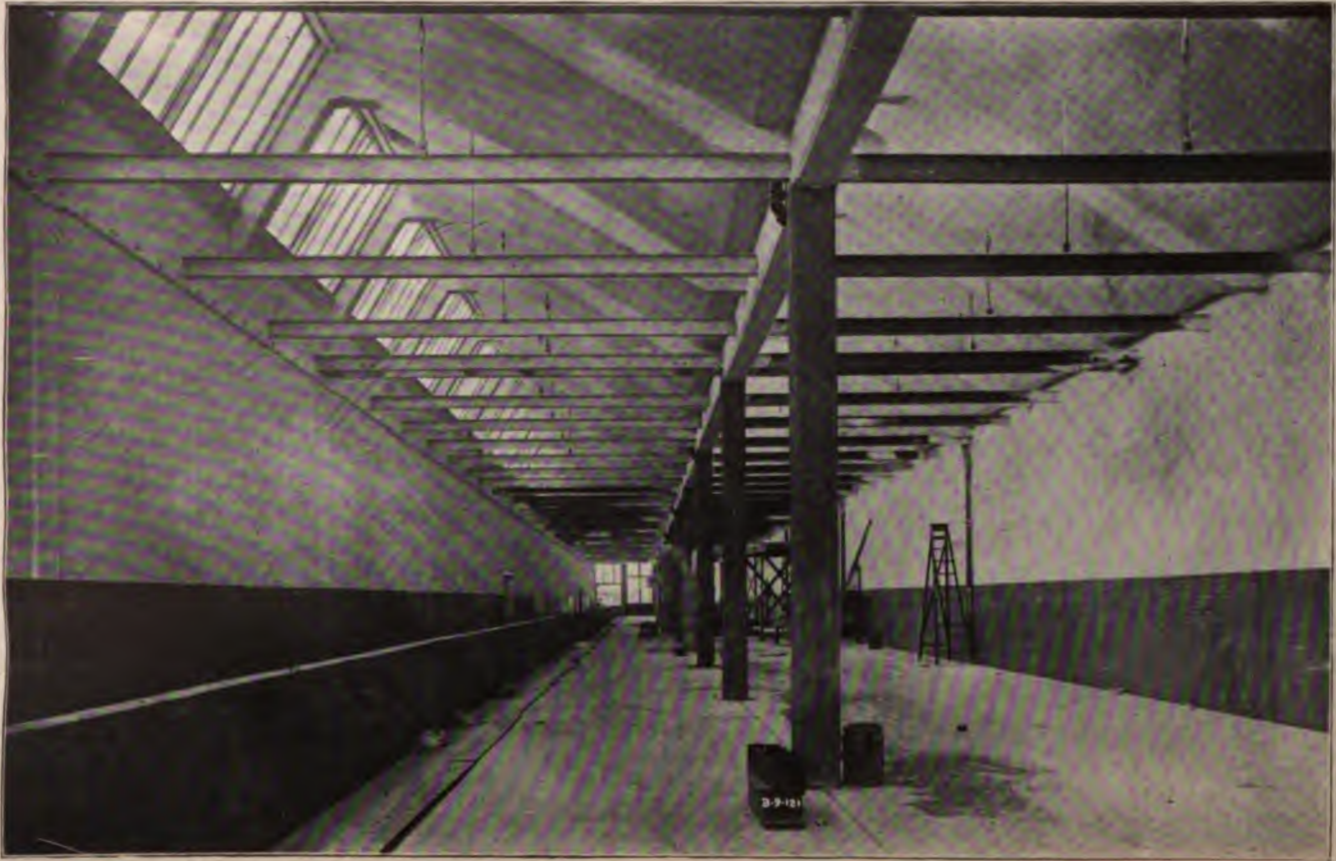


STOCK ROOM IN MANUFACTURING BUILDING OF THE GEORGE N. PIERCE COMPANY, BUFFALO, N. Y.

With the sawtooth roof and windows, side lights are not missed. Kahn System of Reinforced Concrete.

Courtesy of Trussed Concrete Steel Company, Detroit, Mich.

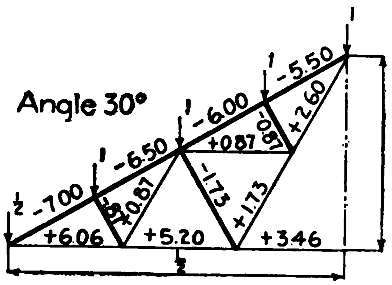


Fig. 24.

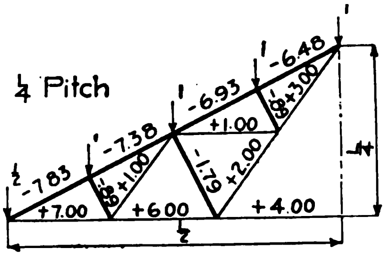


Fig. 25.

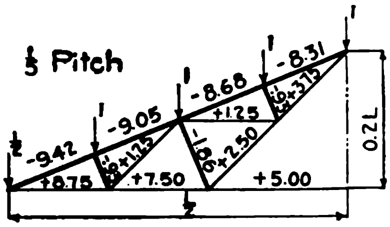


Fig. 26.

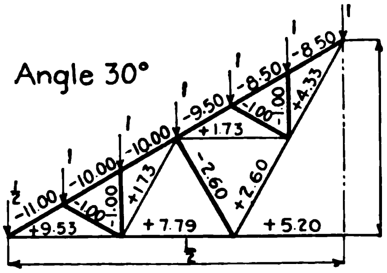


Fig. 27.

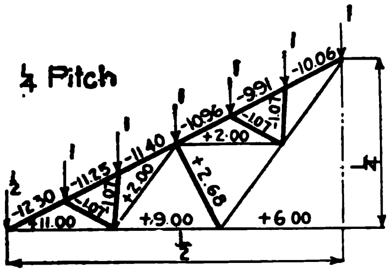


Fig. 28.

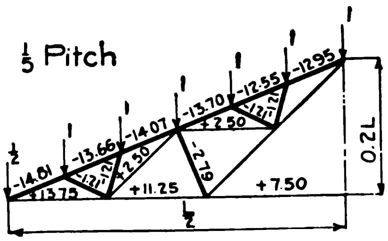


Fig. 29.

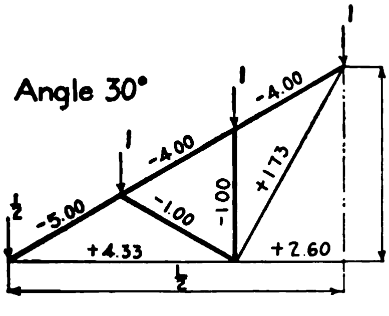


Fig. 30.

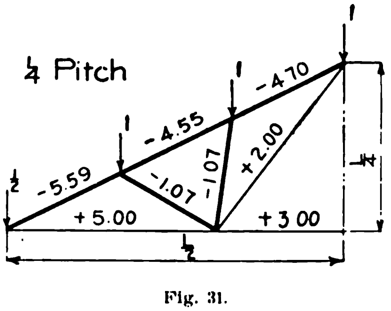


Fig. 31.

Analysis of Stresses in Various Members of Fink Truss Due to Unit-Loads.

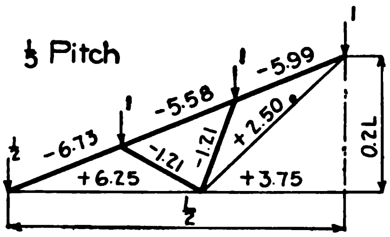


Fig. 32.

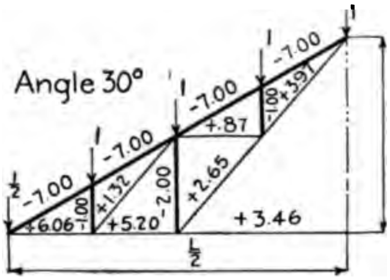


Fig. 33.

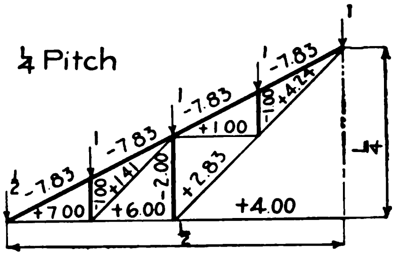


Fig. 34.

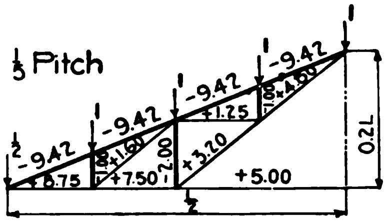


Fig. 35.

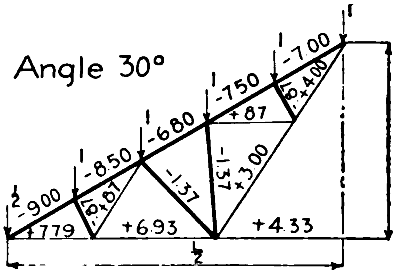


Fig. 36.

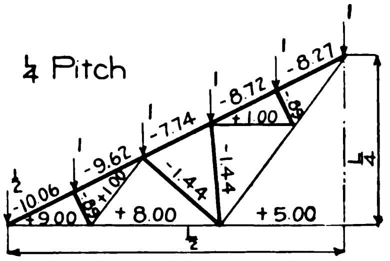


Fig. 37.

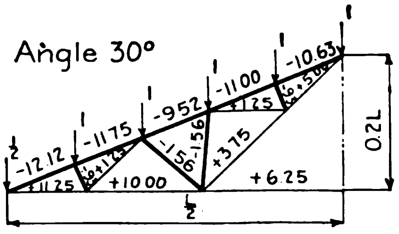


Fig. 38.

Analysis of Stresses in Various Members of Fink Truss Due to Unit-Loads.

dead weight of roof trusses and coverings, but to compute the stresses due to a dead panel load caused by 40 pounds per square foot of horizontal projection. The stresses resulting from this procedure are very nearly equal to those produced by considering the various loads—as snow, dead load, and wind—separately or together. Whenever differences occur, they are on the safe side, except as noted below, and in the next article, in case of the stresses produced by the use of knee-bracing.

The panel load to be used when 40 pounds per square foot of horizontal projection is considered, may be computed from the formula:

$$P = \frac{40 \times a \times l}{n},$$

in which,

a = Distance between trusses, in feet;
l = Span of truss, in feet;
n = Number of panels in top chord of truss.

For example, let it be required to compute the panel load P for the truss of Fig. 24 when the span is 70 feet and the distance between trusses is 16 feet. Here $a = 16$; $l = 70$; and $n = 8$.

$$P = \frac{40 \times 16 \times 70}{8} = 5\,600 \text{ pounds.}$$

The truss would then be computed for a vertical panel load of 5 600 pounds, and the members designed to withstand the stresses thus obtained.

This method is applicable to all spans up to 100 feet when the truss is set on masonry walls (or steel columns built in masonry walls) and the roof covering is of corrugated steel or any of the ordinary materials. Where clay tile or slate are used, 50 pounds should be taken; and in case of concrete slabs, 65 pounds would be about right. It is better practice to compute the stresses due to wind, snow, and dead loads when clay tile, slate, or concrete are used.

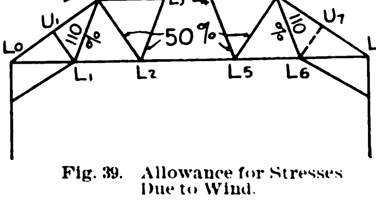


Fig. 39. Allowance for Stresses Due to Wind.

In cases where the roof truss is placed on steel columns and is connected with the column by a knee-brace at the first joint (see Fig. 39), stresses caused by the overturning action of the wind take place in those members shown by heavy lines. In this case the stresses caused by 40 pounds per square foot of horizontal projection are not large enough; but the truss will be safe enough if the stresses as determined by the 40 pounds are increased by the amounts indicated in Fig. 39.

For example, let the truss of Fig. 24 be supported by steel columns and knee-bracing. Let the span be 60 feet, and the distance between trusses 16 feet; and let it be required to compute the stresses in $L_1 U_2$ and $L_3 U_4$. Here $P = (40 \times 16 \times 60) \div 8 = 4\,800$, and the stresses will be:

$L_1 U_2$ $(0.87 \times 4\,800) \times 2.10 = +\ 8\,350$ pounds.
$L_3 U_4$ $(2.60 \times 4\,800) \times 1.50 = +18\,700$ pounds.

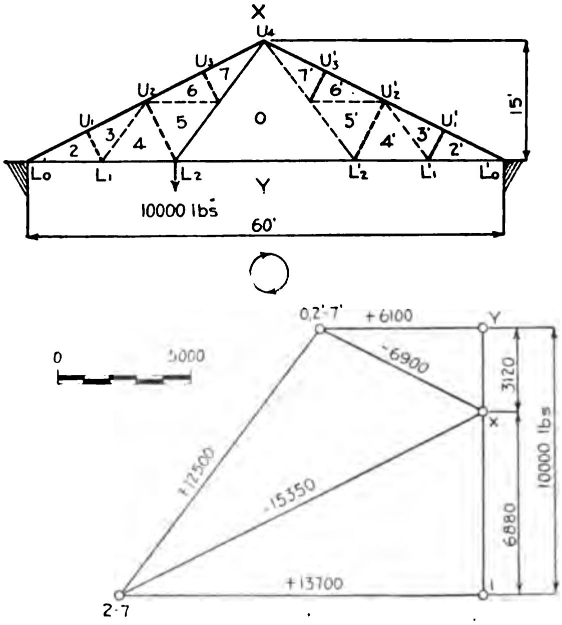


Fig. 40. Fink Truss Loaded with 5-Ton Hoist; also Stress Diagram of Same.

In addition to the above conditions, shafting, heating apparatus, small cranes, and electric wiring and other conductors are often attached to the lower chord of the truss. These cause additional stresses. The case is that of a concentrated load or loads at the lower chord, and the stresses may be computed by the methods given in "Statics."

For example, let a 5-ton hoist be connected as shown in Fig. 40 This hoist runs longitudinally of the shop, or perpendicularly to the plane of the roof truss. The maximum stress in the truss due to this cause will occur when the hoist is directly beneath the truss. The stresses will be those caused by a load of 10 000 pounds at the second panel point of the lower chord. Fig. 40 gives the stress diagram for

TABLE VI

Hoist Stresses in Fink Truss

Member	Stress	Member	Stress
L_0U_4	−15 350	L_2U_4	+12 500
L_0L_2	+13 700	$U_4L'_0$	− 6 900
$L'_2L'_0$	+ 6 100	All Others	0

this condition, and Table VI gives the stress record. From this it is seen that the hoist does not affect all members of the truss. The stresses due to the hoist should be added to those caused by the 40 pounds per square foot of horizontal projection, and the member designed accordingly. Of course, if the stress caused by the hoist decreases the stress caused by the 40 pounds, the member must be designed for the stress due to the 40 pounds.

Note that concentrated loads, as in the case of the hoist, cause different stresses in symmetrical members on the two sides of the truss. In the final design, the members are made the same, being designed for the greatest stress. This is done for the sake of economy in manufacture; and besides, it might be desirable to change the hoist to the other side of the truss.

For Fink trusses with pitches of from ⅙ to ⅓, and spans of less than 100 feet, very light angles are usually required for the members, unless heavy, concentrated loads are placed on the lower chord. The thickness of the connection plates is seldom more than ⅜ inch, the top chord angle seldom greater than 5 by 3½-inch, the lower chord angle seldom greater than 3 by 3-inch; and the web members are usually composed of angles either 2 by 2-inch or 2½ by 2½inch. It appears to be the rule, in present practice, to make the sizes such that the thickness shall be ¼ or $\frac{5}{16}$ inch. Connection plates for spans up to 60 or 70 feet are usually ¼ inch thick, except in the case of that at point L_0.

The stresses in knee-braces depend upon the height and also the width of the building. The stresses may be computed according to the methods of the next article, and the knee-bracing should be designed accordingly. The inspection of a number of plans seems to indicate that the sizes of knee-braces vary from two angles 2½ by 2½ by ¼-inch for spans of 30 feet and a height of building of 35 feet, to two angles 4 by 3 by $\frac{5}{16}$-inch for a span of 70 feet and a height of building to the top of the truss of 75 feet.

In case of roof trusses with the chords nearly parallel (see Fig. 3, p. 2), the stresses, on account of the small depth, are usually quite large, and much heavier members than above mentioned are required. In some cases, 6 by 6-inch angles with 8-inch plates are used, and connection plates of ⅜ to ½ inch are common.

In cases where the trusses are subjected to the action of corrosive gases, the thickness of the members should be made greater than that required by the design alone, since corrosion will decrease the section considerably, and this should be allowed for.

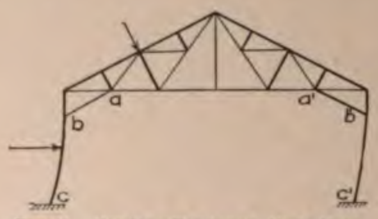


Fig. 41. Bending Tendency, Ends Free.

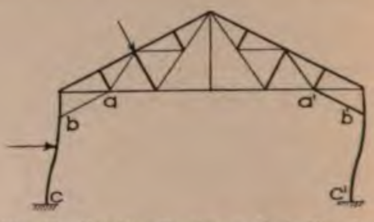


Fig. 42. Bending Tendency, Ends Fixed.

10. **The Steel Truss-Bent.** When a truss is connected to steel columns at its ends and by means of knee-bracing (see Fig. 39), it forms what is called a *steel truss-bent.* The stresses in the truss due to the roof covering and snow loads are the same as when it is supported by a masonry wall; but the wind stresses are different. The wind blowing on the roof and also on the sides of the building, causes stresses in the truss. The wind on the building is transferred to the columns, which, by means of the knee-braces, cause stresses in the truss. The whole bent tends to bend as shown in Fig. 41 if the ends of the columns rest on masonry pedestals. If the ends of the columns are securely bolted to heavy masonry pedestals so that the ends of the post will remain vertical, they will tend to bend as shown in Fig. 42. In the first case, the overturning is resisted by the bending of the post as shown at b and b' (Fig. 41); in the second case, by bending as at b, c, b', and c' (Fig. 42). Since the post is the same size throughout, and the bending caused by the wind the same in both cases, the bend-

ing moment in the post at *b* and *b'* (Fig. 41) is less than what it is at *b* and *b'* (Fig. 42), as in the first case there are only one-half the number of points to withstand the total bending that there are in the second case.

The wind blowing on one side of the building causes a compressive stress in the column on the *leeward* side (the side opposite that on which the wind blows) and a tensile stress in the column on the *windward* side (the same side on which the wind blows). It also creates a bending moment as mentioned above; and this, as well as the direct stresses, must be taken into account when the post is designed. The case is that of a member under direct compression and bending at the same time.

The stresses in the knee-braces and the columns, and the bending in the columns when the ends of the posts are not fixed, may be computed from the following formulæ, in which,

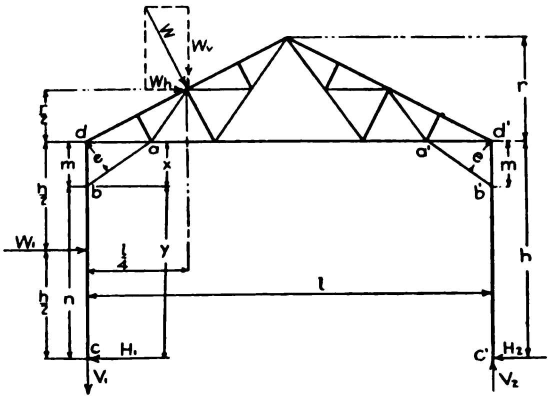


Fig. 43. Notation for Formulæ, Ends Free.

W = Total wind load perpendicular to the roof;
W_h = Horizontal component of W;
W_v = Vertical component of W;
W_1 = Total wind load on the side of the building;
w = Unit wind load normal to the roof;
w_1 = Unit wind load normal to the side of the building;
a = Distance between trusses, in feet.

These and other characters are shown in Fig. 43.

$$W = wa\sqrt{r^2 + \left(\frac{l}{2}\right)^2}$$

$$W_1 = w_1ah$$

$$H_1 = H_2 = \frac{W_h + W_1}{2}$$

$$S_{b'c'} = -\frac{W_1 \frac{h}{2} + W_h \left(h + \frac{r}{2}\right) + W_v \frac{l}{4}}{e} = V_2$$

$$S_{bc} = +\frac{W_1 \frac{h}{2} + W_h \left(h + \frac{r}{2}\right) - W_v \frac{3l}{4}}{e} = V_1$$

$$S_{a'b'} = -\frac{H_2 h}{e}$$

$$S_{ab} = +\frac{H_1 h - W_1 \frac{h}{2}}{e}$$

Bending moment at b = $H_1 - W_1 \left(\frac{h}{2} - m\right)$

Bending moment at b' = $H_2\, n$.

The stresses in the truss caused by the wind are the same as if it were under the action of the normal wind load W, and in addition two concentrated loads equal in intensity and direction to the stresses in the knee-braces and at the same point of application, and two forces E_1 and E_2, which may be computed as follows:

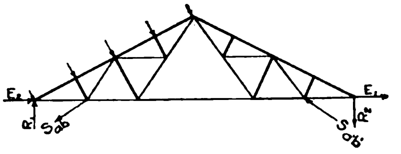


Fig. 44. Application and Direction of the Exterior forces.

$$E_1 = \frac{H_2\, n}{m}$$

$$E_2 = \frac{H_1\, n - W_1 \left(\frac{h}{2} - m\right)}{m}$$

For the points of application for these loads and for their direction, see Fig. 44. The stresses can now be computed by the method of Statics.

The diagram for such a truss-bent is given in Fig. 45. The span is 60 feet, the rise ¼, the distance between trusses 16 feet; and the wind pressure is taken as 18 pounds per square foot normal to the roof surface, and 20 pounds per square foot normal to the sides. In this case, w = 18 pounds; a = 16 feet; r = 60 ÷ 4 = 15 feet; w = 20 pounds; h = 20 feet; n = 14 feet; m = 6 feet; and l = 60 feet. The length of L_0U_4 is readily computed to be 33.5 feet; $L_0\,L_1$, 9.1 feet; and e = 5 feet. The values of the quantities and stresses are computed as follows (see Fig. 46):

W = 18 × 16 $\sqrt{30^2 + 15^2}$ = 9 650 pounds.

W_1 = 16 × 20 × 20 = 6 400 pounds.

W_h = (9 650 ÷ 33.5) × 15 = 4 320 pounds.

W_v = (9 650 ÷ 33.5) × 30 = 8 650 pounds.

H_1 = H_2 = (4 320 + 6 400) ÷ 2 = 5 360 pounds.

$$S_{ab} = +\frac{5\,360 \times 20 - 6\,400 \times 10}{5} = +8\,640 \text{ pounds.}$$

$$S_{a'b'} = -\frac{5\,360 \times 20}{5} = -21\,440 \text{ pounds.}$$

$$E_1 = \frac{5\,360 \times 14}{6} = 12\,520 \text{ pounds.}$$

$$E_2 = \frac{5\,360 \times 14 - 6\,400 \times 4}{6} = 8\,240 \text{ pounds.}$$

The horizontal and vertical components of the stresses in the knee-bracing should now be computed. They are:

For *ab*: horizontal, 7 240; vertical, 4 760 pounds.

For *a'b'*: horizontal, 17 900; vertical, 11 800 pounds.

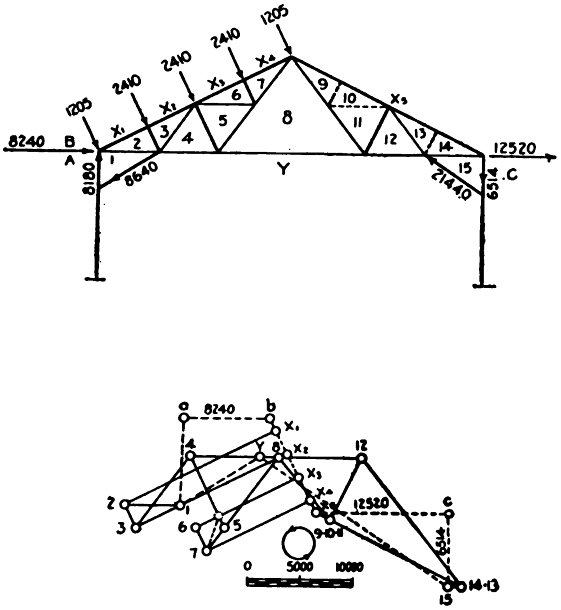


Fig. 45. Stress Diagram of Truss-Bent under Wind Load.

As a check upon the computations, the sum of the values of E_1, E_2, and W_h should be equal to the sum of the horizontal components of the knee-braces. By summing up the above values, it will be seen that they check by 80 pounds, which is less than 0.4 of one per cent and is a close enough check (see Figs. 46 and 47).

To obtain the vertical reactions, proceed as with a simple truss. For R_2, take the center of moments at L_0 (see Fig. 47). Then:

$$R_2 = \frac{1}{60}\left\{8\,650 \times 15 + 4\,320 \times 7.5 + 4\,760 \times 9.1 - 11\,800 \times (60 - 9.1)\right\}$$

$$= -6\,514 \text{ pounds.}$$

The negative sign indicates that the reaction acts downward; that is, the truss must be riveted to the post at L_8, or the end of the post would be lifted off the top of the column.

For R_1, the center of moments is at L_8, and the resulting equation is:

$$R_1 = \frac{1}{60}\left\{-11\,800 \times 9.1 + 8\,650\,(60 - 15) - 4\,320 \times 7.5 + 4\,760\,(60 - 9.1)\right\}$$
$$= +8\,180 \text{ pounds.}$$

The bending moment at b is:

$$M_b = 14 \times 5\,360 - 4 \times 6\,400 = 49\,440 \text{ pound-feet;}$$

and the bending moment at b' is:

$$M_{b'} = 5\,360 \times 14 = 75\,040 \text{ pound-feet.}$$

The forces in their proper direction are now placed on a diagram of the truss (Fig. 47), and the stresses are solved by the method of Statics. The stress diagram is given in Fig. 45, and the stress record in Table VII.

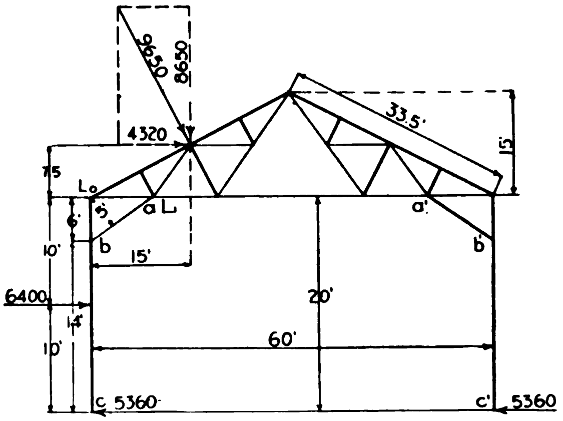


Fig. 46. Position, Direction, and Intensity of Wind Forces, Ends Free.

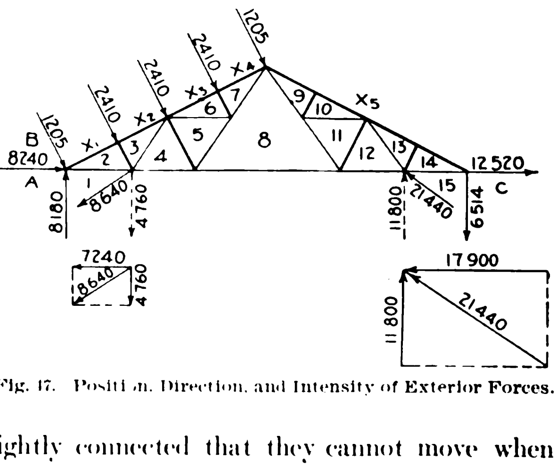


Fig. 47. Position, Direction, and Intensity of Exterior Forces.

The above formulæ are for cases when the columns are free at the lower end. When the columns are not free, they are called *fixed*; that is, they are supposed to be so tightly connected that they cannot move when the post bends as shown in Fig. 42. In such cases the result is the same as if the columns were shortened by an amonut $n \div 2$, and the following formulæ result (see Fig. 48):

TABLE VII

Stress Record of Truss-Bent under Wind Load

Member	Stress	Member	Stress
X-2	−15 700	9-10, 11-12	+ 6 500
X-3	−15 700	12-13, 14	−15 300
X-6	−10 700	Y-4	+ 6 500
X-7	−10 700	Y-8	− 1 900
X-9	+ 1 500	Y-12	− 9 600
X-13	+15 400	13, 14-15	− 1 300
1-2	+ 5 200	1-Y	+ 8 640
2-3	− 2 410	15-Y	−21 440
3-4	+ 8 500	A-1	− 8 180
4-5	− 7 500	c-15	+ 6 514
5-6	+ 2 700	b-c	− 3 457
6-7	− 2 410	b'-c'	− 5 193
7-8	+11 000	9-10-11	0
8-9	− 7 600	13-14	0

$$W = wa\sqrt{r^2 + \left(\frac{l}{2}\right)^2}$$

$$W_1 = w_1 a\left(m + \frac{n}{2}\right)$$

$$H_1 = H_2 = \frac{W_h + W_1}{2}$$

$$S_{ab} = + - \frac{H_1\left(m + \frac{n}{2}\right) - W_1\left(\frac{m + \frac{n}{2}}{2}\right)}{e}$$

$$S_{a'b'} = - \frac{H_2\left(\frac{m}{2} + \frac{n}{4}\right)}{c}$$

$$S_{bc} = + \frac{W_1\left(\frac{m}{2} + \frac{n}{4}\right) + W_h\left(m + \frac{n}{2} + \frac{r}{2}\right) - W_v\frac{3l}{4}}{l}$$

$$S_{b'c'} = + \frac{W_1\left(\frac{m}{2} + \frac{n}{4}\right) + W_h\left(m + \frac{n}{2} + \frac{r}{2}\right) + W_v\frac{l}{4}}{l}$$

$$E_1 = H_2\frac{n}{2m}$$

$$E_2 = \frac{H_1 \frac{n}{2} - W_1 \left(\frac{n}{4} - \frac{m}{2}\right)}{m}$$

Bending moment at $b = M_b = H_1 \frac{n}{2} - W_1 \left(\frac{n}{4} - \frac{m}{2}\right)$

Bending moment at $b' = M_{b'} = H_2 \frac{n}{2}$.

For the truss-bent of Fig. 45, when the columns are fixed at the base, the stresses are the same as if the columns were shortened by an amount $n \div 2$, as above mentioned. The bent would then appear as in Fig. 49, and the values of the various stresses and the quantities, together with their points of application, are:

$W = 18 \times 16\sqrt{30^2 + 15^2} = 9\,650$ pounds, as before.

$W_1 = 13 \times 16 \times 20 = 4\,160$ pounds.

$H_1 = H_2 = \frac{4\,160 + 4\,320}{2} = 4\,240$ pounds.

$S_{ab} = + \frac{4\,240 \times 13 - 4\,160 \times 6.5}{5} = +5\,616$ pounds.

$S_{a'b'} = - \frac{4\,240 \times 13}{5} = -11\,024$ pounds.

$S_{bc} = + \frac{4\,160 \times 7 + 4\,320 \times 20.5 - 8\,650 \times 45}{60} = -4\,526$ pounds.

$S_{b'c'} = - \frac{4\,160 \times 7 + 4\,320 \times 20.5 + 8\,650 \times 15}{60} = -4\,124$ pounds.

$E_1 = \frac{4\,240 \times 7}{6} = 4\,947$ pounds.

$E_2 = \frac{4\,240 \times 7 - 4\,160 \times 0.5}{6} = 4\,600$ pounds.

$M_b = 4\,240 \times 7 - 4\,160 \times 0.5 = 27\,600$ pound-feet.

$M_{b'} = 4\,240 \times 7 = 29\,680$ pound-feet.

The stresses in the bent are then computed in a manner similar to that used when the columns are fixed, E_1, E_2, and the stresses in the knee-braces being attached to the truss as concentrated loads.

Since in this case, E_1, E_2, and the stresses in the knee-braces are less than they are when the columns are free at the base, the wind stresses throughout the truss will be less when the columns are fixed than when they are free.

On account of the difficulty of fixing the ends rigidly, it is advisable always to consider the ends free and to compute the stresses accordingly.

The student is advised not to take the trouble of determining the wind stresses in trusses of steel truss-bents by the method given above, but to use the 40 pounds per square foot of horizontal projection and to correct the stresses as previously mentioned (see Fig. 39).

The formulæ of this article giving the stresses in the knee-bracing and the bending moment in the columns, should be used in all cases, and the posts and knee-braces designed according to the stresses so determined.

In cases where the 40 pounds per square foot is used, the direct stress in each column is:

$$S = \frac{40 \times a \times l}{2};$$

and the column should be designed for this stress, together with the stress due to the bending at the point where the knee-brace joins the column. See "Strength of Materials," pp. 85 and 86.

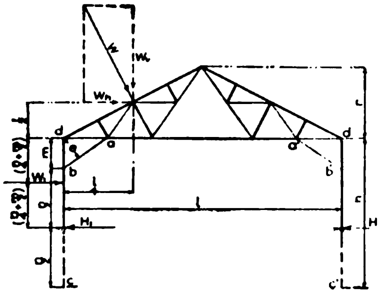


Fig. 48. Notation for Formulæ, Ends Fixed.

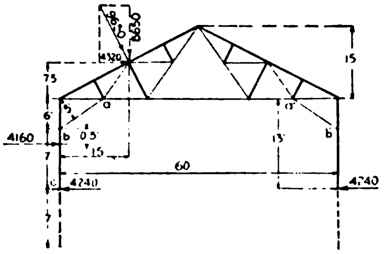


Fig. 49. Position, Direction, and Intensity of Wind Forces, Ends Fixed.

In case a crane is attached to either the truss or the column, the stresses due to its action must be considered in the design.

11. Suspended Loads. Under this head come any loads which may be suspended from the lower chord of the truss. The load may not be actually suspended from the underneath part, but may be placed above, and the connections so arranged as to bring the weight on the lower chord. This weight should preferably be concentrated at a panel point. In case it cannot be brought directly to the panel point, it may be distributed over a portion or all of the panel. In this case the portions distributed to the adjacent panel points are computed, and they are, for purposes of computation, considered as concentrated loads at the panel points. The sections of the chord over which these loads are distributed are in the condition of direct tension and bending, and must be designed for such stresses (see "Strength of Materials," pp. 85 and 86).

The suspended loads may consist of small hand cranes; shafting for transmission of power; heating apparatus, such as steam or hot-air pipes; water or compressed-air tanks; or platforms on which stand the operators for the cranes or hydraulic lifts. Figs. 50 and 51 show trusses with various forms of suspended loads attached.

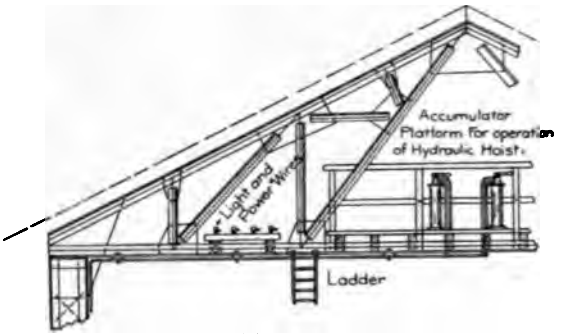


Fig. 50.

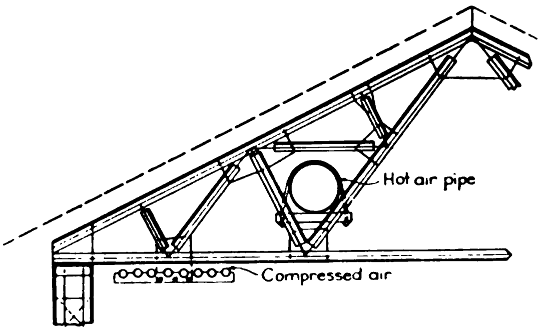


Fig. 51.

Various Kinds of Suspended Loads.

12. **Details of Roof Trusses.** The spans of triangular roof trusses of the Fink type are usually less than 100 feet, and the spans of roof trusses with chords nearly horizontal are seldom greater than 50 feet. For trusses of such spans the details are almost standard. Since these spans and trusses constitute a large majority of those built, only the details of such trusses will be considered in this text.

Where trusses rest on masonry walls or on light columns in masonry walls, provision is made for expansion due to temperature. For trusses up to 75 or 80 feet, slotted holes are placed in the end-bearing, and the bearings rest directly upon another plate. Bolts are fastened to the masonry, and extend upward through the slotted holes and have nuts on their ends. The nuts hold the truss securely to the wall, while the slotted holes allow the bearing to move backward and forward when the temperature falls or rises. *The slotted holes* *should be ⅛ inch in length for every ten feet of span.* The bolts should not be less than ½ inch in diameter, and should be buried in the masonry at least 6 inches. Fig. 52 shows details of an expansion bearing of this character. In case the span of the truss is greater than 75 or 80 feet, a roller or a rocker bearing is used. Figs. 53 and 54 show details of this class of bearings.

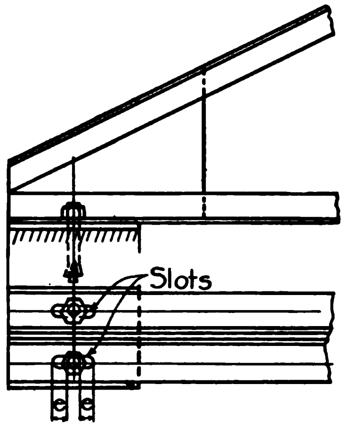


Fig. 52. Slotted-Hole Expansion Bearing.

For convenience in references to the common Fink truss, the following notation will be used: the points in the upper chord are given the letter U, with a subscript corresponding to the number of the joint from the left end. The lower chord and interior joints are given the letter L, with a subscript corresponding to the number of the joint from the left end (see Figs. 24 to 38). The advantage of this system of notation is that it enables one to refer to any particular joint by the use of the letter and its subscript, and its position will at once be apparent to the mind without the use of a figure.

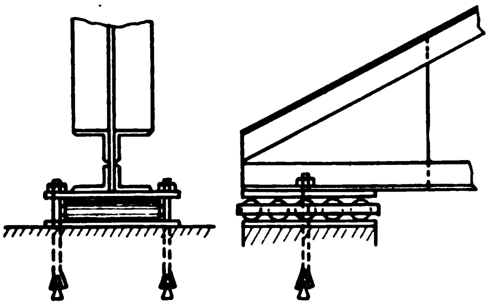


Fig. 53. Roller Expansion Bearing.

If a truss rests on masonry walls, three methods of making the details at L_0 are in common use. These are shown in Figs. 55, 56, and 57. The detail shown in Fig. 55 is the most commonly used; but its use is not advised unless a sufficient number of rivets are placed in the members to take up both the direct stress and that due to the fact that the point of application of the reaction does not coincide with the intersection of the center lines of the chord members.

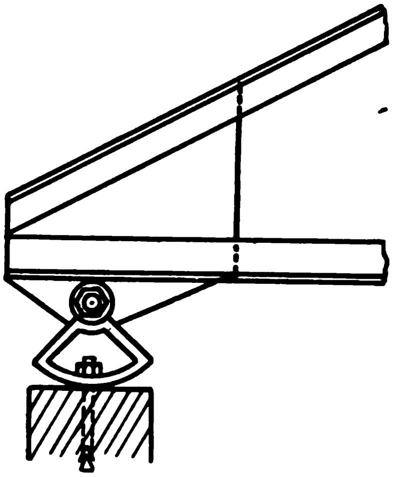


Fig. 54. Rocker Expansion Bearing.

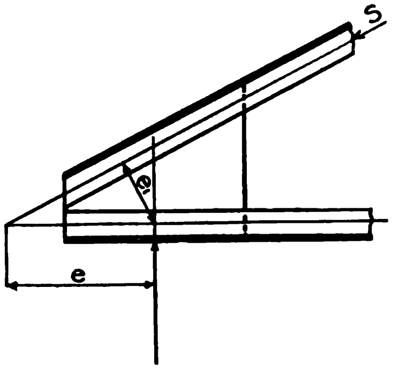


Fig. 55.

In case the bearing shown in Fig. 55 is used, the number of rivets in $L_0 L_1$ may be calculated from the equation:

$$n^2 v - Rn = \frac{6Re}{p},$$

in which,

n = Number of rivets required;
v = Allowable stress on one rivet;
R = Vertical reaction;
p = Rivet spacing, in inches;
e = Distance as shown in Fig. 55.

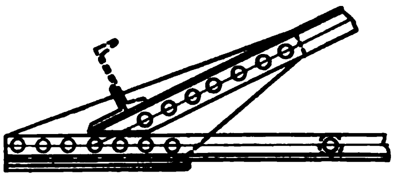


Fig. 56.

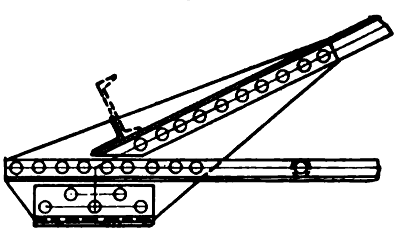


Fig. 57.

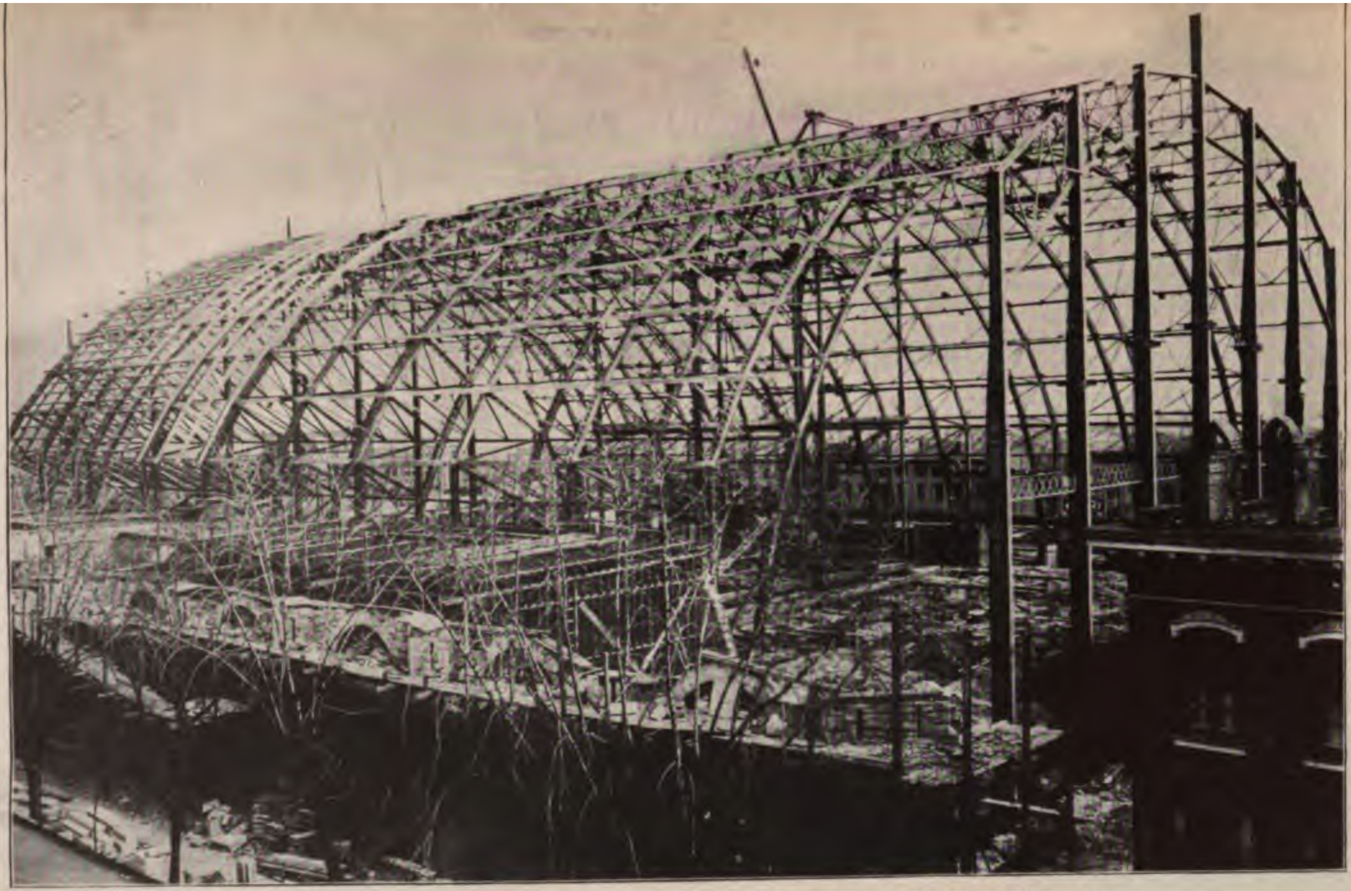
Details of Ends of Roof Trusses.

If the number of rivets in $L_0 U_1$ is desired, it may be calculated from the equation:

$$n^2 v - Sn = \frac{6Se_1}{p},$$

in which S is the stress in $L_0 U_1$, e the distance shown in Fig. 55, and the remaining notation as above.

If the point of application of the reaction coincides with the intersection of the center lines of the top and bottom chords, the number of rivets required to withstand the direct stress, which is the only stress would be equal to the stress in that member divided by the allowable stress in one rivet.



STEEL FRAME OF BUILDING FOR FIFTH REGIMENT ARMORY, BALTIMORE, MARYLAND

Span, 200 ft., centers; length of structure, 300 ft.; total weight of steel work, 1,928,000 lbs.

Courtesy of American Bridge Company.

In order to illustrate the use of the above equation, and to bring out the fact that more rivets are required when the point of application of the reaction does not coincide with the intersection of the upper and lower chords than when it does coincide, an example will be solved. The stresses, the thickness of the connection plate, and the distance of the point of application of the reaction from the intersection of the chord, are as shown in Fig. 58.

It will be assumed that the chords consist of two angles each; and since this is the case, the allowable unit-stress in one rivet will be

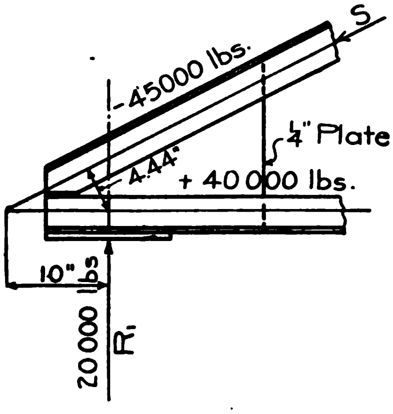


Fig. 58. Data for Example on Page 37.

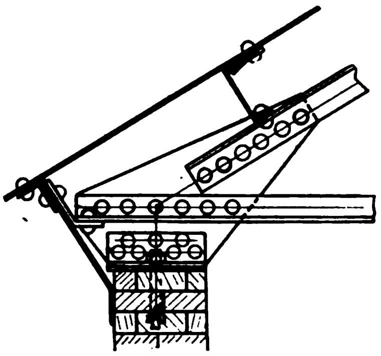


Fig. 59. Eave Detail for Fig. 57.

3 750 pounds, the value of a ¾-inch rivet in bearing in a ¼-inch plate when the allowable unit bearing stress is 20 000 pounds per square inch. If the point of application of the reaction coincides with the intersection of the two chords, the number of rivets required will be:

$$\text{For } L_0\,U_1 \quad \frac{45\,000}{3\,750} = 12.00 \text{ rivets.}$$

$$\text{For } L_0\,L_1 \quad \frac{40\,000}{3\,750} = 10.67 \text{ rivets.}$$

Since the point of application of the reaction does not coincide with the intersection of the chord, the number of rivets required in $L_0\,U_1$ is:

$$3\,750\,n^2 - 45\,000\,n = \frac{6 \times 45\,000 \times 4.44}{3},$$

the spacing being 3 inches; dividing by 3 750, we have:

$$n^2 - 12n = 106.56.$$

Completing the square and solving for n, there is obtained:

$$n = 6 + \sqrt{142.56} = 17.9, \text{ say 18 rivets.}$$

The number of rivets required in $L_0 L_1$ is:

$$3\,750\, n^2 - 20\,000\, n = \frac{6 \times 20\,000 \times 10}{3};$$

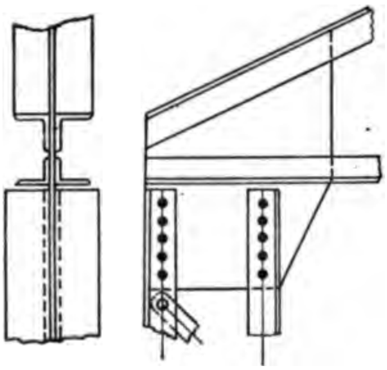


Fig. 60.

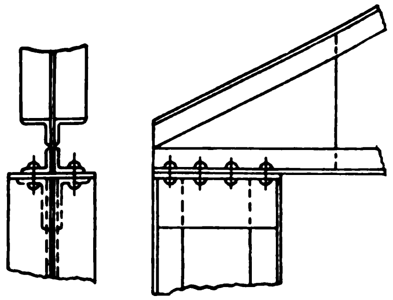


Fig. 61.

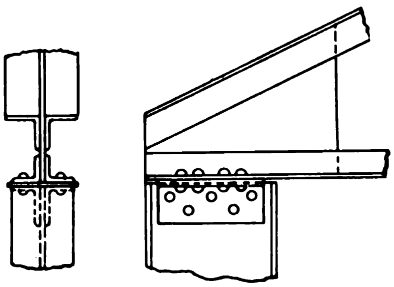


Fig. 62.

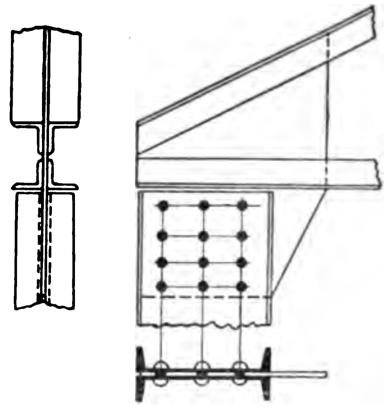


Fig. 63.

Details of Tops of Columns.

and dividing by 3 750 and completing the square, there results:

$$n = 2.67 + \sqrt{113.75} = 13.32, \text{ say 14 rivets.}$$

Inspection of the above results shows that when the point of application of the reaction is placed 10 inches from the intersection of the chords, it requires 6 more rivets in the upper chord and 3 more rivets in the lower chord than would be required if the point of application of the reaction coincided with the intersection of the chords.

The detail just discussed is a very convenient one, and is very commonly used; but in most cases no allowance is made for the additional rivets required because of the fact that the reaction does not coincide with the intersection of the chord members. The student should always compute the rivets by the formulæ given above, since it is very evident that neglect to do so causes the joint to be exceedingly weak, in some cases as much as 50 per cent, as is shown in the case of $L_0 L_1$ in the above problem.

Fig. 56 is excellent, but the length of the bearing plate, which should be as long as the connection plate, is liable to become greater than the width of the wall. In such cases the detail shown in Fig. 57 is to be used. The objection raised to these details is that the end connection plate prevents the placing of a purlin near the end of the roof truss. In case sheathing is used, this objection does not hold good, since the overhanging sheathing will reach to the end of the truss and form a good eave detail, as shown in Fig. 59.

When the roof truss rests on steel columns which are composed of latticed angles, the connections may be made as shown in Figs. 60 and 61. Fig. 60 is preferable, because it gives a more rigid connection than is given by Fig. 61. If the columns consist of two panels placed close together, back to back, the same details may be used. If the column consists of one I-beam or of two channels placed back to back at some distance apart, then details shown in Figs. 62 and 63 may be used.

Where one member is joined to another and makes an angle or is perpendicular to it, then details as shown in Figs. 64 and 65 may be used. It is not good practice to cut the angles as shown at *b* in Fig. 65; *a* is a better detail. No joints should have less than two rivets.

In places where three members meet, and two make the same angle with one of the others, the details should be made as shown in Fig. 66. The leg of the angle which is not joined to the plate should always be upward. This prevents the dust and dirt from becoming mixed with the moisture and running or jarring down into joints at the lower ends of the members.

At L_1 and L_3, square plates (see at left, Fig. 67) should be used where possible. If the stresses are such that more rivets are required in one member than in the other, then the plate should be cut as shown at right in Fig. 67.

At L_2 the splice occurs, since Fink trusses are usually shipped in two parts. In addition to the vertical connection plate, which also acts as a splice plate, the bottom plate is used (see Fig. 68). Rivets shown in black indicate that the holes are left open, the pieces in which they occur are shipped separately, and then are riveted together at the place where the truss is put up.

In some cases where the member L_2 L_5 is long enough to sag considerably, or where it is desired to connect a load (such as a hand hoist) at its center, a vertical U_4 M is run from U_4 and connected to the lower chord. No stress is caused in this member by any load except the load at M, in which case the stress is equal to that load. If a load is at M, it will cause stresses in other members of the truss, the stresses in the truss being the same as if the dead panel load at U_4 were increased by an amount equal to the load at M.

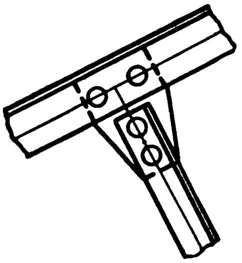


Fig. 64.

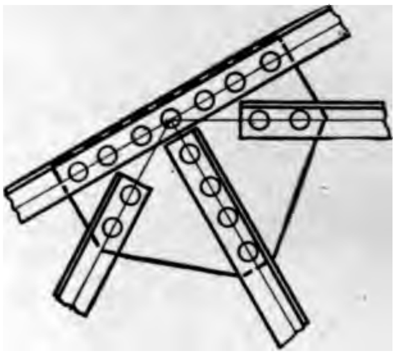


Fig. 66.

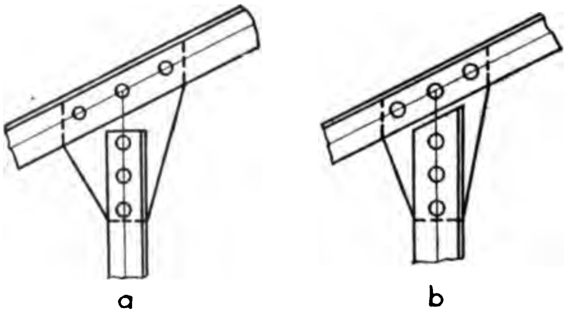
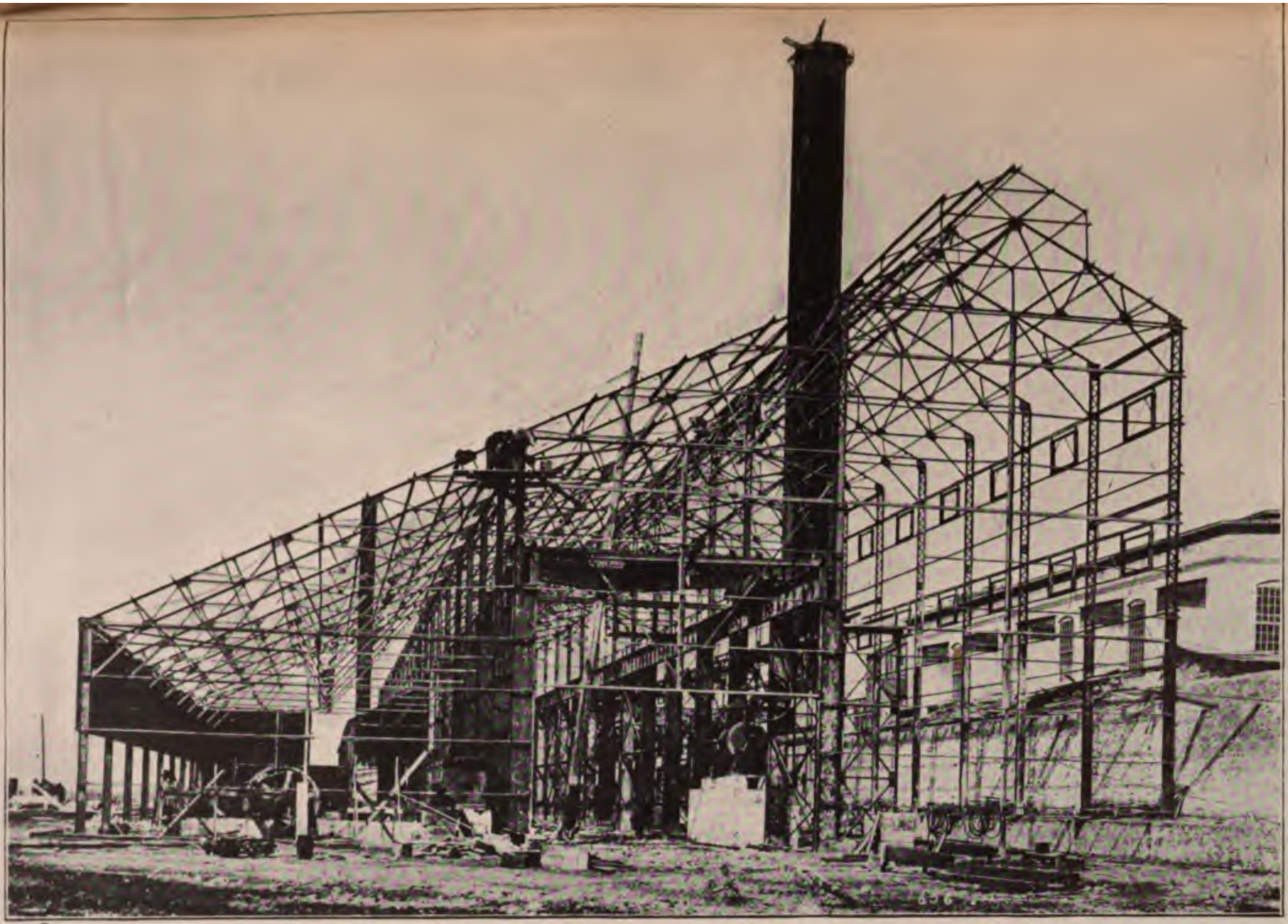


Fig. 65.

Details of Roof Truss Connections. See also Figs. 67 and 68.



SMELTER BUILDING FOR THE UNITED STATES MINING COMPANY, BINGHAM JUNCTION, UTAH

Structure, 100 ft. by 378 ft.; weight of steel work, 870,000 lbs.

Courtesy of American Bridge Company.

The general details of a Fink truss are shown in Plate I (p. 43), Plate II (p. 60), and Plate III (p. 61).

In case the building is devoted to some purpose wherein no smoke or noxious gases are produced, some form of *patent ventilator* may be used. One very excellent make is shown in Fig. 69—called the *Star* ventilator (Merchant & Co., Philadelphia, Pa.). These ventilators are made from 2 to 60 inches in diameter at the lower portion, where they fit to the ridge of the roof. Fig. 70 shows one of them in position on a roof. The number and size of these ventilators depend of course upon the number of times per hour it is desirable to change the air in the shop.

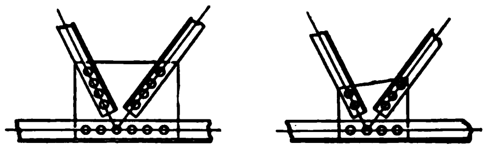


Fig. 67.

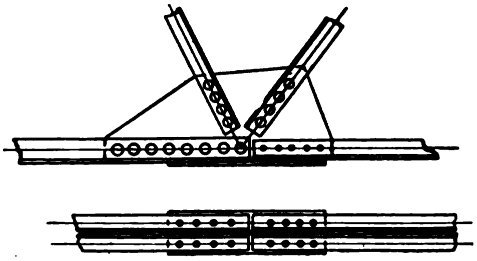


Fig. 68.

Details of Roof Truss Connections. See also Figs. 64, 65, and 66.

In case the shop is for such purposes that smoke, gases, or noxious fumes of any kind are produced, it is desirable to have some channel for ventilation which is considerably larger than those given by the patent ventilators. In such cases the ventilation is usually obtained by a small house-shaped construction called a *lantern*, *monitor*, or *ventilator* (see Fig. 71). The sides of these ventilators may be fitted with *louvres* or windows, or left open. Louvres may be made either of wood or of corrugated or plain bars. For details of monitors and louvres, see Figs. 124, 125, 126.

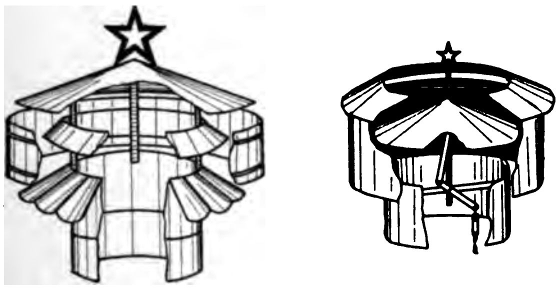


Fig. 69. Details of "Star" Ventilators.

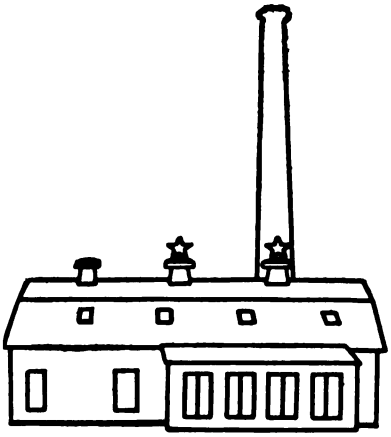


Fig. 70. "Star" Ventilators on a Roof.

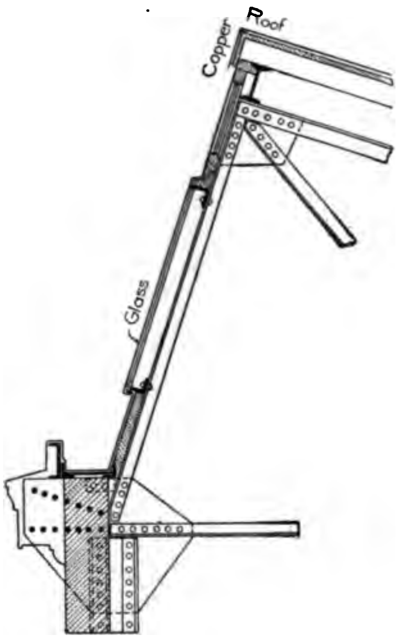


Fig. 72. Detail of Window in Saw-Tooth Roof.

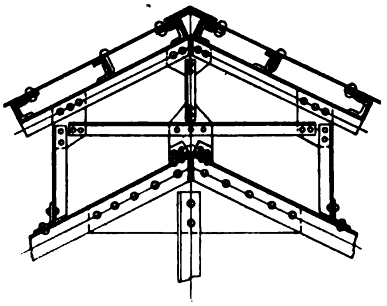


Fig. 71. Detail of a Monitor Ventilator.

In order to admit sufficient light into the building, part of the roof of buildings over 80 feet wide must be made of glass, since the amount of light admitted from the sides of the building is not sufficient to light up those parts of the shop near the center of the trusses. In some cases the saw-toothed truss is used, in which case the entire surface of the short rafter is covered with glass. In case the ordinary triangular roof truss is used, a portion of the roof covering must be made of glass, so put on as to prevent leakage and also to prevent the moisture which forms on the under side of the glass from dropping in the shop. Fig. 72 shows the glass in place on a saw-toothed roof; and Figs. 73 and 74 give the details of several methods of securing glass on the roof so that no leakage or condensation will get onto the shop floor. The glass area should be from $\frac{1}{10}$ to $\frac{1}{4}$ of the floor area.

13. **Specifications for Roof Trusses and Steel Buildings.** In case of an important structure, special specifications are written, embody-

TABLE VIII

Allowable Unit-Stresses, Medium Steel

For Shear	10 000	pounds	per	square	inch.
For Bearing	20 000	"	"	"	"
For Tension	15 000	"	"	"	"
For Bearing of Steel on Masonry	250 to 400	"	"	"	"

For Compression .... $P = 24\,000 - 110\frac{l}{r}$.

In case the stresses are those due to crane loads, the unit-stresses in tension and compression indicated in Table VIII should be reduced ¼ and ½ respectively. Members of the lateral bracing and their connections may be allowed an increase of 25 per cent over the unit-stresses there indicated.

In the equation above given "For Compression," l is the length of the member in inches, and r the least radius of gyration. The ratio of $\frac{l}{r}$ should never be greater than 120.

The *gauge line* or *gauge* is the line on the flange of a shape, on which the rivets are placed. In angles and channels it is located by giving its distance from the back of the shape; in the case of I-beams the distance between two gauge lines on opposite sides of the web is indicated. Some angles have double gauge lines, in which case the rivets are placed first on one and then on the other; this is called *staggering*. Fig. 75 shows gauge lines for various shapes.

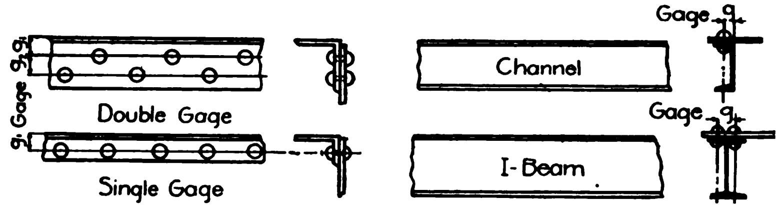


Fig. 75. Gauges for Angles, Channels, and I-Beams.

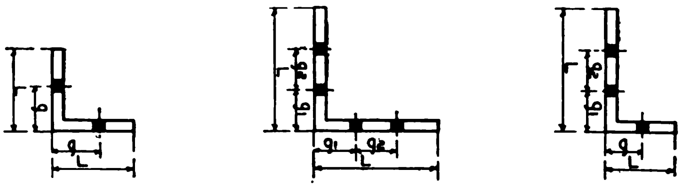
Rivets ¾ inch in diameter are generally used in legs of angles 3 to 4 inches long or greater. For the gauge lines and the maximum sizes of rivets to be used in angles, see Table IX. For similar data for channels and I-beams, see Carnegie Handbook, pp. 177–185.

It is often desirable to express the length in feet instead of inches, in which case the formula becomes:

$$P = 24\,000 - 1\,320\frac{L}{r}.$$

TABLE IX

Gauges and Maximum Allowable Rivets for Angles



L	*g*	Maximum Rivet or Bolt	*L*	*g*	Maximum Rivet or Bolt	*L*	*g*	Maximum Rivet or Bolt
8	4½	⅞	3½	2	⅞	2	1⅛	½
7	4	⅞	3	1¾	⅞	1¾	1	½
6	3½	⅞	2¾	1⅝	¾	1½	⅞	⅜
5	3	⅞	2½	1⅜	⅝	1¼	¾	⅜
4	2¼	⅞	2¼	1¼	⅝	1	$\frac{9}{16}$	¼

L	g_1	g_2	*L*	g_1	g_2
8	3	3	6*	2½	2¼
7	2½	3	5	2	1¾
6	2¼	2½			

* When thickness is ¾ inch or over.

For convenience in designing, the values of $L \div r$ should be plotted as ordinates, and the resulting values of P as abscissæ, on cross-section paper, and the curve drawn in. Then the value of P for any given value of $L \div r$ may be taken at once from the diagram without the labor of substituting in the above formula.

The bearing value of a rivet in a plate of given thickness is equal to the thickness of the plate, *times* the diameter of the rivet, *times* the allowable unit bearing stress. The value of a rivet in single shear is equal to the area of the cross-section of the rivet, *times* the allowable unit shearing stress. The bearing values of rivets of different diameter in plates of different thickness, and the shearing values of rivets of different diameter, are given in Table X, the unit-stresses being as given above.

TABLE X

Bearing and Shearing Values of Rivets

Diameter of Rivet (Inches)	Single Shear (at 10 000 lbs. per sq. in.)	Bearing in Different Thicknesses of Plates (at 20 000 lbs. per sq. in.)							
		1/4 in.	5/16 in.	3/8 in.	7/16 in.	1/2 in.	9/16 in.	5/8 in.	11/16 in.
1/2	1 960	2 500	3 130	3 750					
9/16	2 480	2 810	3 520	4 210	4 920				
5/8	3 070	3 130	3 910	4 690	5 470				
11/16	3 710	3 440	4 290	5 160	6 010	6 880			
3/4	4 420	3 750	4 690	5 630	6 560	7 500	8 440		
13/16	5 180	4 070	5 080	6 090	7 110	8 120	9 150	10 160	
7/8	6 010	4 380	5 470	6 570	7 660	8 750	9 840	10 940	12 040

DESIGN OF A RIVETED ROOF TRUSS

14. Let it be required to design a Fink roof truss of 64 feet span and 1/4 pitch, the distance between trusses being 16 feet. The roof covering is taken as 12 pounds per square foot of roof surface, and the total snow and wind load will be taken as 30 pounds per square foot of horizontal projection. The weight of the steel in the roof truss will be computed from Merriman's formula (see Art. 4, p. 6). The total weight is now found to be:

$$\text{Weight of truss, } \frac{3}{4} \times 16 \times 64\left(1 + \frac{l}{10}\right) = 5\,580 \text{ pounds.}$$
$$\text{Weight of roof cover, } 35.6 \times 2 \times 16 \times 12 = 13\,650 \text{ pounds.}$$
$$\text{Weight of wind and snow } 64 \times 16 \times 30 = \underline{30\,700 \text{ pounds.}}$$
$$\text{Total} \quad 49\,930 \text{ pounds.}$$

Each apex load is therefore 49 930 ÷ 8 = 6 240 pounds. By multiplying this value by each of the stresses as given in Fig. 25, the stress in each member is computed as follows:

$L_0\ U_1 = 7.83 \times 6\,240 = 48\,800$ pounds
$L_0\ L_1 = 7.00 \times 6\,240 = 43\,700$ "
$U_1\ L_1 = 0.89 \times 6\,240 = 5\,580$ "
$U_1\ U_2 = 7.38 \times 6\,240 = 46\,000$ "
$L_1\ U_2$ and $U_2\ L_3 = 1.00 \times 6\,240 = 6\,240$ "
$L_1\ L_2 = 6.00 \times 6\,240 = 37\,450$ "
$U_2\ L_2 = 1.79 \times 6\,240 = 11\,150$ "
$U_2\ U_3 = 6.93 \times 6\,240 = 43\,200$ "
$L_2\ L_5 = 4.00 \times 6\,240 = 24\,950$ "
$L_2\ L_3 = 2.00 \times 6\,240 = 12\,475$ "
$U_3\ L_3 = 0.89 \times 6\,240 = 5\,580$ "
$L_3\ U_4 = 3.00 \times 6\,240 = 18\,725$ "
$U_3\ U_4 = 6.48 \times 6\,240 = 40\,500$ "

In the design of this truss, no material thinner than ¼-inch, and no angles smaller than 2½ by 2-inch, will be allowed.

Fig. 76 shows an outline diagram of the truss, with the stresses placed upon it. A positive sign signifies a tensile stress, and a negative sign signifies a compressive stress. The length of the top chords is $\sqrt{32^2 + 16^2} = 35.6$ feet; and the length of each panel is ¼ of this, or 8.9 feet. The horizontal projection of one panel is ¼ of half the span, or $32 \div 4 = 8$ feet.

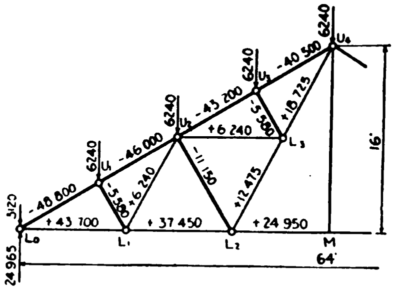


Fig. 76. Stresses in a Fink Truss.

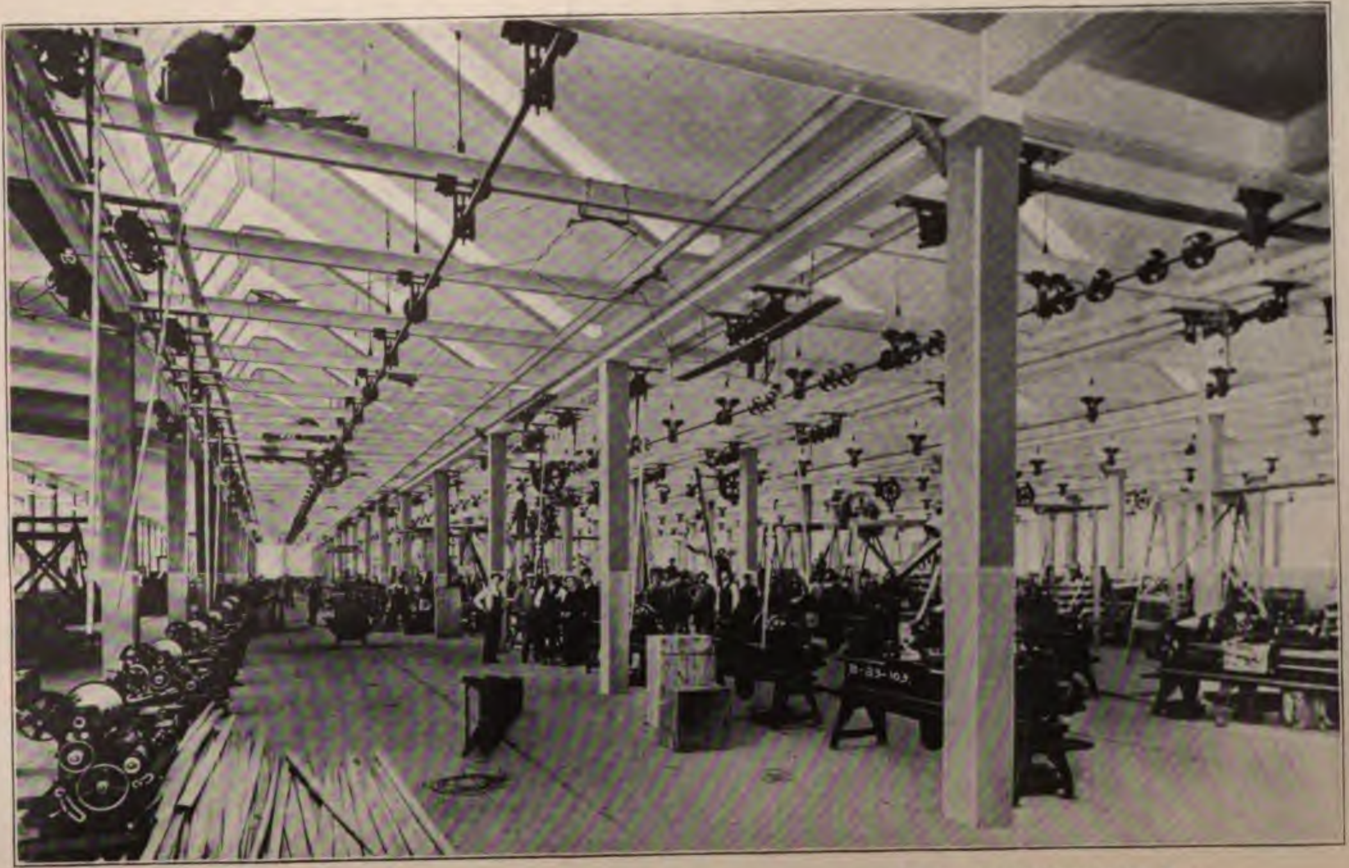
Design of the Purlins. The distance between the trusses is 16 feet, and the distance between the purlins is 8.9 feet; therefore the load coming on one purlin is:

Roof covering, $8.9 \times 16 \times 12 = 1\,710$ pounds
Snow and wind, $8 \times 16 \times 30 = 3\,840$ "
Total $= 5\,550$ pounds

This should be resolved in two components, V and H, perpendicular and parallel to the truss chord. These are determined by the proportions of similar triangles, as follows:

$$V : 5\,550 = 32 : 35.6$$
$$V = 5\,080 \text{ pounds.}$$
$$H : 5\,550 = 16 : 35.6$$
$$H = 2\,490 \text{ pounds.}$$

The bending moment caused by V is $M_v = (5\,080 \times 16) \div 8 = 10\,160$ pound-feet. The bending moment caused by H is $M_H = (2\,490 \times 16) \div 8 = 4\,980$ pound-feet. The stress caused by V is $= \frac{M_v c}{I}$; and the stress caused by H is $\frac{M_H c^1}{I'}$; and there is also the condition that the sum of these two stresses shall not be greater than 15 000 pounds. Since the above formula involves the moment of inertia and half the depth of the beam, a beam must be chosen, and its moment of inertia and half-depth substituted in the above equation, and the equation solved. In case the sum of the stresses is in excess of 15 000 pounds, or very much smaller, a re-computation must be made, using a larger or a smaller beam.



AN INTERIOR VIEW IN MANUFACTURING BUILDING OF THE GEORGE N. PIERCE COMPANY, BUFFALO, N. Y.

Note the system of 8-inch I-beams, spaced 8 ft. 4 in. on centers, designed to carry shafting, motors, fans, etc.; also the stiffeners running from roof to center of each beam, to minimize vibration. Note also the efficient lighting from the double-glazed sawtooth windows above.

Courtesy of Trussed Concrete Steel Company, Detroit, Mich.

A 15-inch 42-pound I-beam will be assumed, and will be examined to see if it fulfils the necessary conditions. The value of I and I' are taken from the Carnegie Handbook, p. 97. The value of c is $\frac{15}{2} = 7\frac{1}{2}$ in the first case, and $\frac{5.50}{2} = 2.75$ in the second case. The quantity 5.50 is the width of the flange of the I-beam. Substituting in the above formula, there results:

$$\frac{10\,160 \times 12 \times 7\frac{1}{2}}{441.8} + \frac{4\,980 \times 12 \times 2.75}{14.62} = 13\,320 \text{ pounds.}$$

The above I-beam could be used; but in case the sheathing is laid closely and nailed tightly, we may consider it acting as a beam of a span of 16 feet, 8.9 feet deep, and of a thickness equal to that of the sheathing, which in this case will be assumed as 1½ inches. The sheathing will then take up the moment caused by the force H; and the purlin will take up the vertical bending moment alone. The stress in the sheathing due to the force H is $\frac{M_H c}{I}$. Here $M = 4\,980 \times 12$; $c = 8.9 \times 12 \div 2$; and $I = \frac{1.5\,(8.9 \times 12)^3}{12}$. Therefore,

$$S = \frac{4\,980 \times 12 \times 8.9 \times 12 \times 12}{2 \times 1.5\,(8.9 \times 12)^3}$$

$= 20.95$ pounds per square inch, which is insignificant.

The vertical bending moment taken up by the purlin is $10\,160 \times 12 = 121\,920$ pound-inches, and this requires a section modulus of $121\,920 \div 15\,000 = 8.14$. By consulting pages 101 and 102 of the Carnegie Handbook, the following is found to be true:

An 8-inch 11.25-pound channel is just too small.

A 7-inch 17.25-pound channel gives the nearest section modulus.

An 8-inch 13.75-pound channel would be lighter and stiffer.

A 9-inch 13.25-pound channel would be still lighter and stiffer; and since it weighs less than any of the others, it will be more economical.

A 9-inch 13.25-pound channel will accordingly be used for the purlins.

On account of one half-panel load coming on the purlin at the ends and ridge of the truss, these purlins must theoretically be only one-half as strong as the other; but, on account of the fact that all purlins must be of the same height, these purlins are made of the lightest weight channel of the same height as the others. In this case it happens that the lightest weight 9-inch channel is required for the intermediate purlins as well as for the end ones. To illustrate the above, suppose that the purlins were required to be 10-inch 25-pound channels, then the end purlins would be made of 10-inch 15-pound channels.

In case sheathing is not used, then some other method must be employed to take up the bending moment due to the force H. The usual method of doing this is to bore holes in the center of the purlins at the middle point of their span, and to connect them with rods which run from one eave up over the ridge and down to the other eave (see Fig. 22).

Design of Tension Members. *For Member* $L_0\ L_1$: The required net area is $43\,700 \div 15\,000 = 2.92$ square inches. By consulting the Carnegie Handbook, p. 118, it is seen that two 3 by 3 by $\frac{5}{16}$-inch angles give a gross area of $1.78 \times 2 = 3.56$ square inches. From this must be subtracted the rivet-hole made by a $\frac{3}{4}$-inch rivet. Since all rivet-holes are punched $\frac{1}{8}$ inch larger in diameter than the rivet, the amount to be substracted from the above gross area is $\frac{5}{16} \times (\frac{3}{4} + \frac{1}{8}) \times 2 = 0.54$, there being two rivet-holes taken out of the section. This gives a total net area of $3.56 - 0.54 = 3.02$ square inches. As this is but slightly larger than the required net area, these angles will be used for this member. Since the stress in this member is the greatest stress in the bottom chord, and since the bottom chord is made of the same section up to the splice at L_2, on account of economical construction, it being cheaper to run the same sized angle throughout than it would be to change the size of each panel and make a splice at each panel point, the size of angle as determined above will be used for the first two panels of the bottom chord at each end.

For Member $L_2\ L_5$: The required net area is $24\,950 \div 15\,000 = 1.67$ square inches. From Carnegie Handbook, p. 115, two angles $2\frac{1}{2}$ by 2 by $\frac{1}{4}$-inch give a gross area of $2 \times 1.06 = 2.12$ square inches; and taking out two $\frac{5}{8}$-inch rivets, the net area is $2.12 - \frac{1}{4}\ (\frac{5}{8} + \frac{1}{8}) \times 2 = 1.74$ square inches. This coincides very closely with the required area, and this angle will be used. Even if this angle should have been in excess of the required area, it would still be necessary to use it, since it is the smallest angle and of the least thickness allowed.

For Member $L_3 U_4$: The required net area is 18 725 ÷ 15 000 = 1.25 square inches. Two angles 2½ by 2 by ¼-inch give a gross area of 2.12 square inches, and a net area of 1.74 square inches, as above computed. Although they give an area considerably larger than that required, nevertheless they must be used, since they are the smallest allowed.

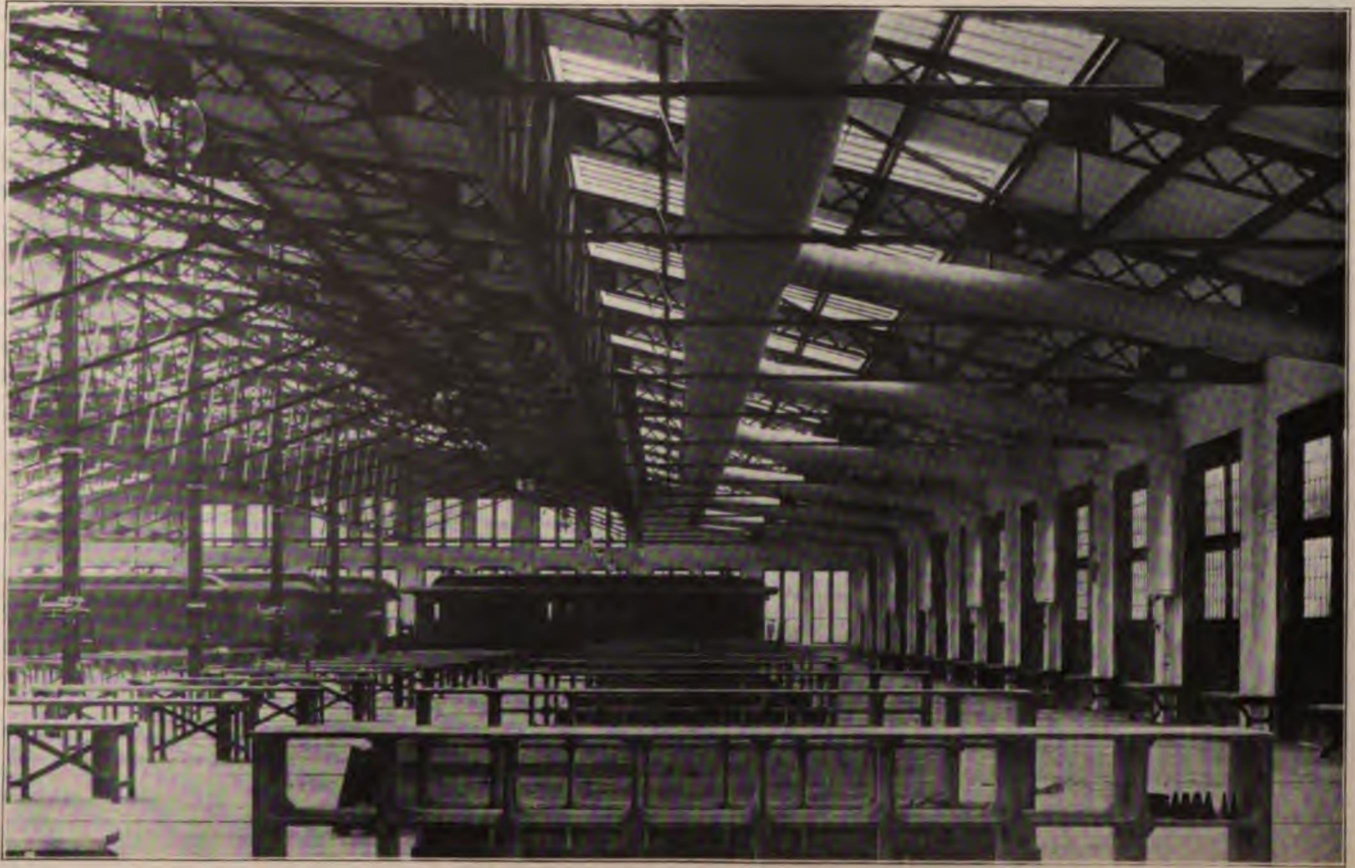
For Members $L_1 U_2$ and $U_2 L_3$: The required net area is 6 240 ÷ 15 000 = 0.42 square inch. One angle 2½ by 2 by ¼-inch gives a gross area of 1.06 square inches. The amount to deduct from this is $\frac{1}{4} \times (\frac{5}{8} + \frac{1}{8}) = 0.19$ square inch, one ⅝-inch rivet-hole being taken from the section. This gives a net area of 1.06 − 0.19 = 0.87 square inch, which shows this angle to be sufficient.

Since the member $U_4 M$ has no other use than to prevent the bottom chord from sagging, it will be made of the lightest angle allowed. It will therefore be made of one angle 2½ by 2 by ¼-inch.

The member $L_2 L_3$ is made of the same section as the member $L_3 U_4$, since this is more economical than to change the section and to make a splice at L_3. It will be made of two angles 2½ by 2 by ¼-inch.

Design of the Compression Members. The general method of procedure in the design of compression members is, first, to assume a cross-section, and then to determine the unit compressive stress allowable by inserting the length of the member and the radius of gyration of the assumed section in the formula given for the unit allowable compressive stress; then divide the stress in the member by the unit allowable compressive stress determined as above. This will give the required area. If this required area is equal to, or slightly less than, the area of the cross-section assumed, the section assumed will be the correct one. If the required area as computed above is greater than the area of the section, then a larger section must be assumed and the operation repeated. Usually only two operations are required in order to obtain a section whose area is correct. It should be noted that the area of the rivet-holes is not deducted from the section in compression members, since the rivet fills up the rivet-hole and makes a section as strong in compression as it was in the first place. Care should be taken to assume a section whose radius of gyration is equal to or greater than the length of the member divided by 120. This is due to the fact that $l \div r$ should not be greater than 120. Compression members of roof trusses for the usual spans are made of two angles placed back to back. The radius of gyration of such a section is equal to the radius of gyration of one angle, if it is referred to an axis perpendicular to the legs which are placed together. If it is referred to an axis through the center of the section and parallel to the legs which are placed together, it is equal to some value other than the radius of gyration of one angle. The radii of gyration for pairs of angles placed either directly back to back or a small distance apart, are given on pages 144 to 146 of the Carnegie Handbook and in Table XI, and should be used in the design. The value of the radius of gyration for sizes of angles other than those given, may be obtained by interpolation.

For example, let it be required to determine the radius of gyration of two 5 by $3\frac{1}{2}$ by $\frac{1}{2}$-inch angles placed $\frac{1}{2}$ inch apart and back to back, the 5-inch legs being horizontal (see p. 146, Carnegie Handbook). Since this value is not given in the tables, it must be interpolated from the values given for r_2 for the above sized angle, which are $\frac{5}{16}$ inch and $\frac{7}{8}$ inch thick. The difference between the two thicknesses given is $\frac{7}{8} - \frac{5}{16} = \frac{9}{16}$ inch. The difference between the two values given for the radius of gyration is $2.55 - 2.44 = 0.11$. This gives a difference of $.11 \div 9 = 0.0122$ for each $\frac{1}{16}$ inch difference in thickness in the angle. The difference between the thickest angle and the angle under consideration is $\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$, or $\frac{6}{16}$. Therefore the amount to be subtracted from the radius of gyration of the thickest angle is $6 \times 0.0122 = 0.0732$; and the radius of gyration for two angles placed back to back as above stated is $2.55 - 0.07 = 2.48$. In case one angle is used for a member in compression, the least rectangular radius of gyration must be used; and if two angles are employed, placed back to back, care should be exercised to use the least radius of gyration; and if the angles have unequal legs, those legs should be placed back to back, which will make the rectangular radii of gyration as nearly equal as possible. The values of the radii of gyration will indicate whether the short legs or the long legs should be placed together. The tables given in the Carnegie Handbook give the radii of gyration for angles spaced at distances $\frac{1}{2}$ inch and $\frac{3}{4}$ inch apart; but since the connection plates of roof trusses are usually $\frac{1}{4}$ inch or $\frac{3}{8}$ inch thick, the values of the radii of gyration should be given for angles spaced $\frac{1}{4}$ inch and $\frac{3}{8}$ inch apart. Such values are given in Table XI.

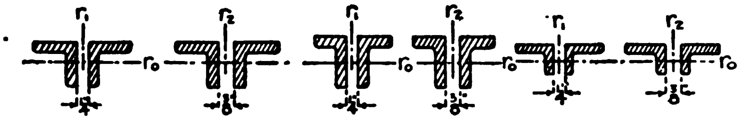


INTERIOR OF PASSENGER-CAR PAINTING SHOP

Showing large amount of glass area per unit of floor area. Fire monitors are shown on each column. The shop is heated with hot air; and the main air-duct, together with the smaller ducts which empty the air directly into the shop, are to be seen at the upper right-hand side. For another view of same shop, see page 282.

TABLE XI

Radii of Gyration of Angles Placed Back to Back



Equal Legs			Unequal Legs				
Size (Inches)	r_1	r_2	Size (Inches)	r_1	r_2	r_1	r_2
2 × 2 × 3/16	0.93	0.98	2½ × 2 × 3/16	0.88	0.92	1.19	1.24
2 × 2 × 7/16	0.98	1.03	2½ × 2 × ½	0.94	0.99	1.25	1.30
2½ × 2½ × ¼	1.14	1.19	3 × 2½ × ¼	1.09	1.13	1.40	1.45
2½ × 2½ × ½	1.19	1.24	3 × 2½ × 9/16	1.15	1.20	1.46	1.51
3 × 3 × ¼	1.34	1.39	3½ × 2½ × ¼	1.04	1.09	1.67	1.72
3 × 3 × ⅝	1.41	1.46	3½ × 2½ × 11/16	1.13	1.18	1.75	1.80
3½ × 3½ × ⅜	1.56	1.61	3½ × 3 × 5/16	1.30	1.35	1.61	1.66
3½ × 3½ × 13/16	1.65	1.70	3½ × 3 × 13/16	1.40	1.45	1.71	1.76
4 × 4 × 5/16	1.76	1.80	4 × 3 × 5/16	1.25	1.30	1.88	1.93
4 × 4 × 13/16	1.85	1.89	4 × 3 × 13/16	1.35	1.40	1.97	2.02
6 × 6 × 7/16	2.58	2.63	5 × 3 × 5/16	1.17	1.22	2.42	2.47
6 × 6 × ⅞	2.66	2.70	5 × 3 × 13/16	1.27	1.32	2.52	2.57
			5 × 3½ × ⅜	1.42	1.46	2.36	2.41
			5 × 3½ × ⅞	1.51	1.56	2.45	2.50
			6 × 3½ × ⅜	1.34	1.39	2.90	2.95
			6 × 3½ × ⅞	1.44	1.49	3.00	3.05
			6 × 4 × ⅜	1.58	1.62	2.83	2.87
			6 × 4 × ⅞	1 67	1.71	2.92	2.97

r_0 = in all cases, the radius of gyration of one angle referred to neutral axis parallel to the horizontal leg as shown above.

For Member $L_0 U_1$: Two angles 3½ by 3 by 5/16-inch, long legs spaced back to back, and ¼ inch apart, will be assumed. The least radius of gyration is 1.10, and the length is 8.9 feet. The area of this section is 2 × 1.93 = 3.86 square inches. The unit allowable compressive stress is:

$$P = 24\,000 - \frac{110 \times 12 \times 8.9}{1.10} = 13\,400 \text{ pounds.}$$

The required area is 48 800 ÷ 13 400 = 3.65 square inches. Since the angles given are of somewhat larger area than that required, it might be well to examine the next smallest angle.

Two angles 3½ by 2½ by 5/16-inch, with a radius of gyration

1.11 and a total area of 3.56 square inches, will be assumed. The unit allowable compressive stress is:

$$P = 24\,000 - \frac{110 \times 12 \times 8.9}{1.11} = 13\,510 \text{ pounds.}$$

The required area is 48 800 ÷ 13 510 = 3.61 square inches. Since the required area is greater than the given area, it shows that these angles are too small. Two angles 3½ by 3 by 5/16-inch will therefore be used for this member, and also for all the members of the top chord, since it is more economical to run the same size throughout than to change the section and make splices at all the upper chord panel points.

For Member $U_2\,L_2$: The length of this member is easily computed from similar triangles, and is found to be 8.9 feet. Two angles 2½ by 2 by 5/16-inch, with the long legs back to back, give a total area of 1.62 square inches and a radius of gyration of 0.78. The unit-stress is computed and found to be 8 950 pounds. The required area is 11 150 ÷ 8 950 = 1.25 square inches. These two angles would be used, but the least allowable radius of gyration is 8.9 × 12 ÷ 120 = 0.89. This is seen to be considerably greater than the radius of gyration given above, and therefore these angles cannot be used, according to Specifications. By consulting the tables, it is seen that two angles 3 by 2½ by ¼-inch are the smallest angles that will give a radius of gyration nearest to the required amount (0.89) and still be standard size angles. Angles marked with a star in the tables are special angles, and can be procured only at a cost greatly in excess of the others, and then only with great delay in delivering except when large quantities are ordered. It may be said that standard angles should never be used.

For Members $U_1\,L_1$ and $U_3\,L_3$: The length of these members is 4.45 feet. The radius of gyration must therefore not be less than 4.45 × 12 ÷ 120 = 0.45. One angle 2½ by 2 by ¼-inch, with an area of 1.06 square inches and a least rectangular radius of gyration of 0.59, will be assumed. The allowable unit compressive stress is:

$$P = 24\,000 - \frac{110 \times 12 \times \frac{8.9}{2}}{0.59} = 14\,050 \text{ pounds.}$$

The required area is 5 580 ÷ 14 050 = 0.40 square inch. The angle chosen gives a much larger area than that required; but since it is the smallest one allowed by the Specifications, it must be used.

Many designers do not place a limit on the value of the radius of gyration, but simply use the compressive formula, and any section whose radius of gyration will bring the required area near to its own area. This should not be the case, since the formula here given is not applicable when the value of $l \div r$ is greater than 120.

Top and Bottom Lateral Bracing. Since the stresses in the lateral bracing are not susceptible of a well-defined mathematical analysis, it cannot be rationally designed. Experience indicates that it should be as in Article 7. The lower chord bracing will therefore consist of single angles 3 by 2½ by $\frac{5}{16}$-inch; and the upper chord bracing, of 3 by 3 by $\frac{5}{16}$-inch angles. This bracing should not be placed between every truss, but should be placed as indicated on the stress sheet, Plate I. If one ¾-inch rivet is taken out of the section of the bottom lateral bracing, it will give a net area of $1.62 - 0.27 = 1.35$ square inches; this could withstand a stress of $1.35 \times 15\,000 \times 1.25 = 27\,000$ pounds, which is the stress the bracing is assumed to carry, and which is to be used in determining the number of rivets for the connection. The stress in the top lateral bracing may be assumed to be the same.

Determination of Number of Rivets Required. It is to be remembered that ⅝-inch rivets are to be used in the 2½ and 2-inch legs of the angles, and ⅜-inch rivets in all larger legs. Field rivets are to have a value equal to ¾ of a shop rivet. Connection plates ¼ inch thick are to be used in all cases, except where the number of rivets required will be greater than 10. In such cases, use a ⅜-inch connection plate. The correct number of field rivets may be determined by multiplying the required number of shop rivets by $\frac{4}{3}$.

Whenever two angles back to back join on a plate, the number of rivets is governed by the bearing on the connection plate; and when one angle is joined to a plate, the number of rivets is governed by single shear if the rivet is ⅝ inch in diameter, and by single shear if the rivet is ¾ inch in diameter and the plate is over ¼ inch thick. The bearing and shearing value of the rivets are taken from Table X, p.47.

Lower End of $L_0 U_1$: Rivets ¾-inch. Plate ⅜-inch.
$48\,800 \div 5\,630 = 9$ shop rivets required.
Upper End of $U_3 U_4$: Rivets ¾-inch. Plate ⅜-inch.
$40\,500 \div 5\,630 = 8$ shop or 10 field rivets
Upper End of $U_4 L_3$: Rivets ⅝-inch. Plate ⅜-inch.
$18\,725 \div 4\,690 = 4$ shop or 6 field rivets.

Lower End of $L_2\,L_3$: Rivets ⅝-inch. Plate ¼-inch.
$12\,474 \div 3\,130 = 4$ shop rivets.
Each End of $U_2\,L_2$: Rivets ⅝-inch. Plate ¼-inch.
$11\,150 \div 3\,130 = 4$ shop rivets.
Each End of $L_1\,U_2$ and $U_2\,L_3$: Rivets ⅝-inch. Plate ¼-inch.
$6\,240 \div 3\,070 = 2$ shop rivets.
Each End of $U_1\,L_1$ and $U_3\,L_3$: Rivets ⅝-inch. Plate ¼-inch.
$5\,580 \div 3\,070 = 2$ shop rivets.

Where U_1L_1 and U_3L_3 join the top chord, two rivets will be required in the top chord.

Since the components of the two diagonals meeting at U_2 are parallel and equal, and opposite to the stress in $U_2\,L_2$, no rivets will be required, theoretically, to hold the plate to the top chord. A sufficient number, however, must be put in to take up the vertical reaction of the purlin. This number is $5\,550 \div 3\,130 = 2$ shop rivets. In practice a greater number are usually put in to prevent vibration and to fill out the plate.

At L_3 a sufficient number of rivets must be placed in $L_2\,U_4$ to take up the difference in stress between $L_3\,U_4$ and $L_2\,L_3$. The number required is $(18\,725 - 12\,475) \div 3\,130 = 3$ shop rivets.

At the end L_0 of the member $L_0\,L_1$, there is a horizontal stress of 43 700 pounds, and a vertical force equal to the reaction, which is $49\,930 \div 2 = 24\,965$ pounds (see Fig. 76). The force acting on the rivets in this member is the resultant of these two forces, and is:

$$\sqrt{43\,700^2 + 24\,965^2} = 50\,300 \text{ pounds.}$$

Since the rivets are ¾-inch and the plates ⅜-inch, the number of rivets required is $50\,300 \div 5\,630 = 9$ shop rivets. This number should be placed symmetrically with respect to the intersection of the two chords. In case the point of application of the reaction had not coincided with the intersection of the chords, the number of rivets must be computed according to the formula on page 36.

For the joint at L_1, a sufficient number of rivets must be put in, in order to take up the difference in stress between the members $L_0\,L_1$ and $L_1\,L_2$. The number required is $(43\,700 - 37\,450) \div 3\,750 = 2$ shop rivets.

The purlins have a horizontal shear at each end, of $H \div 2 = 2\,490 \div 2 = 1\,245$ pounds. This requires $1\,245 \div 4\,420 = 1$ shop rivet or 1 field rivet, to keep them from sliding down on the top chord. Clip angles 5 by 3½ by ⅜-inch will be used as shown in Plate I.

These help in the erecting of the purlins,. since they are shop-riveted to the truss and therefore hold the purlins in place while they are being field-riveted to the truss and to the clip angles (see Fig. 77).

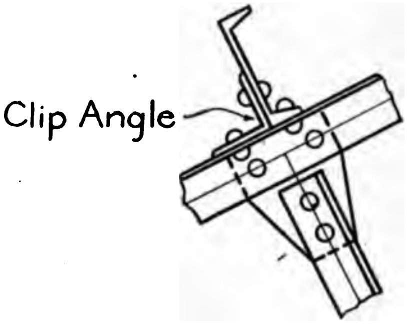


Fig. 77. Detail Showing Clip Angle Connection.

Rivets in Lateral Bracing. The plates of the lateral bracing should be made ¼ inch thick. The 3-inch leg of the angle will be placed against the plate. Rivets ¾ inch in diameter can then be used, and the strength of the joint will be governed by bearing in the ¼-inch plate. The stress for which the rivets are to be determined is given on p. 55. It is 27 000 pounds. The number of field rivets in bearing in ¼-inch plate, required to withstand the stress, is (27 000 ÷ 4 420) × $\frac{4}{3}$ = 9. The size and shape of the plate can be determined only while making the detailed drawing (see Plate III, p. 61).

Design of the Splice. The general details of the splice will be as shown in Fig. 68. The plate underneath will be made ¼ inch thick, the same thickness as the vertical connection plate at this point. Note that the member on the left-hand side of the splice must have ¾-inch shop rivets, and the member on the right-hand side must have ⅝-inch field rivets. The total number of rivets on either side of the splice must be sufficient to take up the entire stress of the member through which they are driven. If eight ⅝-inch field rivets are driven through the horizontal legs and the bottom splice plate, and five ⅝-inch field rivets are driven through the vertical plate and legs of the angles (see Fig. 78), the total strength of the joint, remembering that the rivets are ⅝-inch, will be:

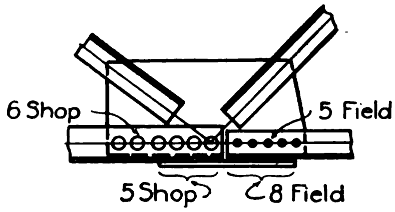


Fig. 78. Detail of Lower Chord Splice.

$$
\begin{array}{rl}
8 \times \frac{2}{3} \times 3\,070 = & 16\,370 \text{ pounds.} \\
5 \times \frac{2}{3} \times 3\,130 = & 10\,430 \text{ pounds.} \\
\hline
\text{Total} & 26\,800 \text{ pounds.}
\end{array}
$$

Note that the rivets through the bottom splice plates are governed by single shear; and those through the vertical plate, by bearing in the plate. Since 16 370 pounds is the value of the rivets through the bottom splice plate, this amount will be transmitted to the other side, where it must be taken up by shop rivets. Bearing in the plate governs the number of $\frac{3}{4}$-inch shop rivets required. This number is 16 370 ÷ 3 750 = 5. Since 16 370 pounds of the stress in the member $L_1 L_2$ is taken up by these 5 shop rivets, the remainder, 37 450 − 16 370 = 21 080 pounds, must be taken up by the rivets through the vertical connection plate. This requires 21 080 ÷ 3 750 = 6 shop rivets.

Since 16 370 pounds is transmitted from one side of the splice to the other by means of the bottom splice plate, this plate should be 16 370 ÷ 15 000 = 1.09 square inches in net section. The net width, the plate being $\frac{1}{4}$ inch thick, is 1.09 ÷ 0.25 = 4.36 inches. If two $\frac{3}{4}$-inch rivet-holes are taken out of the section, the entire width of the plate must be $4.36 + 2\left(\frac{3}{4} + \frac{1}{8}\right) = 6.11$, say 7 inches wide. The length of the plate must be sufficient to get in the number of rivets, and this length is determined in detailing.

Design of the Masonry Plate. If this truss rested upon a masonry wall, it would require a bearing of (49 930 ÷ 2) ÷ 250 = 100 square inches. The width of the plate cannot be less than twice the width of the legs of the bottom chord angle, nor should it extend outside the legs of the chord angle more than 3 inches on each side. The masonry plate will be assumed as 12 inches wide, in which case it must be 100 ÷ 12 = 8.34, say $8\frac{1}{2}$ inches long. The thickness should be $\frac{1}{2}$ inch.

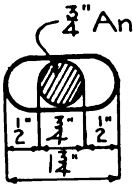


Fig. 79. Slotted Hole for Truss of Fig. 76.

Temperature Allowance. Slotted holes must be put in one end of the truss, to allow for a variation of 150 degrees in temperature. A common rule is to allow $\frac{1}{8}$-inch expansion for every ten feet of span. The total allowance for expansion is $64 \times \frac{1}{8}$ = say, 1 inch. Since the bolts which go through this hole are $\frac{3}{4}$ inch in diameter, the hole must be long enough to allow for $\frac{1}{2}$ the expansion on each side. The width of the hole should be $\frac{1}{4}$ inch greater than the diameter of the bolt (see Fig. 79).

Connections to the Posts. If the truss rests upon posts at the end, sufficient rivets must be driven through the posts and the end connection plates to take up the end reaction, which (see page 38) is 24 970 pounds. Since the rivets are field rivets, this will require 24 970 ÷ 3 750 = 7. This number is to be used in case the posts are built in masonry walls. In case the truss has knee-braces and the walls of the building consist of steel framework, the reaction due to the wind must be added to the above.

15. **The Stress Sheet.** This should also be somewhat of a general drawing, showing the details. It should give an outline sketch of the building, showing bays, the distances between trusses, and the bracing in the plane of the top and bottom chords. See Plate I, p. 43, which is a stress sheet of the truss designed in Article 14. While not necessary, it is very convenient to have the required number of rivets noted upon the stress sheet.

16. **The Detail Drawing.** The stress sheet, in the matter of sizes, gives general dimensions only. It would be impossible for the shop men to make a truss from the stress sheet.

The shop or detailed drawings must be prepared by the draftsman. These drawings must show the exact number of rivets, and their positions, the dimensions of every plate, member, and purlin. The placing of the dimensions so that it will be unnecessary to add or subtract in order to get another desired dimension, is quite an art, and can be attained only through experience or from the study of correctly detailed work. Plates II and III give the shop drawings for a roof truss and the bracing. These are made according to the latest and best practice, and a thorough study of them will be a help to an intelligent design of the trusses.

All members and plates which are to be riveted together in the field should be given a mark. This mark should be painted on the member or plate, and also marked on the *Marking* or *Erection Diagram* (see Plate IV). This diagram is a sketch, with the pieces in their proper position and the correct mark placed upon them. For example, if it is desired to rivet into place the first panel of the lower lateral system, the men look on the marking diagram and see that plates Pl_7, Pl_8, and Pl_9, and the laterals $BL1$, $BL2$, and $BL3$ are required. They would then go to the place where all the trusses are piled up, and pick out the plates and members with these marks upon them. They would then rivet Pl_7 at L_0, Pl_8 or Pl_9 at L_2, then $BL1$,

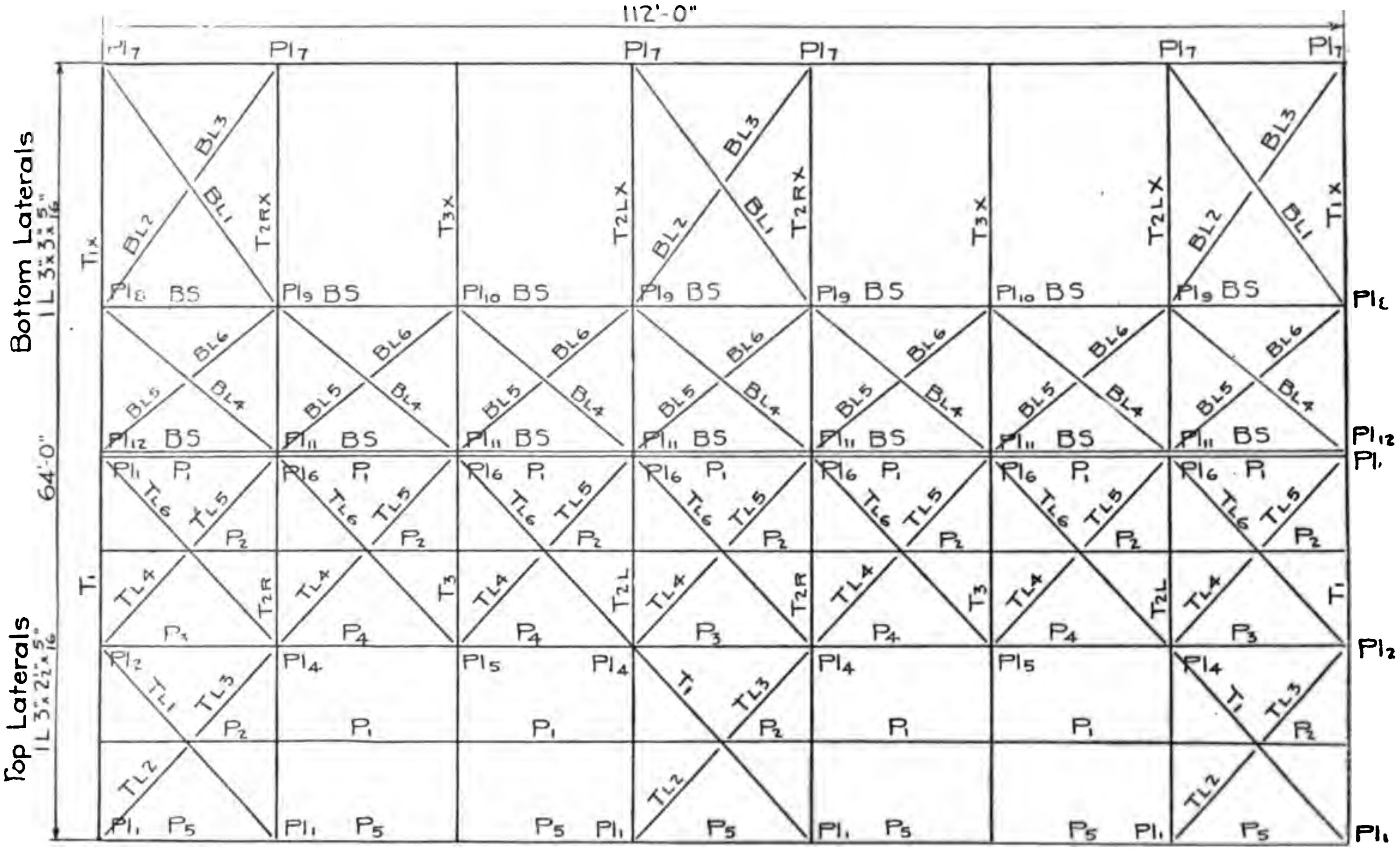


Plate IV. Marking or Erection Diagram.

then *B L* 3, and finally *B L* 2, all of which are shown on the Marking or Erection Diagram.

Care should be taken to give each piece that is different from others in any way whatsoever, a different mark. For instance, the purlins are the same size, and differ only in length and on account of the fact that one has holes in the bottom flange (see Plate III).

Plate IV gives the roof marking and erection diagram for the roof trusses of Plates II and III. Note that the roof truss on Plates II and III is not the same as that of which Plate I is a stress sheet.

17. **Estimate of Cost.** A rough estimate of the cost of steel in the roof may be obtained by multiplying the weight of the purlins, in pounds, by 2½ cents; then adding to this the result obtained by multiplying the weight of all the steel in the trusses and the bracing by 3½ cents. This will give the cost of the steel work in place with two coats of paint. This will give the cost closely enough for an Engineer's estimate; but should a Contractor desire to bid, a detailed estimate should be made as indicated in the remainder of this article.

The cost of the roof covering may be approximately determined according to the prices given in Article 5, but may be more accurately obtained by asking a Contractor for a figure which his experience will indicate as correct.

Paint of various kinds may be bought in open market. Table XII gives some of the kinds used in painting structural steel, together with the amount of surface one gallon will cover.

TABLE XII

Surface Covered per Gallon of Paint*

PAINT	SQUARE FEET	
	1 coat	2 coats
Iron Oxide (powdered)	600	350
" " (ground in oil)	630	375
Red Lead (powdered)	630	375
White Lead (ground in oil)	500	300
Graphite " " "	360	215
Black Asphalt	515	310
Linseed Oil	875	...

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*Pencoyd Handbook, 1898, p. 293.

One gallon of paint will give two tons of structural steel the first coat, or $2\frac{1}{2}$ tons the second coat. The cost of one coat of paint in the shop is 45 cents, and two coats after erection $1.80 per ton of structural steel.*

The detailed estimate of the cost of steel includes several items which are given in Table XIII. In each case the weight of the steel on which the work is done must be multiplied by the unit-cost, and the sum total of all the costs will be the total cost of the entire roof or building. Table XIII gives the various operations which go to make up the cost, and also the unit-costs. Note that the costs vary considerably. This table is given as a rough guide. In order to analyze intelligently the cost in this manner, great experience or access to the cost records of some structural steel company is necessary.

TABLE XIII

Analysis of Cost of Roof Trusses and Mill Buildings†

OPERATION		COST PER TON
Raw Material		$37.00 to $40.00
Work done at Rolling Mills (mill work)		3.00 " 7.00
Work done in Bridge Shops	Columns	14.00 " 20.00
	Trusses	12.00 " 25.00
	Girders	12.00 " 25.00
Work done in Drafting Room	Purlins	.30 " 1.00
	Trusses and Buildings	2.00 " 8.00
Painting		1.50 " 3.00
Shipping (depends upon freight rates)		....
Erection		5.00 " 15.00

It is not to be supposed that all of the operations indicated in Table XIII are made on one piece. Usually pieces which have mill work done on them require no shop work. In such cases a saving of freight is effected, since the material may be shipped directly from the mill to the place of erection.

MILL BUILDING

18. **Definitions and Description.** A *mill building* consists of a roof supported either on steel columns, on steel columns built in

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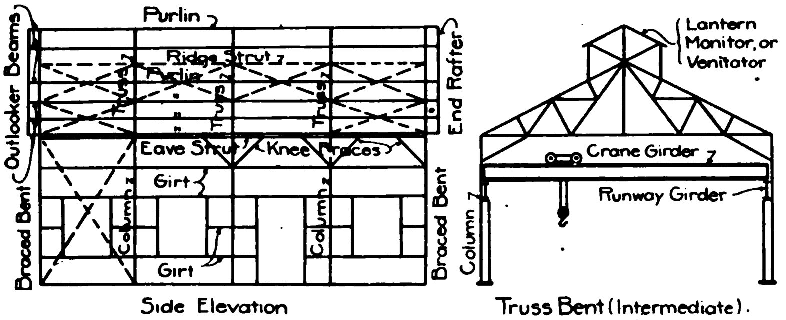
*Pencoyd Handbook, 1898, p. 293.

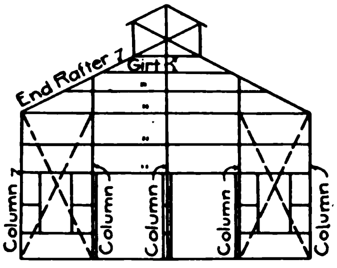
†Compiled from Ketchum's "Steel Mill Buildings."

masonry walls, or on masonry walls alone. The roof may consist of any of the forms of roof trusses that have previously been mentioned; and the roof covering, which may rest on purlins, or on rafters and purlins, may consist of any of the roof coverings mentioned in Article 5. In case the roof is supported on steel columns, the columns are connected at their tops by a strut called the *eave-strut*; and they are also connected at certain distances throughout their height by horizontal members called *girts*. The building may or may not have a monitor ventilator on top. See Fig. 80 for general form of mill buildings, together with the names of the various parts.



Side Elevation
Dotted Lines Indicate Lateral Bracing

Truss Bent (Intermediate).



Braced End Bent

Fig. 80. Physical Analysis of a Mill Building.

The eave-struts and the girts are used as a framework on which to place the covering for the walls of the building. This covering may be of wood, of wire lath and plaster, or of corrugated steel. The eave-strut may also act as the end purlin.

Since the majority of mill buildings have their roofs and sides covered with corrugated steel, the remainder of this text will be devoted to mill buildings with this kind of covering.

19. Types of Buildings. Mill buildings may be classified according to their width and the number of bays which they have. A building may consist of one center bay (see Fig. 81). In this case

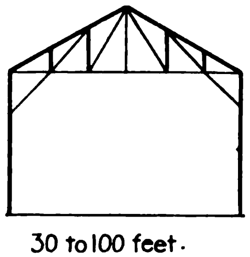


Fig. 81.

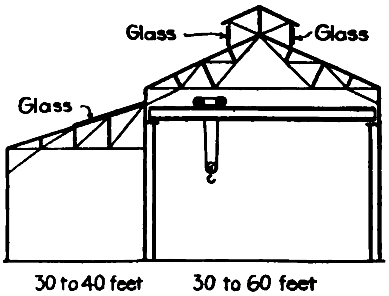


Fig. 82.

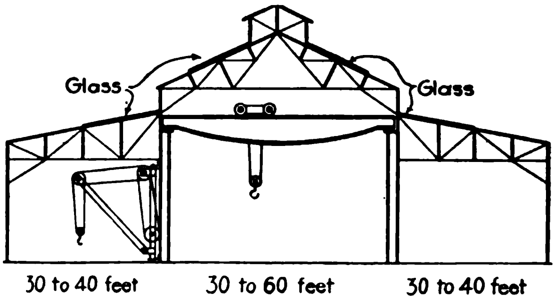


Fig. 83.

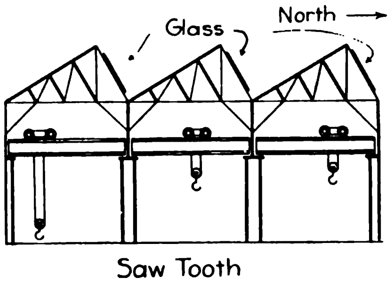


Fig. 84.

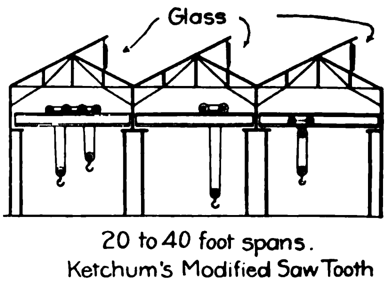


Fig. 85.

Cross-Sections of Mill Buildings.

the span may vary from 30 to 100 feet. Usually side windows give sufficient light, no skylights being required in the roofs or monitor if the span is less than 80 feet.

The building may consist of one center bay and one or two side bays, as shown in Figs. 82 and 83. The truss of the center bay is usually of the Fink type, and in most cases is supplied with skylights and lights in the monitor. The side trusses for the most part consist of that type in which the chords are nearly parallel. The center bay is generally not more than 60 feet in span. This is due to the fact that the crane girder would be unnecessarily heavy if a longer span were used. The side spans are usually from 30 to 40 feet.

In case it is desirable to have the building wider than 150 feet, and still have it lighted by natural light, the common saw-tooth roof (Fig. 84) or Ketchum's modified saw-tooth (Fig. 85) is used. In such cases the bays are seldom greater than 40 feet. Cranes may be placed in one or all of the bays. One great advantage of this type of roof is that it gives a good light uniformly throughout the entire shop; and at any time it is desired to widen the shop, additional bays may be added. The shop may also be lengthened by adding additional trusses at the end. Of course, shops of the first two types mentioned may be widened by addition of extra bays; but the connection to the old work will be unsatisfactory, and skylights will have to be placed in the roofs both of the old bays and of the new ones. For views showing the interior of shops, see pages 77 to 84.

20. **General Requirements.** The general requirements of a mill building depend in detail on the purpose for which it is intended. The requirements which are common to all classes of buildings are ventilation, good light, and transportation facilities both inside and outside the shop.. The question of light and ventilation is discussed on pp. 42 and 66. In regard to transportation facilities, it may be said that either the building should be placed so close to a railway track that the material may be unloaded by means of a crane and hauled into the shop, or the track must run into the shop so that the material may be unloaded and placed on the stock floor by means of a crane in the center bay and wall jib-cranes (see Fig. 109, p. 83) or by means of hand trucks.

21. **Layout.** The purpose for which the building is intended, and its relative location in regard to transportation facilities, will determine its layout. For manufacturing purposes, it should be so laid out that the materials will always pass *forward* in going from the raw material to the finished product. In general it may be said that the engines, machines (lathes, milling machines, drill presses, shears,

punches, etc.), and stock room, should be in the side bays; and the laying out, erection, and shipping floors should be placed in the center bay in the order mentioned. Fig. 86 gives a layout of a concern manufacturing frogs and switches.

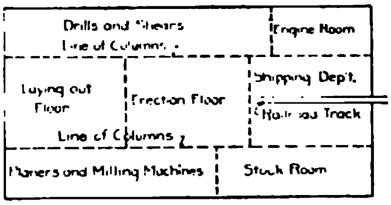


Fig. 86. Layout of a Frog and Switch Company's Building.

22. Framing. The framing of a mill building consists of the *roof framing*, which has been discussed in the preceding articles; the *columns*, which will be discussed in the next article; and the *girts* and *eave-struts*.

Eave-struts are a detail of cornice design. Various forms and methods of connections are shown in Article 29, p. 95, and the student is referred to this article.

Girts may be made of wood, angles, or channels. They should be designed for a pressure of from 20 to 30 pounds to the square foot on the side of the building. The spacing of the purlins depends upon the thickness of the corrugated steel used. On account of the fact that corrugated steel can be procured in lengths up to 10 feet and for spans of 5 feet, the stress per square inch due to 30 pounds per square foot is about 25 000 pounds. In No. 24 gauge corrugated steel, the spacing of the girts is limited to 5 feet or less.

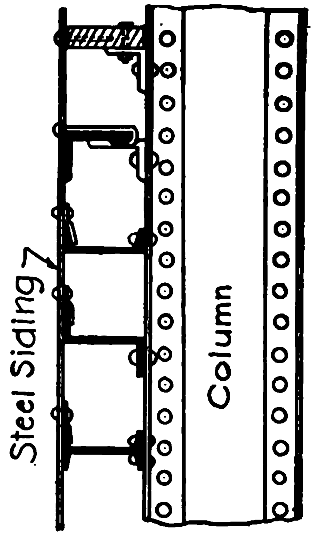


Fig. 87. Methods of Connecting Corrugated Steel to Girts.

Corrugated steel may be fastened to the girts by barbed roofing nails in case the girts are wood, or by clinch nails in case the girts are angles, or by clips fastened with rivets or $\frac{3}{16}$-inch stove bolts $\frac{3}{8}$ inch long. Nails and clinch nails should be spaced about 8 to 12 inches apart. Clips are made of No. 16 gauge steel from $\frac{5}{8}$ inch to $2\frac{1}{2}$ inches wide, and are spaced 8 to 12 inches apart. Fig. 87 shows girts, together with the method of attaching the corrugated steel. The number of nails, rivets, and stove bolts in a pound is to be found in the handbooks of various manufacturers.

Window-frames in mill buildings are, in general, similar to those placed in frame or brick buildings. These frames are fastened either directly to an iron framing or to wood nailing-pieces placed on the iron framing. The windows may be glazed in the usual fashion by means of putty, or may have the glass held in place by some of the methods shown in Fig. 73, p. 44. Windows in the side of the shop may be so fixed that they may be raised and lowered as the ordinary dwelling window; or they may slide horizontally; or, again, they may be fixed so that they cannot be moved. The windows in the monitors are usually fixed with a swinging sash which can be operated from the floor of the shop (see Fig. 89).

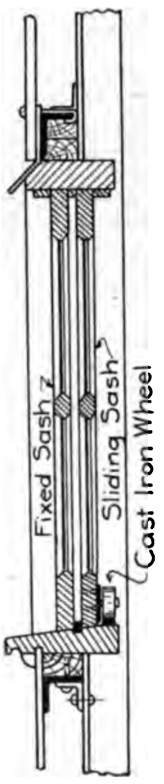


Fig. 88. Section of a Sliding-Sash Window.

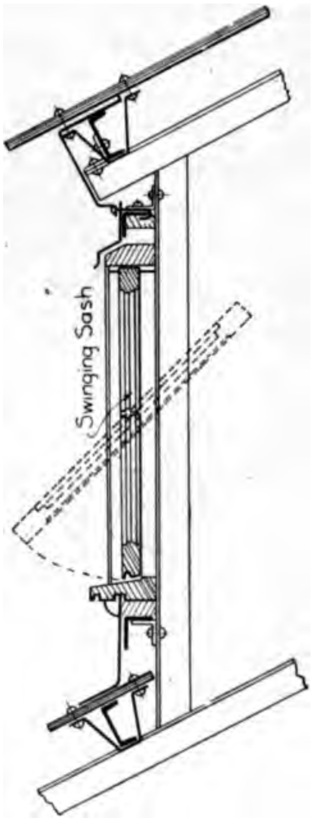


Fig. 89. Section of a Monitor Swing Window.

The glass in the windows may be the common window glass, common plate glass, ribbed or corrugated glass, wire glass, or prisms. Of these varieties, the prisms and the ribbed or corrugated are the best, since they give a more uniform light and are not easily broken. Wire glass, which is made of wire netting moulded in the middle of the sheet of glass, gives a very good light, and has the additional advantage that it does not crack and fall out under the action of fire and water. It is considered fireproof. Common window glass does not diffuse the light so well as most of the other glasses. It is very liable to fracture, and for this reason the inside of the window should be covered with wire netting. Prisms are made by the American Luxfer Prism Company, of Chicago. They may be obtained up to 84 inches in width and 36 inches in height. The width is parallel to the saw teeth. Figs. 88 and 89 give sections of windows, showing the framing. Attention is called to the fact that the roof on the monitor should overhang sufficiently to prevent the water from dropping upon the swinging window when it is fully opened.

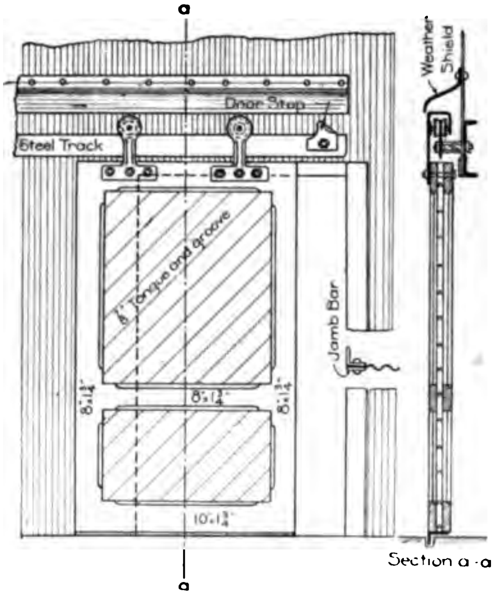


Fig. 90. Detail of a Wooden Door.

Doors may consist entirely of wood, of a frame of angles covered with corrugated steel, or of corrugated steel alone. The first two classes may be so fixed that they will slide, as in the folding doors of residences; open outward like a common door; lift vertically; or, in case they are made entirely of corrugated steel, roll up like a window-shade. This latter door is a patented one. Shop doors are seldom made to open outward or inward, on account of the space required—a space which can be devoted to better purposes. Figs. 90, 91, and 92 show details of the above doors.

Let it be required to design the girt when the trusses are 16 feet apart and the girts are 5 feet center to center. The moment is $\frac{5 \times 16 \times 30 \times 16 \times 12}{8} = 57\,600$ pound-inches; and the required section modulus is $\frac{57\,600}{15\,000} = 3.84$. By inspecting the tables in the Carnegie Handbook, pp. 97 to 119, it is found that the following shapes will be sufficient:

Shape	Section Modulus
One 5-inch 9.75-pound I-beam	4.80
One 6-inch 8.00-pound channel	4.30
One $4\frac{1}{16}$ by $3\frac{1}{8}$-inch 10.3-pound zee-bar	3.91
One 6 by 4 by $\frac{7}{16}$-inch 14.3-pound angle	3.83

From this it is evident that the channel is the most efficient and economical.

23. **Columns.** Columns may consist of almost any combination of shapes, either latticed or connected by plates. Some of the most common cross-sections are shown in Fig. 93, those illustrated in *b* and *c* being used to a great extent. The advantage of these forms is that they give a small radius of gyration about the axis *b–b*, and a larger one about the axis *a–a* (see Fig. 94). This is especially desirable, since, in addition to the direct stress due to the weight of the cranes, roof truss, and covering, the column must withstand the moment due to the wind and to the eccentricity of the runway girder. Both of these moments tend to bend the column around the axis *a–a*. The bending moment due to the eccentricity of the runway girder is equal to the reaction of the girder, *times* the distance from the center of the column (see Figs. 95 and 96). In case the details of the column are as given in Fig. 96, the direct load due to the reaction of the truss and its covering produces a moment due to its eccentricity. This moment is $R_1 \times e_1$. Since R_1 acts on the opposite side of the center of the column from the point of action of R_2, it tends to counteract the effect of the moment due to the eccentricity of the runway girder. The total moment due to eccentricity is $M_e = R_1 \times e_1 - R_2 \times e$. If

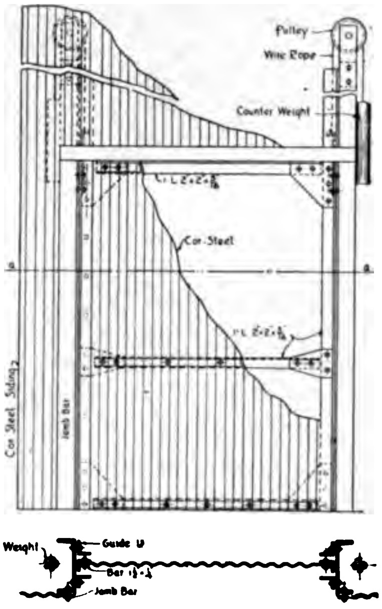


Fig. 91. Detail of Vertical Hoist Door.

the first term of this equation is less than the last term, the compressive stress on the side of the column with the runway girder is increased, and *vice versâ*. The stress in the column from the runway girder to the roof is that due only to the vertical reaction of the roof and the bending due to the wind. In that part of the column below the crane girder, the stress is that due to the direct action of the weight of the roof; its eccentricity, if there be any; the direct action and eccentricity of the runway girder; and the bending moment due to the wind. The bending moment due to the wind is less in this part of the column than it is at the foot of the knee-bracing, but it is customary to consider it the same.

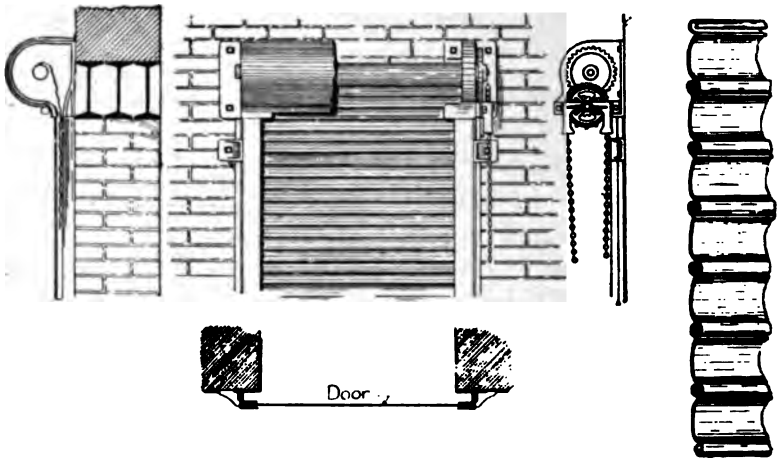


Fig. 92. Detail of Rolling Door of Corrugated Steel.
Courtesy of Kinnear Mfg. Co., Columbus, Ohio.

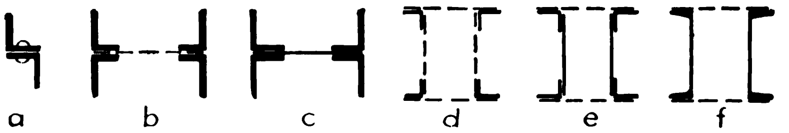
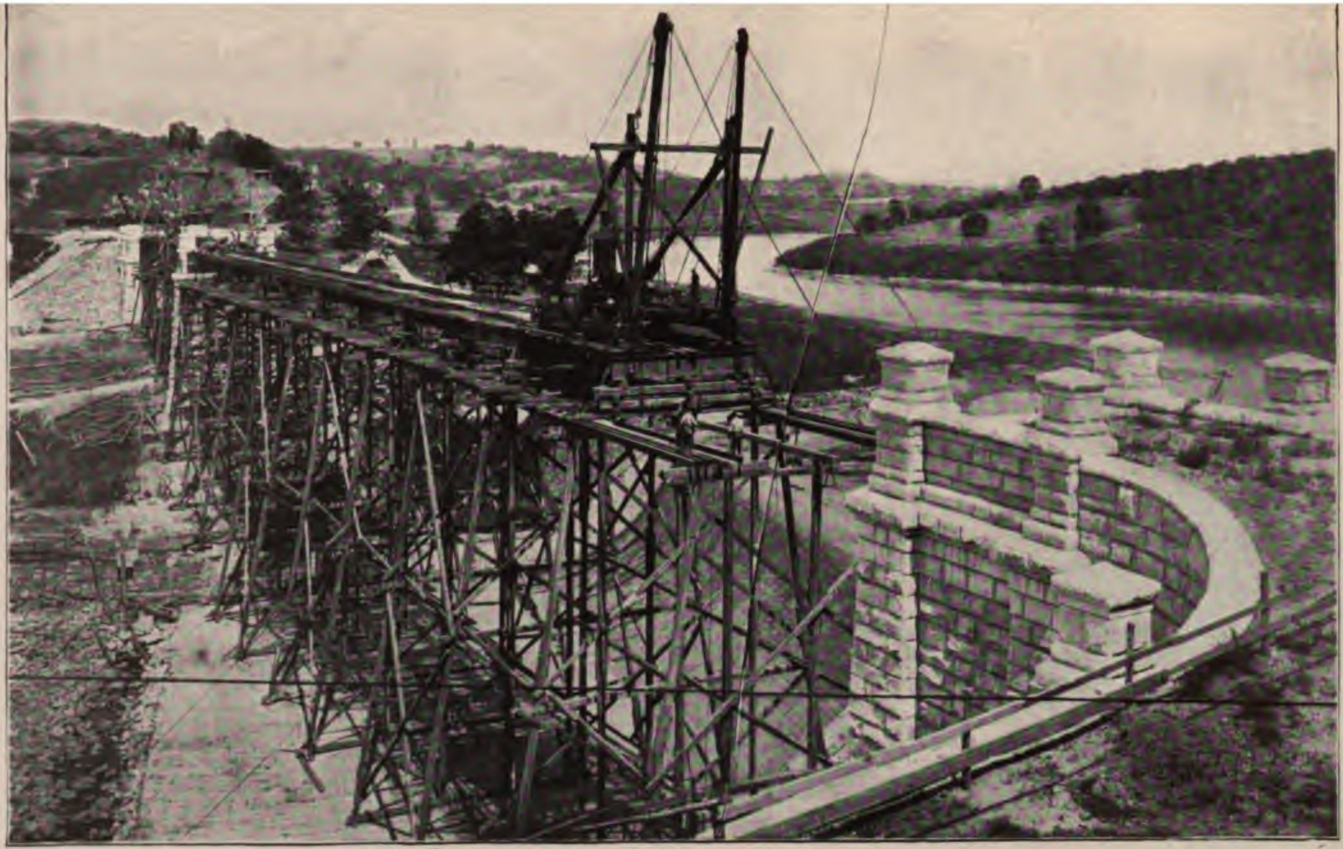


Fig. 93. Cross-Sections of Columns.

In order to prevent eccentric stresses due to the reaction of the runway girder, an extra column to carry the crane girder is placed alongside the roof column (see Figs. 100 and 117, pp. 75 and 88). This is much used by A. F. Robinson, Bridge Engineer of the Atchison, Topeka & Santa Fé Railroad System, who claims it to be a very



FALSEWORK FOR BRIDGE OVER OLD CROTON DAM AT CROTON LAKE

Waterworks system of New York City.

YORK
RARY

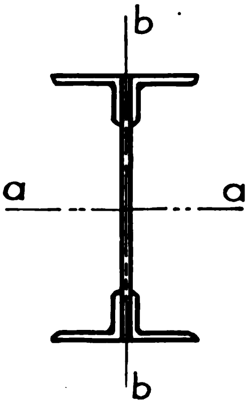


Fig. 94. Illustrating the Two Radii of Gyration.

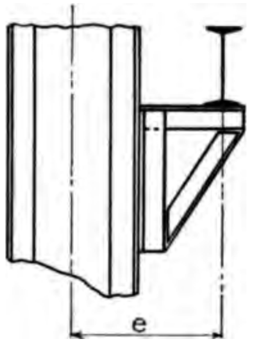


Fig. 95. Runway-Girder Eccentricity.

economical and efficient detail. One advantage of this is, that, if at any time it is desirable to use a heavier crane, this column can be removed and a stronger one put in its place, without in any way affecting the remainder of the building.

In order to illustrate the design of a column, let it be required to design a column with detail as shown in Fig. 95, the height being 20 feet, the distance of the runway girder from the face of the column 8 inches, the direct stress 15 600 pounds, and the bending moment due to the wind 924 000 pound-inches. The reaction of the runway girder is 20 000 pounds. The stress due to the bending moment caused by the wind and the eccentricity of the runway girder must be worked out by formula 8, "Strength of Materials," p. 86; and to this must be added the direct stress caused by the weight of the roof and the crane-girder reaction.

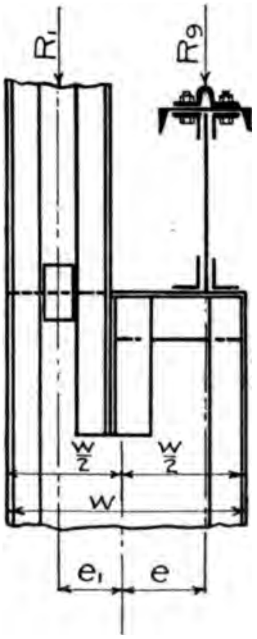


Fig. 96. Runway-Girder and Roof-Truss Eccentricity.

Since, according to Article 13, the unit-stress must be reduced one-half in determining the section to withstand stresses due to crane loads, the moment due to the crane loads and also its direct action must be

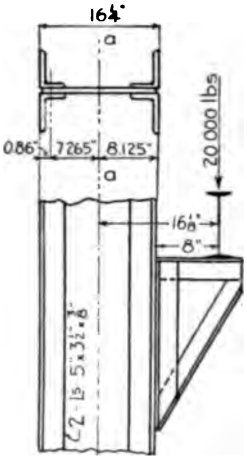


Fig. 97. Illustrating Problem on Page 73.

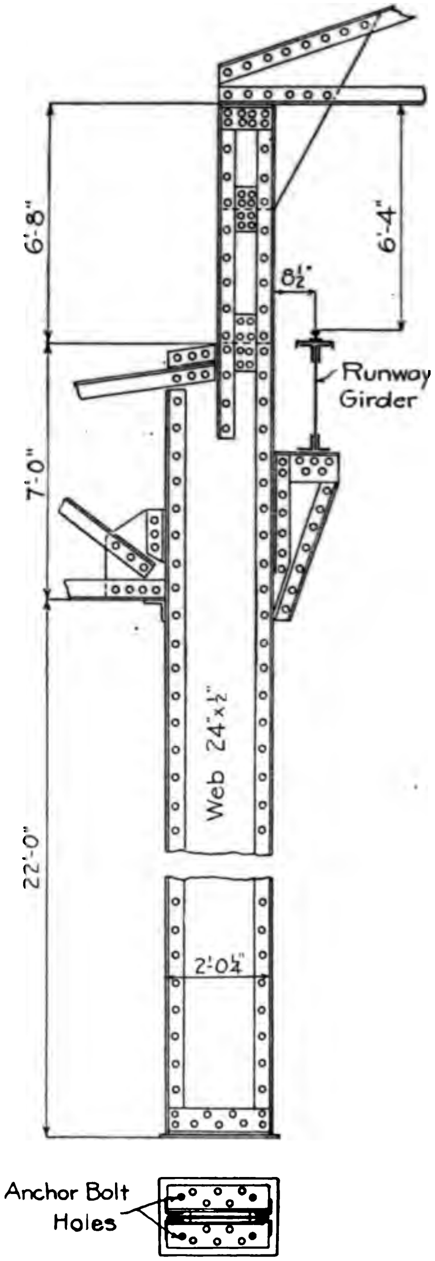


Fig. 98. Detail of Column.

multiplied by 2 in order that the same formula for the unit-stress may be used throughout in the design of the column.

Let four 5 by 3½ by ⅜-inch angles with a 16 by ⅜-inch web plate be assumed, and placed as shown in Fig. 97. These angles have an area of 3.05 square inches each; and a moment of inertia parallel to the long leg, of 3.18. Then (see "Strength of Materials," pp. 48–58), the moment due to crane reaction is 20 000 × (8.125 + 8) = 322 500 pound-inches. Accordingly, in using this in the formula, it will be 2 × 322 500 = 645 000 pound-inches; and this, added to the 924 000 pound-inches due to the wind, will make a total bending moment of 1,569,000 pound-inches.

$$I_{a-a} = 4 \times 3.18 + 4 \times 3.05 \times 7.265^2 + \frac{\frac{3}{8} \times 16^3}{12} = 784.72,$$

$$r_{a-a} = \sqrt{\frac{784.72}{4 \times 3.05 + \frac{3}{8} \times 16}} = 6.56$$

The allowable unit-stress is:

$$P = 24\,000 - 110 \times \frac{20 \times 12}{6.56} = 19\,975 \text{ pounds per square inch.}$$

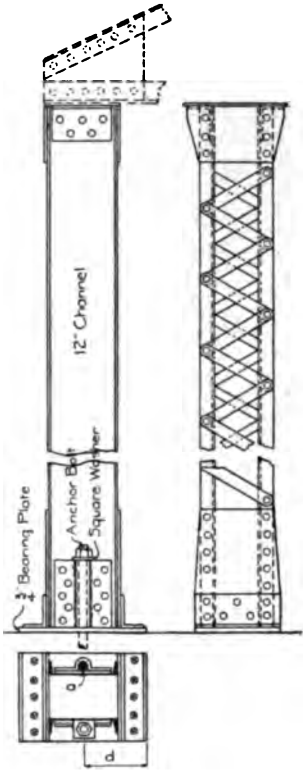


Fig. 99. Detail of Column.

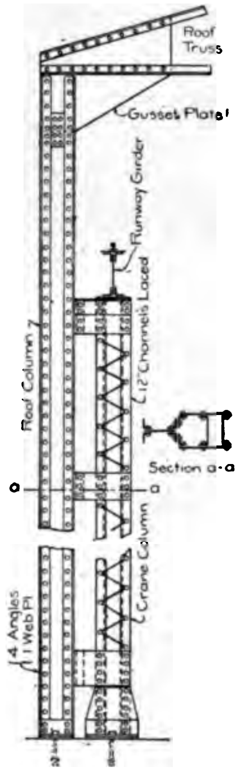


Fig. 100. Detail of Column.

The actual unit-stress (see "Strength of Materials," p. 86), is:

$$S = \frac{15\,600 + 2 \times 20\,000}{4 \times 3.05 + \frac{3}{8} \times 16} + \frac{1\,569\,000 \times \frac{16.25}{2}}{784.72 - \frac{(15\,600 + 2 \times 20\,000)(20 \times 12)^2}{10 \times 28\,000\,000}}$$

$$= 3\,024 + 16\,420$$

$$= 19\,444 \text{ pounds per square inch.}$$

Since this is slightly less than the allowable stress, 19 975 pounds per square inch, this section is the correct one.

Details of columns are shown in Figs. 98, 99, and 100. In case the column is considered fixed at its base, the base of the column is usually made as shown in Fig. 99. Long bolts deeply imbedded in the masonry are run up through the holes a; a heavy washer is placed over the bolt, and the nut screwed down tightly. Each bolt must be designed to withstand a stress of $H_2 \times n \div 2d$ (see Figs. 43 and 99).

24. **Knee-Braces.** The determination of the stresses in knee-bracing has been made in Article 9. Knee-braces consist of two angles placed back to back, and are joined to the column and roof truss as shown in Fig. 101 and in the figures showing cranes. They must be designed to withstand the greatest compressive stress; and must also be examined to see if they are safe in tension, since they are under either tensile or compressive stresses according to the direction in which the wind blows.

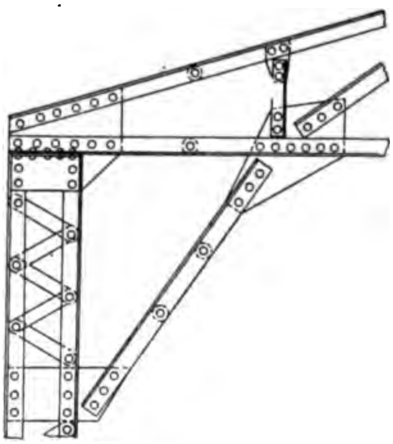


Fig. 101. Detail of Knee-Brace and Connections.

The knee-brace for the truss-bent of Article 9 will now be designed. The maximum compressive stress is 21 440 pounds. The radius of gyration must be at least 131 ÷ 120 = 1.09. Two angles $3\frac{1}{2}$ by 3 by $\frac{5}{16}$-inch, placed back to back with their longer legs $\frac{1}{4}$ inch apart, will be assumed, since they are the smallest size to be used with $r = 1.09$ or greater. The radius of gyration about an axis perpendicular to the longer legs is 1.10; and the allowable unit-stress is $P = 24\,000 - \frac{110 \times 131}{1.10} = 10\,900$ pounds per square inch, the length being 131 inches. The required area is $\frac{21\,440}{10\,900} = 1.97$ square inches. Since this is less than the given area, and since the angles are the smallest allowed, these angles are sufficient. The maximum tensile stress is 8 640 pounds, and the required net area is $8\,640 \div 15\,000 = 0.58$ square inch. The net area given by the angles is $3.86 - 0.55 = 3.31$ square inches, two $\frac{3}{4}$-inch rivet-holes being taken out. This shows the angles to be amply safe in tension, and they will therefore be used for the section of the knee-braces.

25. **Runway Girders.** The runway girders extend from column to column on each side of the bay in which the girder runs. An inspection of the figures of this article will give a clear idea of their position and their details. Along these girders run the wheels which support the end of the crane. The crane may be a small hoist, as indicated in Figs. 102, 103, and 104, in which case the crane girder consists of a simple I-beam supported by two wheels at the ends, and these are placed close together. In other cases the crane consists of

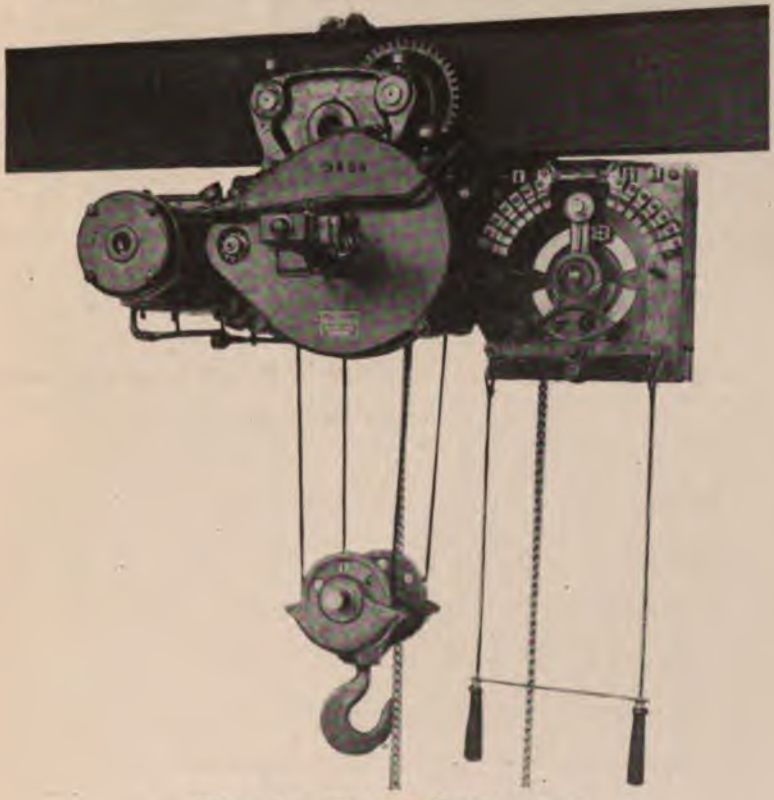


Fig. 102. Electric I-Beam Hoist, One-Motor.
Courtesy of Case Manufacturing Company, Columbus, Ohio.

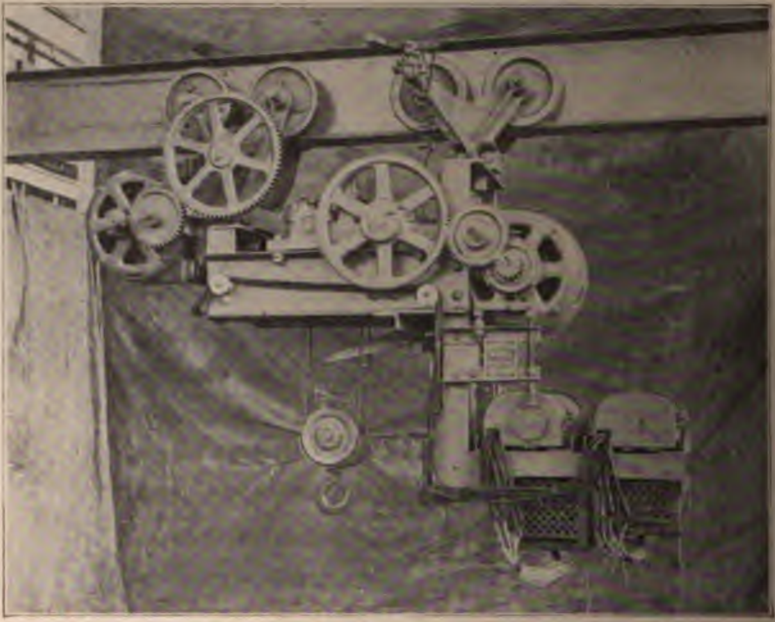


Fig. 103. Electric I-Beam Hoist.
Courtesy of Maris Brothers, Philadelphia, Pennsylvania.

two girders placed side by side, upon which runs the carriage carrying the hoist. This type of crane is supported upon four to eight wheels (see Figs. 105, 106, 107, 108, 109, and 110).

The maximum bending moment and shear in a runway girder

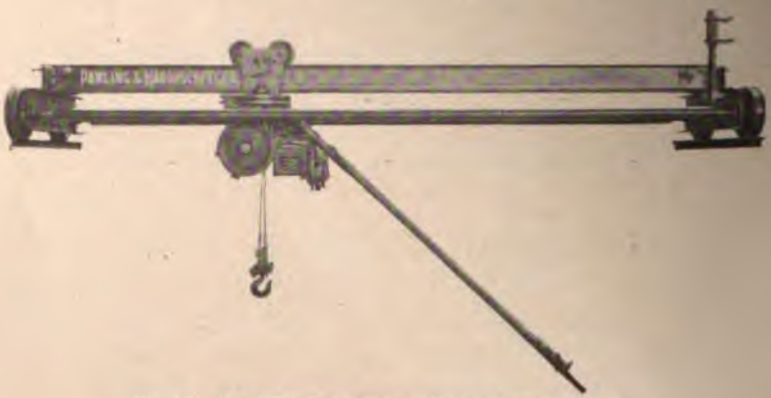


Fig. 104. Small Electric I-Beam Hoist; Capacity 500 Lbs.
Courtesy of Pawling & Harnischfeger, Milwaukee, Wis.

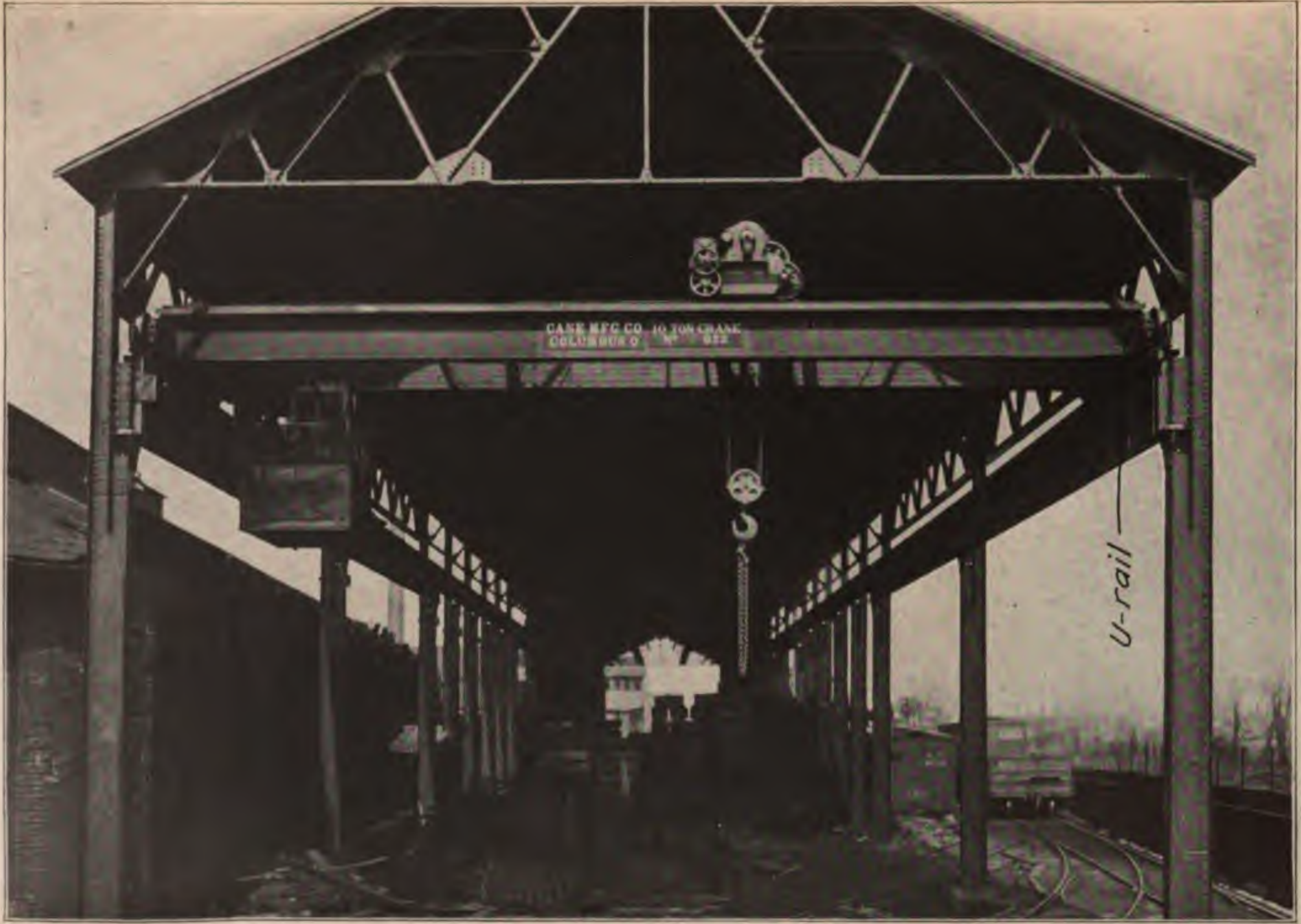


Fig. 105. Ten-Ton Three-Motor Electric Traveling Crane.
Courtesy of Case Manufacturing Company, Columbus, Ohio.

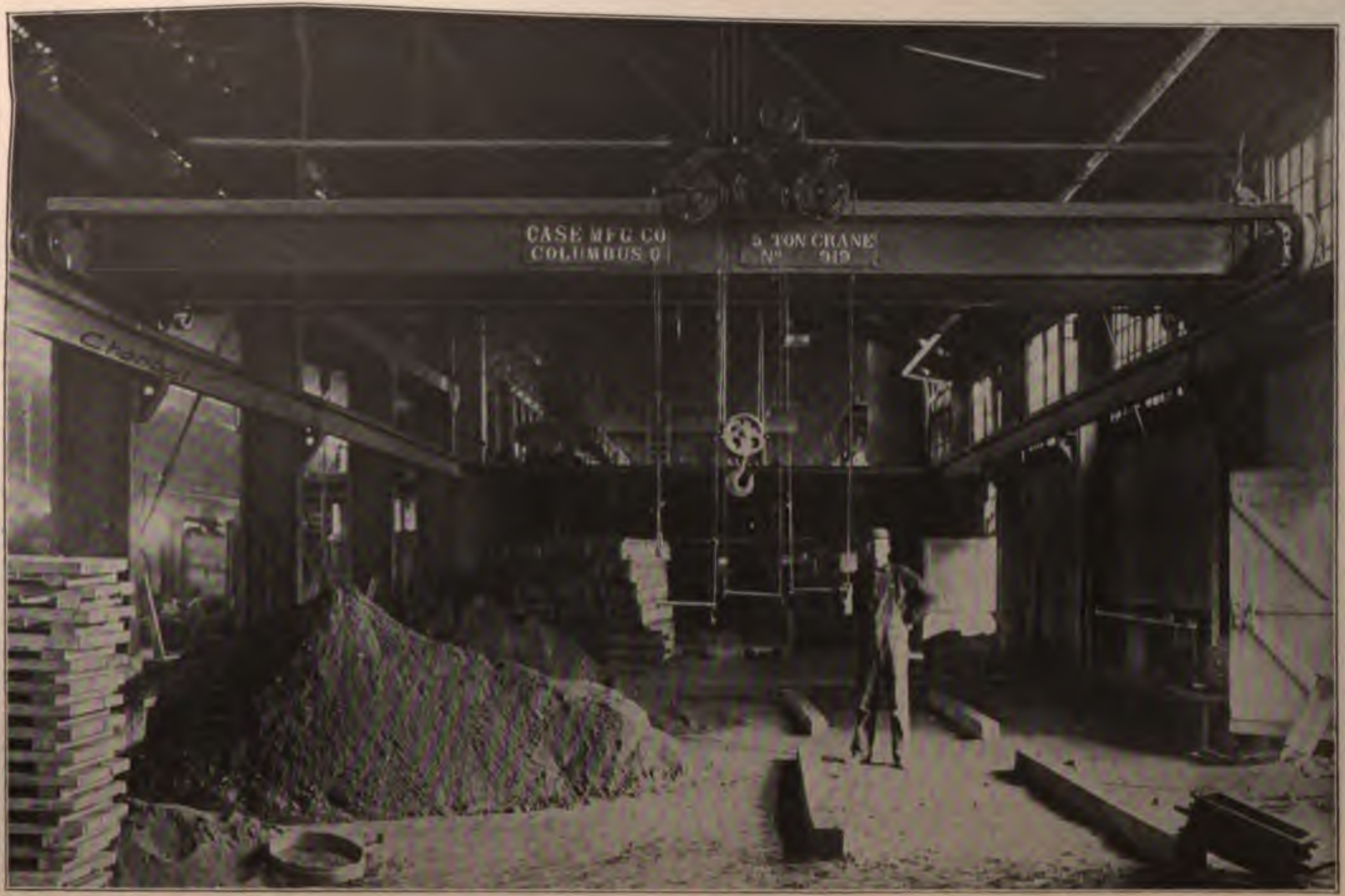
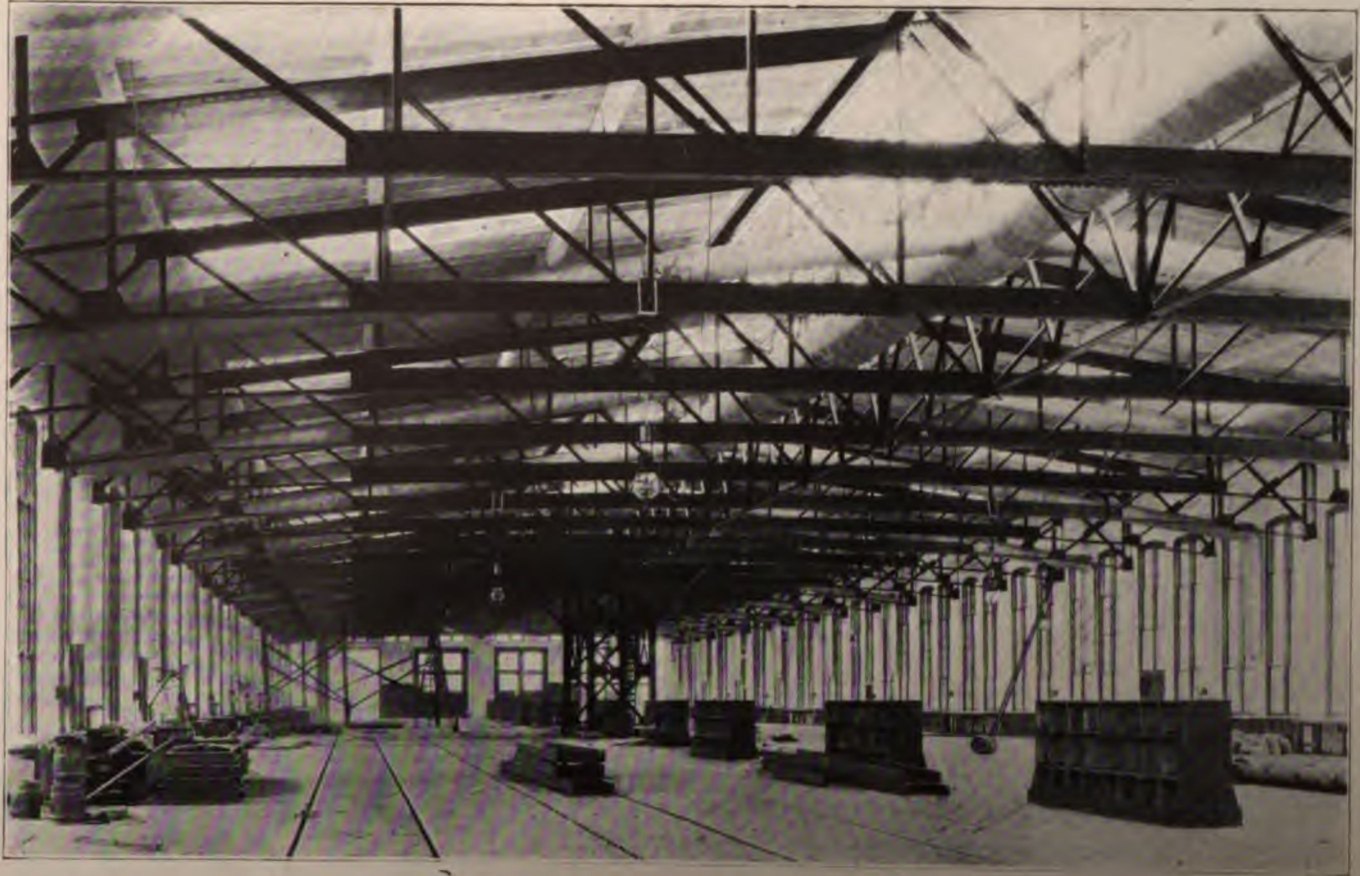


Fig. 106. Five-Ton Three-Motor Electric Traveling Crane, Operated from Floor.
Courtesy of Case Manufacturing Company, Columbus, Ohio.



INTERIOR OF FREIGHT-CAR REPAIR SHOP

Showing pleasing effect of giving bottom chord of trusses a slight rise, instead of making them horizontal. Certain sections of the chords are composed of angles and a plate, this being necessary because they may at some time be subjected to bending, as well as to the direct stress by reason of suspended loads. Note the comparatively small number of arc lights per unit of floor area.

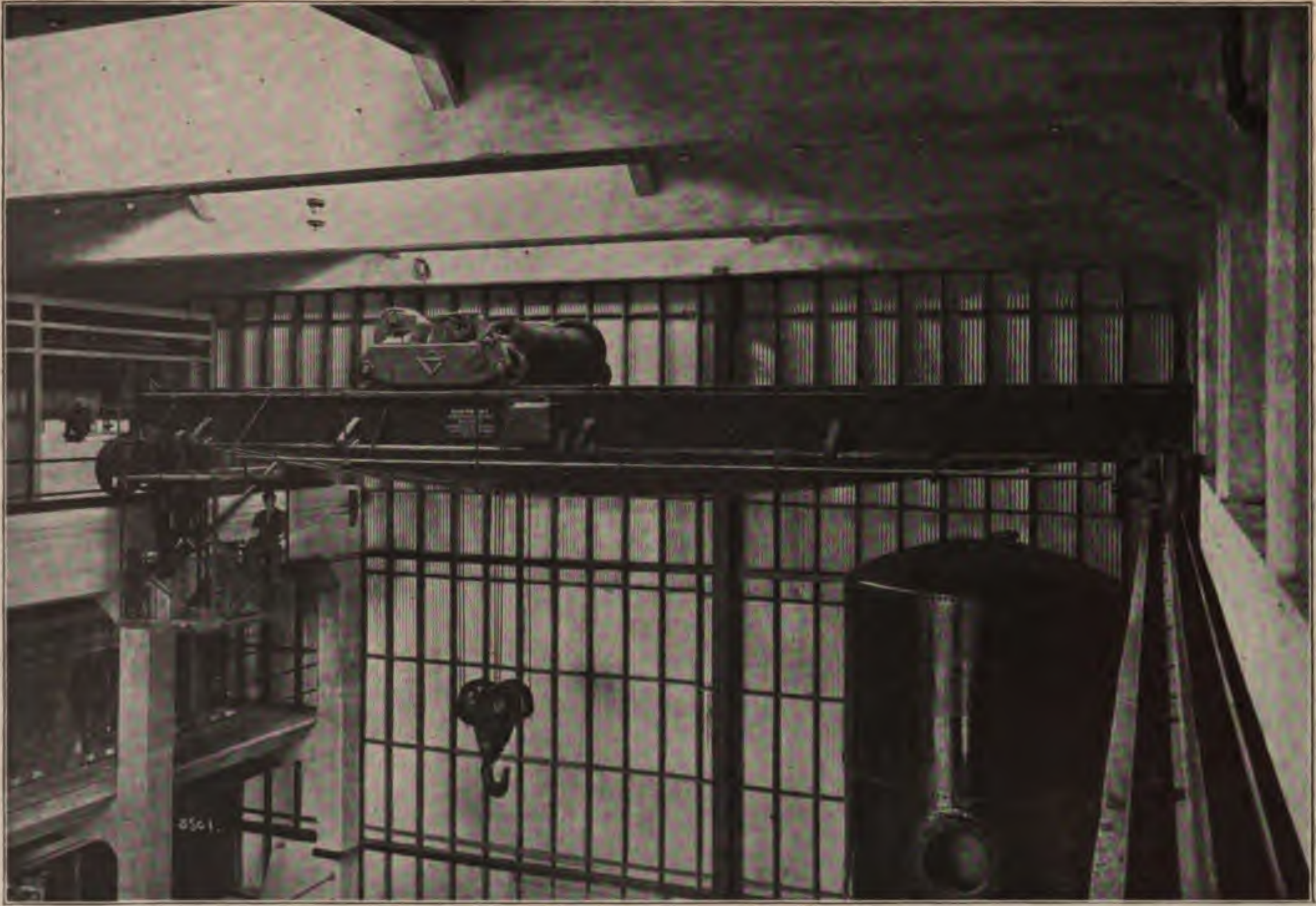


Fig. 107. Fifty-Ton Three-Motor Electric Traveling Crane.
Courtesy of Northern Engineering Works, Detroit, Mich.

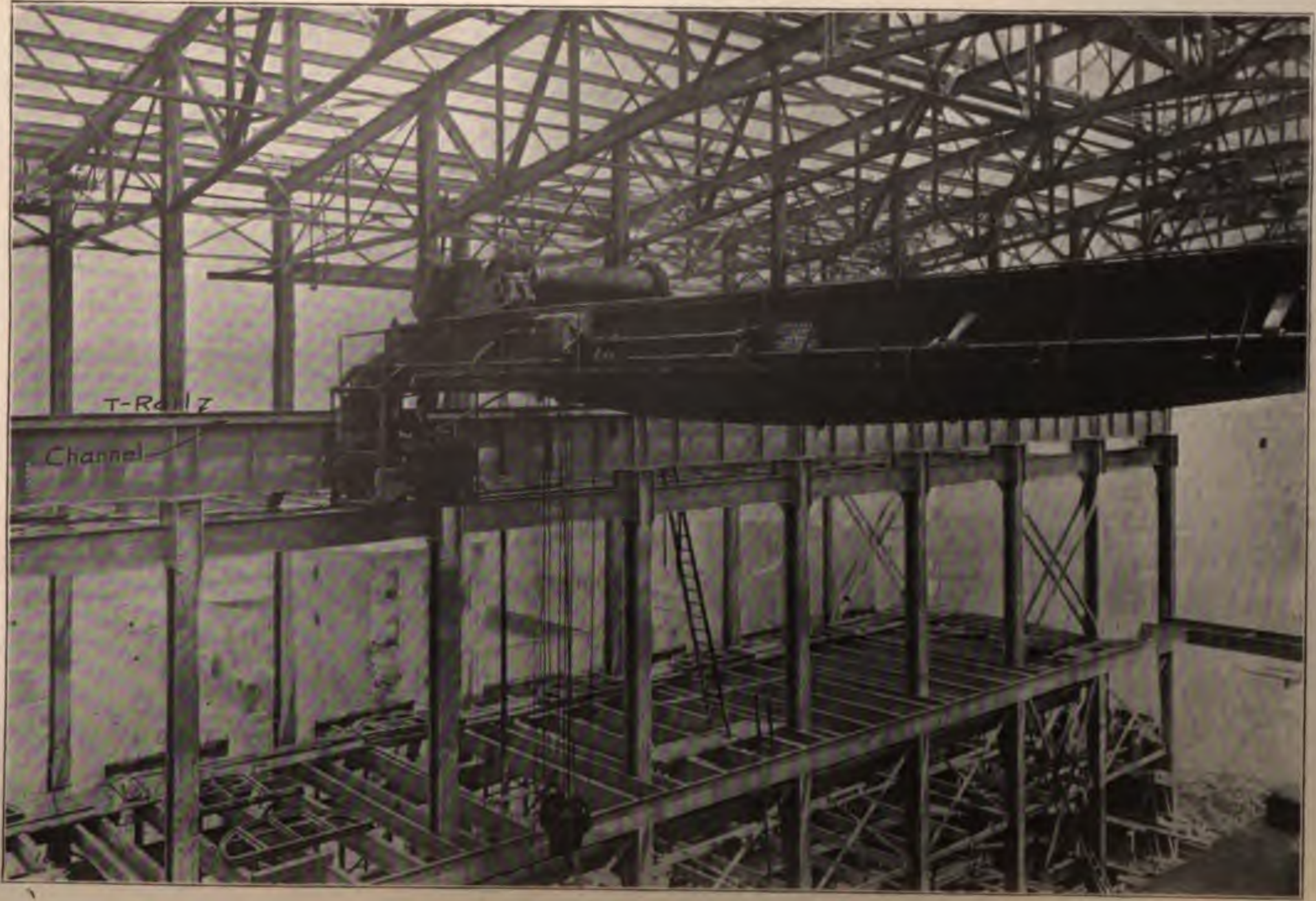


Fig. 108. Fifty-Ton Four-Motor Electric Traveling Crane.
Courtesy of Northern Engineering Works, Detroit, Mich.

will depend upon the capacity and span of the crane, and also upon the distance apart of the wheels at its ends. Where the bending moment is not too great, the runway girders may be composed of channels or I-beams (see Figs. 103, 104, and 106). In case the moment is too great to make the use of these possible, the runway girders are composed of plate-girders (see Figs. 105, 107 to 110).

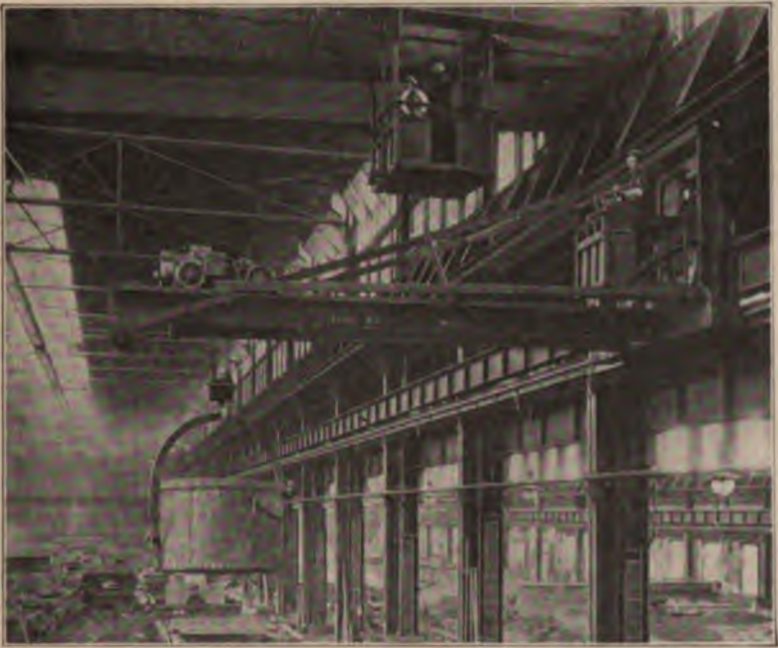


Fig. 109. Wall Jib-Crane, Electrically Operated; Capacity, 10,000 Lbs.
Courtesy of Pawling & Harnischfeger, Milwaukee, Wisconsin.

Plate-girders consist of a flat plate called a *web plate*, which has riveted to it at its upper and lower edges two angles, or two angles and one plate, called the *cover-plate*. The angles are called *flange angles*; and the two angles together, and the cover-plate when used, are called the *flanges*. At certain distances along its length, equal to or less than its depth, vertical angles are riveted on opposite sides of the web plate. These are called *stiffeners*, their function being to stiffen the web under the action of the shear. See Fig. 111 for a general view of a plate-girder, together with the names of the various parts.

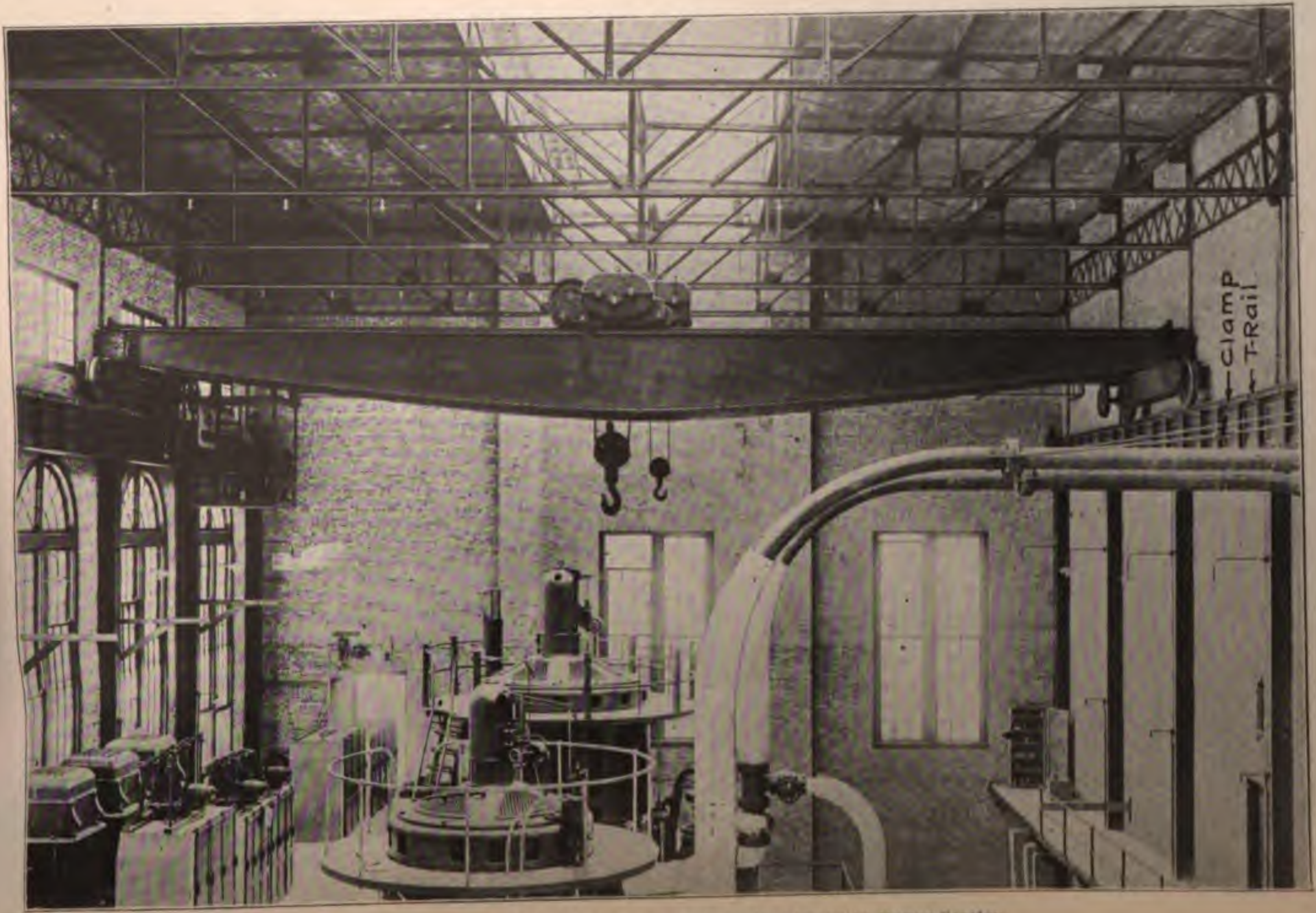


Fig. 110. Thirty-Ton Four-Motor Electric Traveling Crane in Power Station.
Courtesy of Northern Engineering Works, Detroit, Mich.

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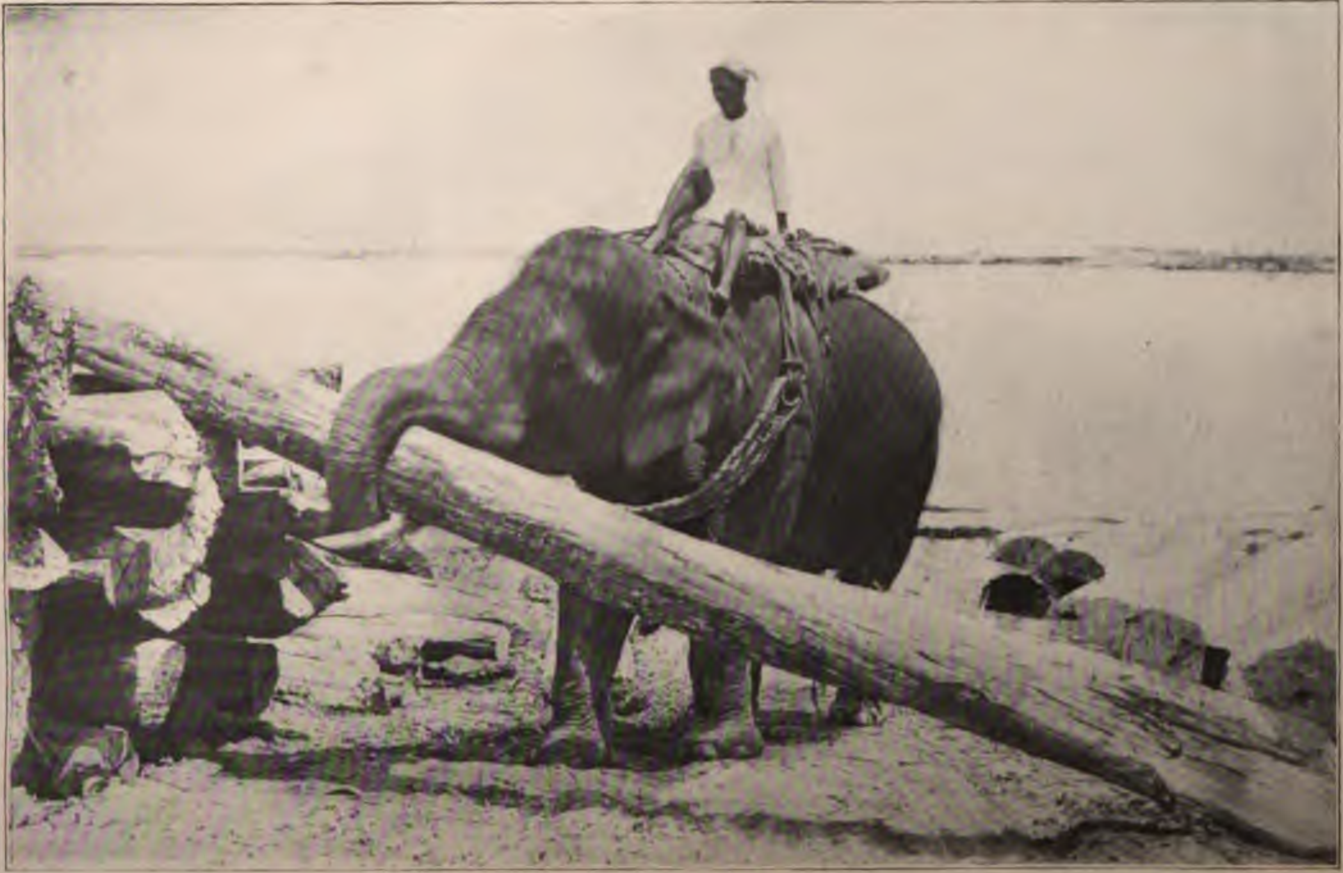


AN ENGINEER'S ASSISTANT AT WORK IN INDIA

Elephant piling timber.

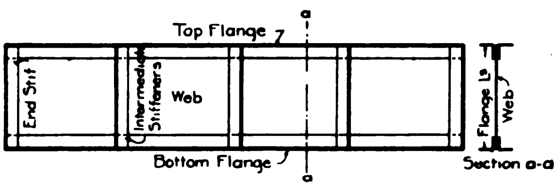


Fig. 111. Plate-Girder Notation.

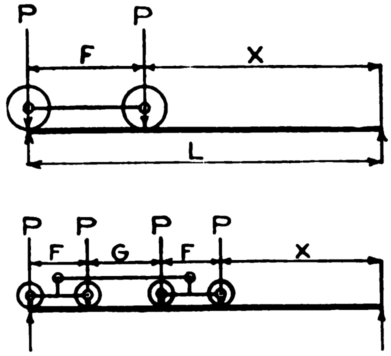


Fig. 112. Position of Crane Truck for Maximum End Shear of Runway Girder.

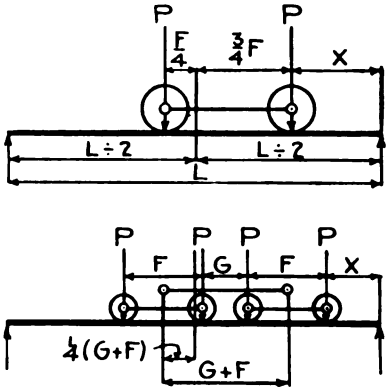


Fig. 113. Position of Crane Truck for Maximum Moment in Runway Girder.

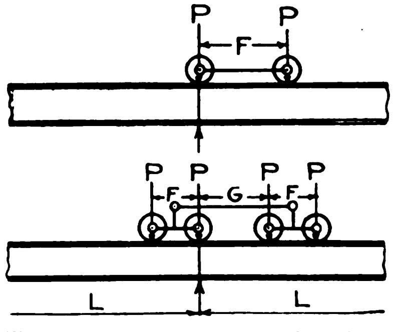


Fig. 114. Position of Crane Truck for Maximum Reaction on Column.

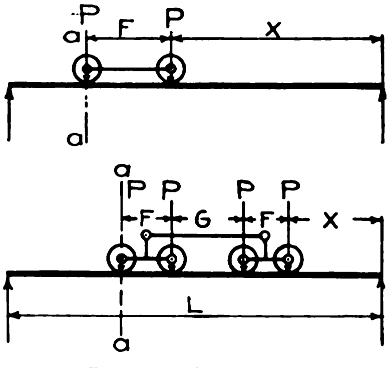
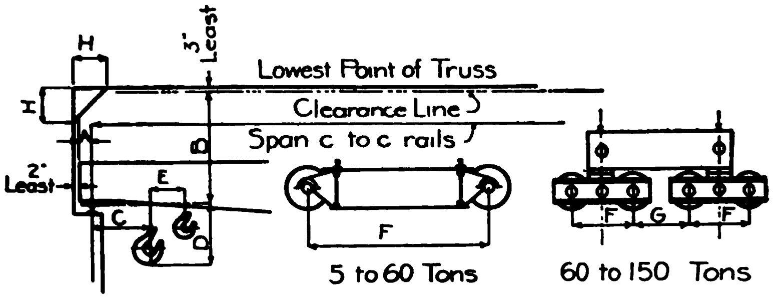


Fig. 115. Position of Crane Truck for Maximum Shear at any Section.

The maximum shear in the runway girder will occur when the crane wheels are in the position shown in Fig. 112; and the maximum moment will occur under the wheel nearest the middle of the span, when the wheels are in the position shown in Fig. 113. The maximum reaction of the runway girders on the column will occur when the wheels are in the position shown in Fig. 114. In order to

TABLE XIV

Maximum Crane Reaction



Capacity (tons)	Span (feet)	A in.	B ft. in.	C ft. in.	D ft. in.	E ft. in.	F ft. in.	G ft. in.	Wheel Load (in lbs.)	Tram Rail Am. Std.	H ft. in.
SINGLE DRUM. TYPE C											
5	40	7	4- 5½	1- 8 2- 0	7		7- 6½		10 000	3½-1⅞- 40	1- 5
	60	8	4- 7½	1-11 2- 1	5		9- 0		12 500		
	80	8	4- 8½	1-11 2- 2	4		9- 3½		15 500		
10	40	8	5- 0½	1- 0 2- 4	9		8- 5½		16 500	3½-1⅞- 40	1- 0
	60	8	4-11¾	1-11½ 2- 4	10		9- 7¼		19 500		
	80	8	5- 0½	1-11½ 2- 4½	9		9- 9½		23 000		
15	40	8	5- 4¼	2- 0 2- 9	1-3½	2- 0	9- 6		24 500	3⅞-2⅛- 50	1- 3
	60	8	5- 4¼	2- 0 2- 9	1-3½	2- 0	10- 7½		28 000		
	80	8	5- 5½	2- 0 2- 9	1-2¼	2- 0	10- 9½		31 500		
20	40	8	5- 5½	2- 0 2- 9	1-6¼	2- 0	9- 6		30 000	3⅞-2⅛- 50	1- 3
	60	8	5- 5½	2- 0 2- 9	1-6¼	2- 0	10- 7½		34 000		
	80	9	5- 6½	2- 0½ 2- 9	1-5¼	2- 0	11- 1½		38 000		
DOUBLE DRUM. TYPE A											
25	40	9	6- 0¼	2- 6	3-9¼	2- 7½	11- 0½		38 600	4¼-2⅜- 60	1- 3
	60	9	6- 0¼	2- 6	3-9¼	2- 7½	11- 3½		42 400		
	80	10	6- 2¼	2- 6	3-7¼	2- 7½	11- 8½		47 000		
30	40	10	6- 3¾	2- 7	3-8½	2-10	11- 4½		45 600	4¼-2⅜- 60	1- 3
	60	10	6- 3¾	2- 7	3-8½	2-10	11- 6½		49 600		
	80	10	6- 3¾	2- 7	3-8½	2-10	12- 0		54 600		
40	40	11	7- 4½	3- 1½	4-2½	3- 8½	12- 8		59 600	5 -2½- 80	1- 6
	60	11	7- 4½	3- 1½	4-2½	3- 8½	13- 0		64 000		
	80	11	7- 4½	3- 1½	4-2½	3- 8½	13- 5		70 000		
50	40	13	7-11½	3- 6	4-0½	3- 9	13-10½		77 000	5¾-2¾-100	1- 6
	60	13	7-11½	3- 6	4-0½	3- 9	13-10½		84 600		
	80	13	7-11½	3- 6	4-0½	3- 9	14- 0		92 000		
60	40	14	8- 7¼	4- 1	4-3¾	4- 4½	15- 2		88 000	5¾-2¾-100	1-11
	60	14	8- 7¼	4- 1	4-3¾	4- 4½	15- 2		94 000		
	80	14	8- 7¼	4- 1	4-3¾	4- 4½	15- 4		103 000		
60	40	12½	10- 3¼	4- 0	2-7¾	4- 4½	3- 6	5-4	44 000	5¾-2¾-100	
	60	12½	10- 3¼	4- 0	2-7¾	4- 4½	3- 6	5-4	47 000	6 -4 -150	
	80	12½	10- 3¼	4- 0	2-7¾	4- 4½	3- 6	5-4	51 500		
75	40	12½	11- 3	4- 6	3-8½	4- 0½	5- 0	6-0	55 000	5¾-2¾-100	2- 2
	60	12½	11- 3	4- 6	3-8½	4- 0½	5- 0	6-0	60 000	6 -4 -150	
	80	12½	11- 3	4- 6	3-8½	4- 0½	5- 0	6-0	64 000		
100	40	16½	13- 2¼	4- 1	2-0	SPECIAL	5- 0	6-0	83 000	6 -4 -150	4- 7
	60	16½	13- 2¼	4- 1	2-0	"	5- 0	6-0	86 000		
	80	16½	13- 2¼	4- 1	2-0	"	5- 0	6-0	89 000		
150	40	17	15- 6¼	6- 0						4 -150	4- 7
	60	17	15-								
	80	17	15-								

TABLE XV
Typical Electric Cranes

CAPACITY (Tons)	SPAN (Ft.)	WHEEL BASE F	[illegible] P	A + 2 in.	B	[illegible] For Plate-Girders	I-Beams
5	40	8 ft. 6 in.	12 000	[illegible]	[illegible]	[illegible]	[illegible]
	60	9 " 0 "	13 000	[illegible]	[illegible]	[illegible]	[illegible]
10	40	9 " 0 "	19 000	[illegible]	[illegible]	[illegible]	[illegible]
	60	9 " 6 "	21 000	[illegible]	[illegible]	[illegible]	[illegible]
15	40	9 " 6 "	26 000	[illegible]	[illegible]	[illegible]	[illegible]
	60	10 " 0 "	29 000	[illegible]	[illegible]	[illegible]	[illegible]
20	40	10 " 0 "	33 000	12 "	[illegible]	[illegible]	[illegible]
	60	10 " 6 "	36 000	12	[illegible]	[illegible]	[illegible]
25	40	10 " 0 "	40 000	12 "	[illegible]	[illegible]	[illegible]
	60	10 " 6 "	44 000	12	[illegible]	[illegible]	[illegible]
30	40	10 " 6 "	48 000	12	[illegible]	[illegible]	[illegible]
	60	11 " 0 "	52 000	12 "	[illegible]	[illegible]	[illegible]
40	40	11 " 0 "	64 000	14 "	[illegible]	[illegible]	[illegible]
	60	12 " 0 "	70 000	14	[illegible]	[illegible]	[illegible]
50	40	11 " 0 "	72 000	14	[illegible]	[illegible]	[illegible]
	60	12 " 0 "	80 000	14 "	[illegible]	[illegible]	[illegible]

obtain the maximum shear at any section, as $a-a$, the load should be placed as shown in Fig. 115; and the maximum shear will then be the left reaction, which is $R = 2P\left(x - \frac{F}{2}\right) \div l$, for two wheels; and $R = 4P\left(x + F + \frac{G}{2}\right) \div l$, for four wheels.

The values of P for traveling cranes of various capacities and spans may be obtained upon writing to the various crane manufacturing companies, whose addresses will be found in the advertising sections of the engineering periodicals. The distances between wheels may be obtained from their catalogues, which may be had upon application. The values of P, and the distances between wheels for cranes of various spans and capacities, are given in Table XIV, which is made from information furnished through the courtesy of Pawling & Harnischfeger, Milwaukee, Wisconsin.

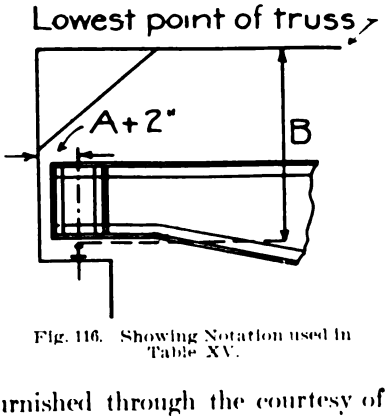


Fig. 116. Showing Notation used in Table XV.

The values in Table XV are taken from the "Transactions" of

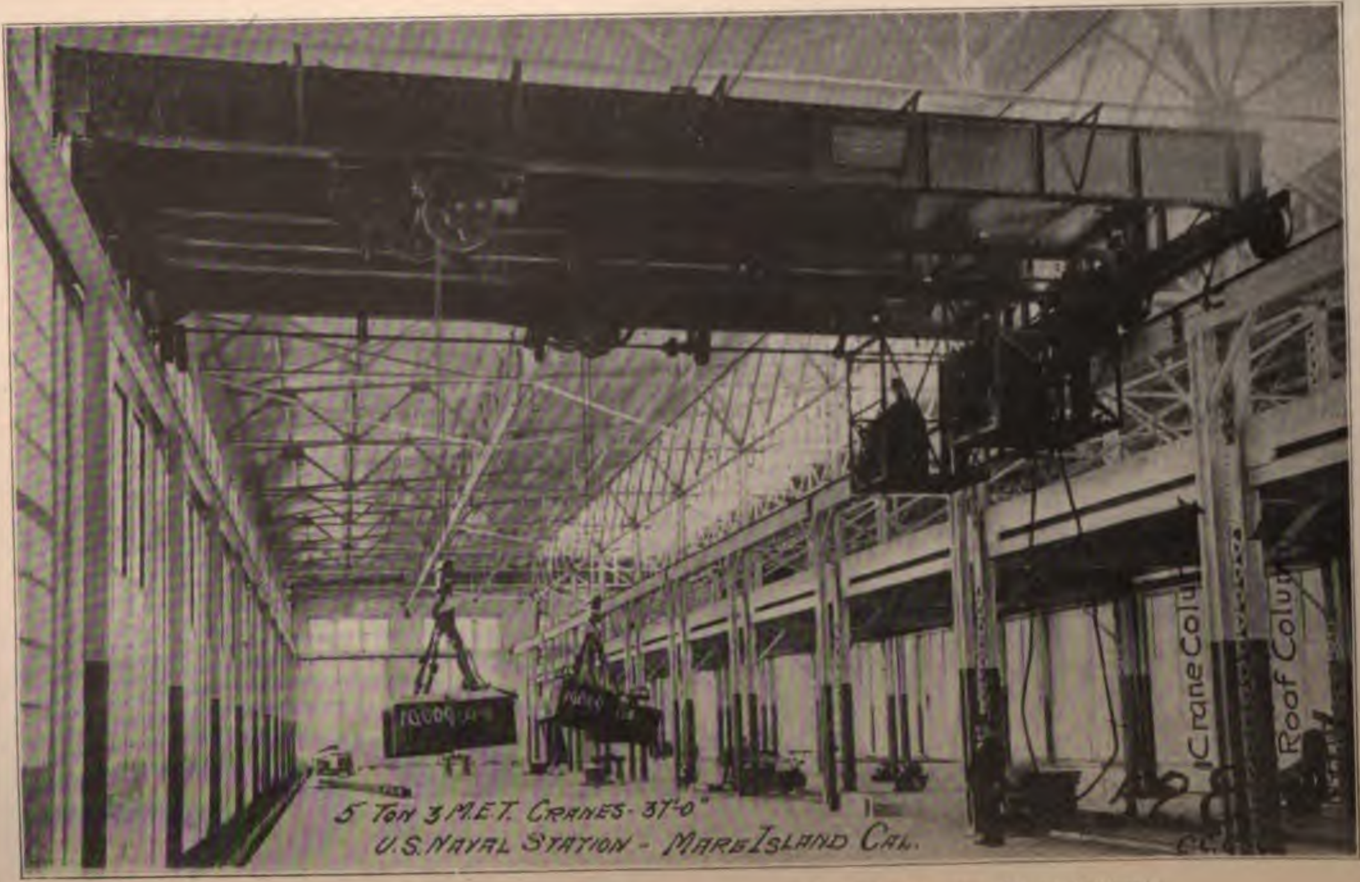

5 TON 3 M.E.T. CRANES-37'-0"
U.S. NAVAL STATION - MARE ISLAND CAL.
Crane Colu
Roof Colu

Fig. 117. Five-Ton Electric Traveling Cranes Installed at U. S. Naval Station, Mare Island, California.
Courtesy of Cleveland Crane & Car Company, Cleveland, Ohio.

the American Society of Civil Engineers, Vol. 54, p. 400, 1905. They are for typical traveling electric cranes, and are proposed by Mr. C. C. Schneider, one of the most distinguished of structural engineers.

The side clearance *B* from the center of the rail, and the vertical clearance of the beam from the top of the rail, are given in this table (see Fig. 116). These values for the cranes of different manufacturers may be obtained from their catalogues; and they must be known, in order that the crane shall not interfere with the columns or the roof trusses.

If the runway girder is composed of an I-beam, a channel is usually riveted to its top; and on this the rail on which the crane wheels move is fastened down at intervals (see Fig. 107) of about $2\frac{1}{2}$ or 3 feet. Figs. 106 and 117 show details of this kind of girder. Note that the rails are U-shaped (see Fig. 105). This rail is used extensively, although in many cases the common T-rail is used and is fastened down by means of clamps around the edge of the flange of the girder (see Fig. 110).

In case plate-girders are necessary for runway girders, they must be designed. The depth of these girders should be $\frac{1}{15}$ to $\frac{1}{6}$ of the distance between trusses or columns—that is, $\frac{1}{10}$ of their span; The depths must be in the even inch. For example, if the trusses were 16 feet apart, the depth of the girder would be 16 ÷ 10 = 1.6 feet, which is equal to 19.2 inches. The depth of the girder must then be made 20 inches, since, if it were made 19 inches, it would be difficult to obtain a web plate 19 inches wide, for the mills do not as a rule have plates of odd-inch widths in stock.

The thickness of the web plate is given by the formula:

$$t = \frac{V_0}{S_s\, d};$$

but in no case shall it be less than $\frac{5}{16}$ inch. In this formula, V_0 is the maximum end reaction of the runway girder. It is equal to *R* as given by the formula on p. 87, when x is equal to $l - F$, and d is the depth of the girder, which is equal to the depth of the web plate, and S_s is the unit allowable shearing stress.

The flanges are composed of two angles, placed with the long legs horizontal in case unequal-legged angles are used. The required net area of *one flange* is given by the formula:

$$A = \frac{M_m}{S_t\,(d-2)},$$

in which M_m is the moment obtained when the wheels are in the position shown in Fig. 113, S_t is the unit allowable tensile stress, and d is the width of the web plate. If the area A has been computed, two angles must be found from the tables in the Carnegie Handbook, such that when one $\frac{7}{8}$-inch or $\frac{3}{4}$-inch rivet-hole, as the case may be, is taken out, each angle will give a net area equal to or slightly in excess of the area A.

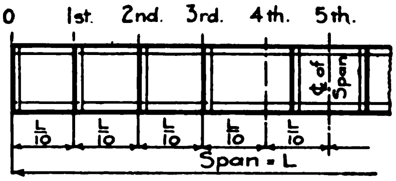


Fig. 118. Position of Tenth-Points.

These flange angles must be riveted to the web by rivets placed a certain distance apart. For convenience of manufacture, the girder is divided into ten equal parts, and the rivet spacing between any two of these divisions—or *tenth-points*, as they are called—is kept the same. These tenth-points are numbered (see Fig. 118). The rivet spacing in the first division is the same as that computed for the end of the girder, which is the zero tenth-point; the rivet spacing in the second division is the same as that computed for the 1st tenth-point; and so on. The rivet spacing at any point is given by the formula:

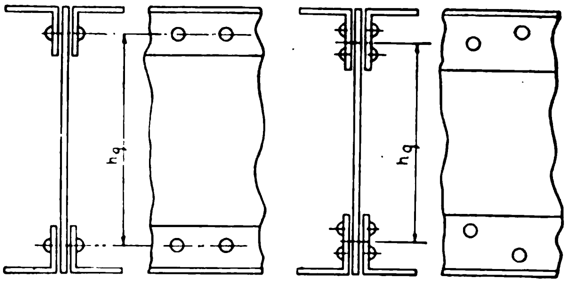


Fig. 119. Determination of Distance between Gauge Lines.

$$S = \frac{r}{\sqrt{\left(\frac{V_x}{h_g}\right)^2 + \left(\frac{P}{30}\right)^2}},$$

in which,

V_x = Maximum shear at the point;
r = Maximum allowable stress on one rivet; this will be the bearing value of the rivet in the web plate (see Table X, p. 47);
P = Maximum reaction of one crane wheel (see Table XIV or XV);
h_g = Distance between gauge lines of the angles.

In case there are two gauge lines on the angle, then the distance h_g is the distance between centers of these gauge lines (see Fig. 119).

Table IX, p. 46, gives the gauge lines for different lengths of angle legs. If S gives a value less than 2⅝ inches, the leg of the angle against the web must be 5 inches or more, on account of practical limitations of manufacture.

26. *Examples.* In order to illustrate the preceding methods, two problems will be worked out.

1. Design a runway girder for a 5-ton crane of 40-foot span, the wheel loads and wheel base being as given in Table XV, p. 87, and the distance between trusses 20 feet.

In order to produce the maximum moment, the wheel must be placed as shown in Fig. 120. The left reaction is 12 000 (2.125 + 10.00 + 3.625) ÷ 20 = 9 450. The moment under wheel 1 is 9 450 × 7.875 × 12 = 894 000 pound-inches, which requires a section modulus of 894 000 ÷ 15 000 = 59.60. Looking in the Carnegie Handbook, pp. 97 and 98, it is seen that a 15-inch 42-pound I-beam with a section modulus of 58.9 will be sufficient, since the section modulus is less than 2½ per cent under that required.

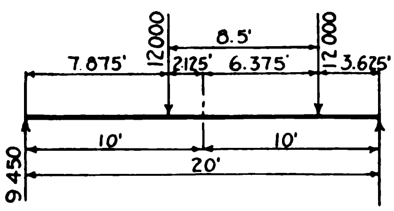


Fig. 120. Position for Maximum Moment for Problem 1 on Page 91.

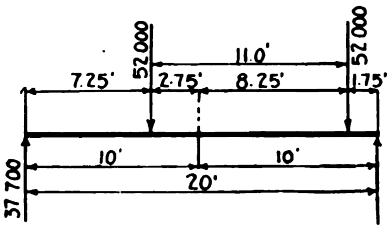


Fig. 121. Position for Maximum Moment for Problem 2 on Page 91.

2. Design a runway girder for a 30-ton crane of 60-foot span, the wheel loads and wheel base being as given in Table XV, and the distance between trusses 20 feet.

The wheels are placed in position as shown in Fig. 121. The left reaction is $\frac{52\,000\,(12.75 + 1.75)}{20} = 37\,700$ pounds; and the maximum moment, which occurs under wheel 1, is 37 700 × 7.25 × 12 = 3 285 000 pound-inches. The maximum shear occurs when the wheels are in position as shown in Fig. 112, p. 85, and is 75 400 pounds. The required thickness of the web is $\frac{75\,400}{10\,000 \times 24} =$ 0.314 inch, the depth being 20 ÷ 10 = 2 feet = 24 inches. The web will be made 24 inches wide and ⅜ inch thick.

The required net flange area is $\frac{3\,285\,000}{15\,000 \times (24 - 2)} = 9.97$ square inches for two angles, or 4.99 square inches for one angle. An angle 6 by 6 by ½-inch gives a gross area of 5.75 square inches and a net area of 5.75 − 0.50 = 5.25 square inches, one ⅞-inch rivet-hole being taken out of the section. Since this area coincides quite closely with the required area and is larger, it will be used. A 6 by 3½ by ⅝-inch angle would have been better in regard to area, but the rivet spacing is less than 2⅝ inches at the end, and this required a double gauge line and therefore a leg 5 inches or over.

The maximum shears at the tenth-points are now computed, and are tabulated as follows:

$V_0 = 75\,400$ pounds.
$V_1 = 65\,000$ "
$V_2 = 54\,600$ "
$V_3 = 44\,200$ "
$V_4 = 33\,800$ "
$V_5 = 26\,000$ "

The value of the shear to be used in any particular case is given above. In this case, $P = 52\,000$ pounds; $v = 6\,570$ pounds, ⅞-inch rivets being used; and h_g is $24\tfrac{1}{4} - (2 \times 2\tfrac{1}{4} + \frac{2\tfrac{1}{2}}{2}) = 18\tfrac{1}{2}$ inches. The rivet spacing for the first division or first two feet of the span is:

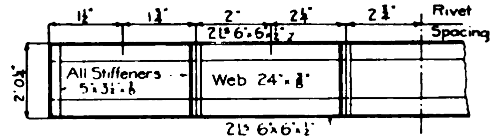


Fig. 122. Stress Sheet of Runway Girder of Problem 2 on Page 91.

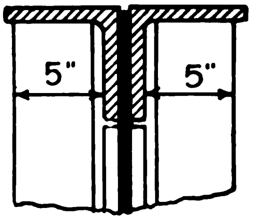


Fig. 123. Determination of Size of Stiffener.

$$S = \frac{6\,570}{\sqrt{\left(\frac{75\,400}{18\tfrac{1}{2}}\right)^2 + \left(\frac{52\,000}{30}\right)^2}} = 1.495, \text{ say } 1\tfrac{1}{2} \text{ inches.}$$

The rivet spacing for the other divisions may be computed by the student. It is given in Fig. 122. The web of the girder should be stiffened as shown in the figure, by angles placed as there indicated. The thickness of the angles should not be less than $\frac{5}{16}$ inch, nor greater than ½ inch. The size of the angles should be such that the

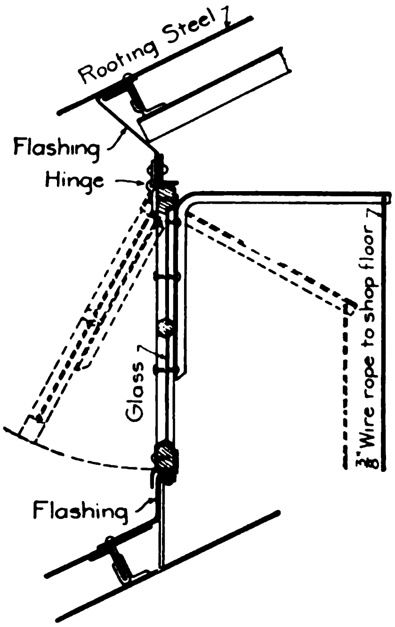


Fig. 124. Section of Glass Louvres in Monitor.

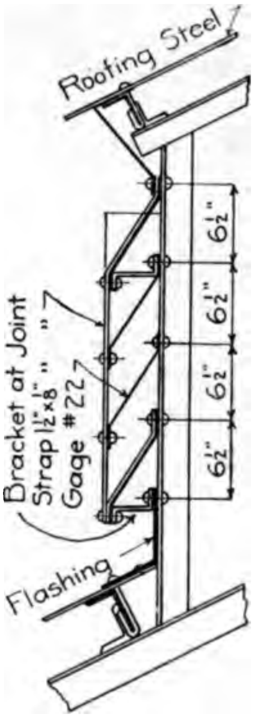


Fig. 125. Section of Metal Louvres in Monitor.

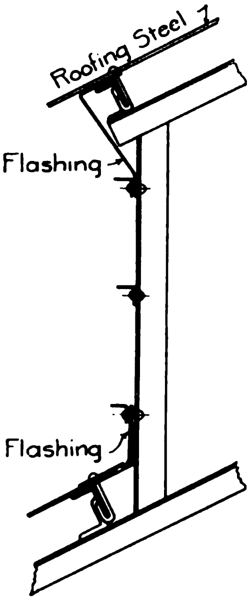


Fig. 126. Section of Open Monitor.

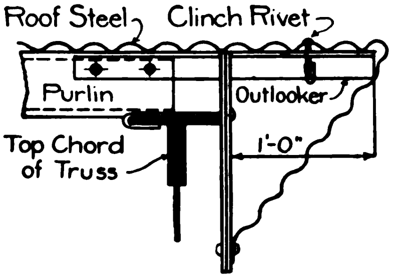


Fig. 127. Gable Details for Corrugated Steel.

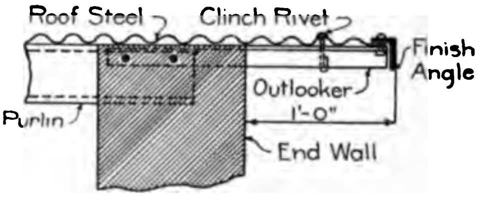


Fig. 128 Gable Details for Corrugated Steel.

outstanding leg does not reach beyond the leg of the flange angle (see Fig. 123). This makes their size as shown in Fig. 122. The crane rail may be connected directly to these and the flange angles; or a channel may be placed over the flange angles and riveted to them in a manner similar to that employed in the case of I-beams, the crane rail being fastened to that. If this latter detail is employed, the area of the channel is reckoned as forming part of the upper flange; and the net area of the angle must then be equal to the required net area, less the net area of the channel.

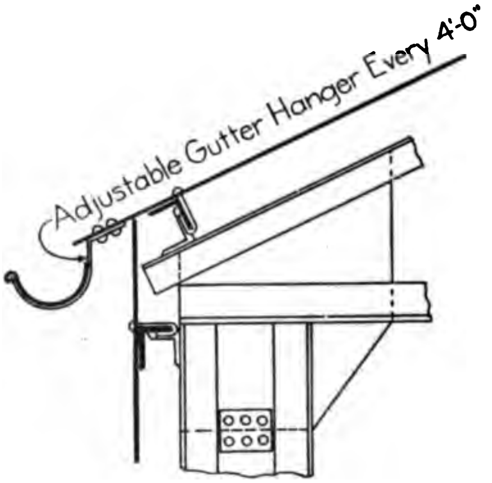


Fig. 129. Cornice Details for Steel Roof. See also Figs. 130 and 131.

27. **Ventilators.** Mill buildings may be ventilated by means of small circular ventilators such as shown in Fig. 69, p. 41, placed at certain intervals along the ridge or peak of the roof, or by means of monitors as shown in Fig. 71. The sides of these monitors may be fitted with swinging glass windows, with wooden or metal louvres, or, in case a large amount of ventilation is required, may be simply left open. Figs. 124, 125, and 126 give details of monitors, and show how they are connected to the trusses.

28. **Gable Details.** The gable is the end of the roof at that end of the building which is parallel to the roof trusses. Since this extends beyond the plane of the side of the building, some method must be employed in connecting the outer edge with the wall of the building, in order to keep out the rain and wind. Figs. 127 and 128 give several details which are efficient and at the same time economical.

29. **Cornice Details.** The cornice is that edge of the roof which is perpendicular to the planes of the roof trusses. In addition to being necessarily so constructed as to keep out the wind and the elements, it must have in many cases some form of gutter connected to it, which takes the water off the roof. This gutter should be con-

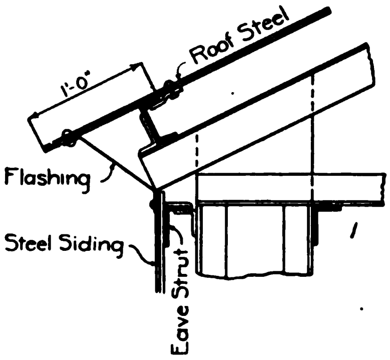


Fig. 130. Cornice Details for Steel Roof. See also Figs. 129 and 131.

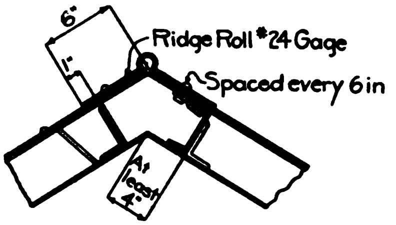


Fig. 133. Ridge Roll.

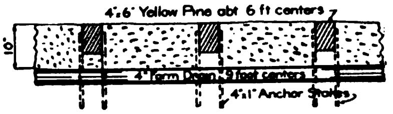

4"x6" Yellow Pine abt 6 ft centers
10"
4" Form Down 9 feet centers
4"x1" Anchor Stakes

Fig. 134. Detail of Cinder Floor.

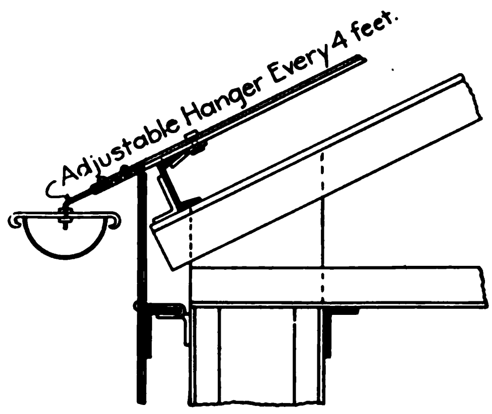


Fig. 131. Cornice Details for Steel Roof. See also Figs. 129 and 130.

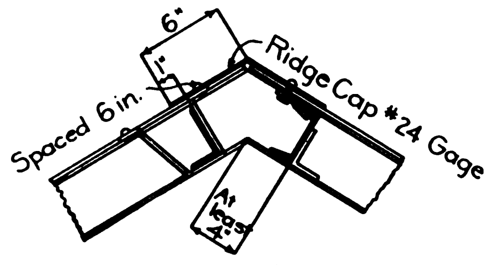


Fig. 132. Ridge Cap.

nected at intervals of every three bays—or, in case this exceeds 50 feet, every two bays—with a pipe or conductor to carry the water to the ground. Gutters, as a general thing, are semicircular or nearly so; and for ordinary spans they should not be less than 6 inches wide. Conductors should not be less than 5 inches in diameter. It is not to be supposed that the water entirely fills either the conductors or the gutters. The sizes are made so as to allow for any obstruction such as dirt or ice. Gutters should preferably have a pitch of one inch in every 10 feet. Figs. 129 to 131 give details of cornices with various forms of gutters attached.

The ridge, or peak of the roof, is usually covered with a plain sheet of metal, in which case it is called the *ridge cap*; or with a metallic roll with flared sides, in which case it is called a *ridge roll.* Figs. 132 and 133 show cross-sections of a ridge cap and a ridge roll.

30. **Floors.** The floor of the shop depends very largely upon the purpose for which the building is intended. It may consist of earth, cinders, boards, concrete, or sheet steel. In cases where men are required to work standing, cinders or boards give the best results. Earth floors will wear into holes in places where the men stand, and concrete or steel makes them foot-weary on account of its inelasticity. Where heavy machinery is installed, and men are seldom present except for a short time at certain periods, concrete makes an ideal floor. Figs. 134, 135, and 136 show details of various kinds of floors. Fig. 137 gives a detail of the floor in the Steam Engineering Laboratory of the University of Illinois. This consists of channels imbedded in concrete. These channels, which are placed in pairs a small distance apart, run both lengthwise and crosswise of the shop. The advantage of this form of construction is that machinery can be placed anywhere on the shop floor and quickly bolted into place by means of T-bolts, a detail of which is shown in the figure.

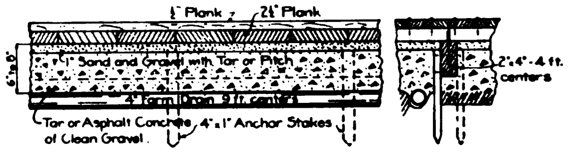

½" Plank
2½" Plank
6" to 8"
1" Sand and Gravel with Tar or Pitch
4" Farm Drain 9 ft. centers
Tar or Asphalt Concrete of Clean Gravel.
4" x 1" Anchor Stakes
2" x 4" - 4 ft. centers

Fig. 135. Detail of Asphalt or Coal-Tar Concrete Floor.

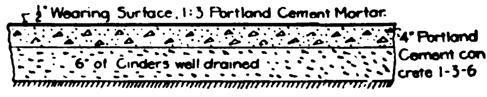

½" Wearing Surface. 1:3 Portland Cement Mortar.
4" Portland Cement concrete 1-3-6
6" of Cinders well drained

Fig. 136. Detail of Concrete Floor.

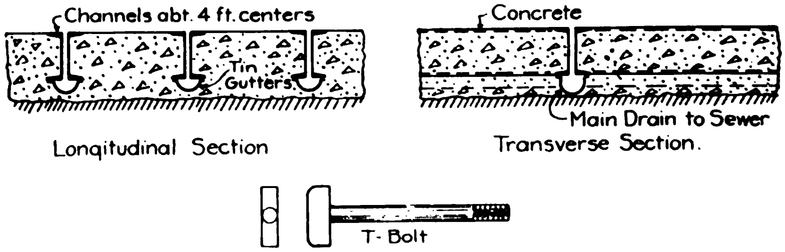


Fig. 137. Detail of Floor of Steam Laboratory of the University of Illinois.

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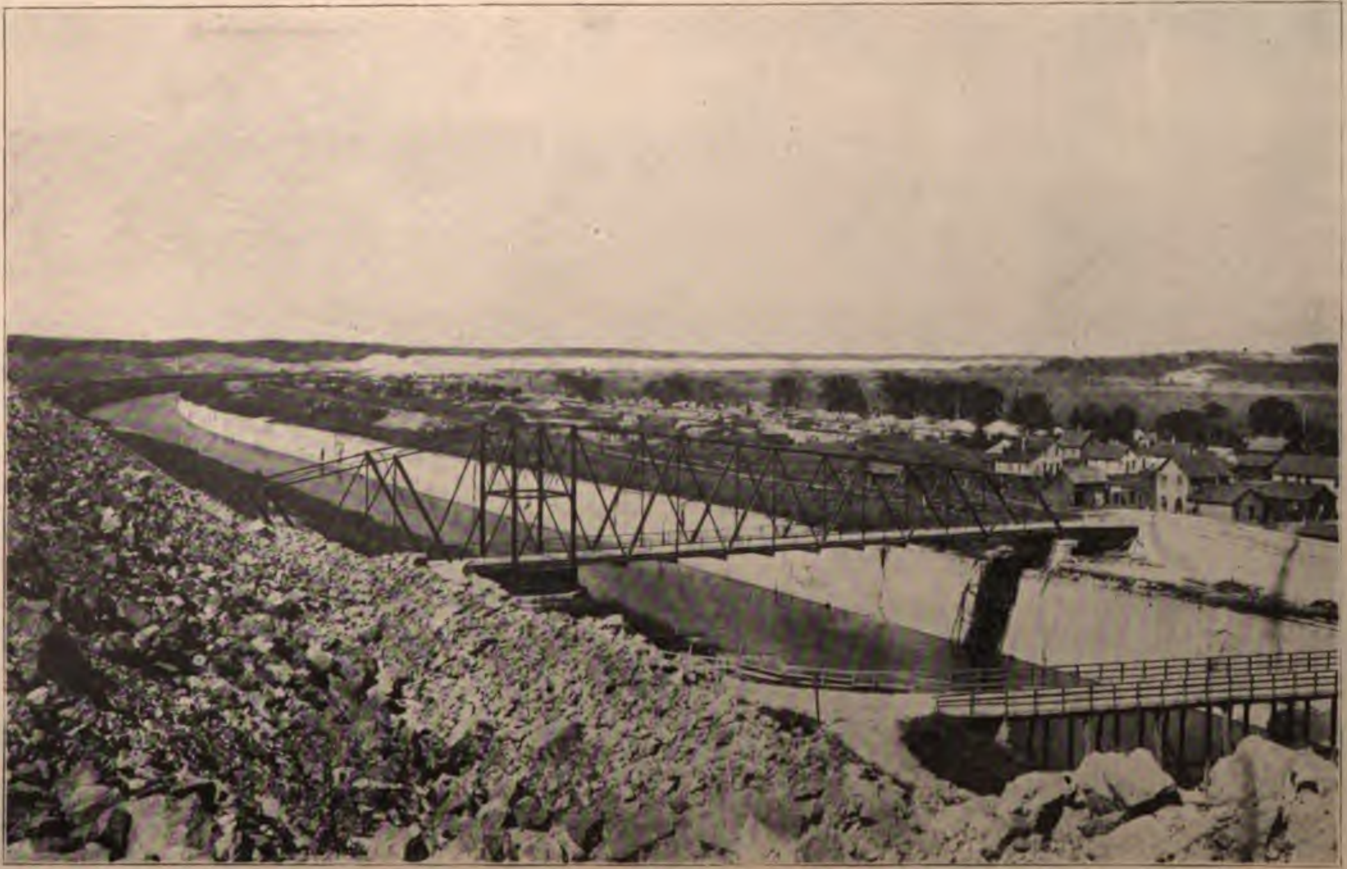

CURVE IN CHICAGO DRAINAGE CANAL NEAR ROMEO, ILLINOIS

The white streak along the bank is all stone excavated from the channel. The limestone taken out in the construction of this waterway, which is practically a 25-mile extension of the harbor of Chicago, has a market value, for concreting, paving, etc., of about $30,000,000.

COST-ANALYSIS ENGINEERING

Definition. Cost-Analysis Engineering is that branch of Engineering which has for its object the analysis of costs of construction or of operation, with a view to effecting a greater economy of production, and with a view to securing accuracy in estimating the probable cost of projected structures or operations.

The Modern Manager a Cost-Analysis Engineer. It takes few men to design machines and structures, but it takes many men to superintend their operation. Therefore the great field of activity for the engineer of the future is in the field of operation rather than of design.

Until very recent years, engineers have rested satisfied with being designers of labor-saving appliances. Now, however, they are beginning to assume the broader and more profitable function of operating the plants which their brains have created. To handle the ordinary industrial enterprise successfully, involves:

First, the application of engineering ability in selecting and improving the machines used;

Second, managerial ability in organizing the workmen, and in stimulating them to produce a large output economically;

Third, advertising ability to sell the product.

The man who combines in himself the maximum sum of these three abilities is the man best adapted to succeed as the executive of an industrial enterprise. Since the introduction of systems of cost analysis and unit-payments for work done, engineers have become best qualified to act as managers of manufacturing plants. We include contracting and railroading among manufacturing industries, for the contractor manufactures structures, and the railroader manufactures transportation.

Before cost analysis had been developed to its present stage of excellence, the successful manager of men was usually one who had relied upon his lynx eyes and his knowledge of the weaknesses of human nature. He was often a man who owed his success largely to the fear he could inspire in his subordinates. He was domineering; he held his men to their tasks; he was, indeed, an industrial captain;

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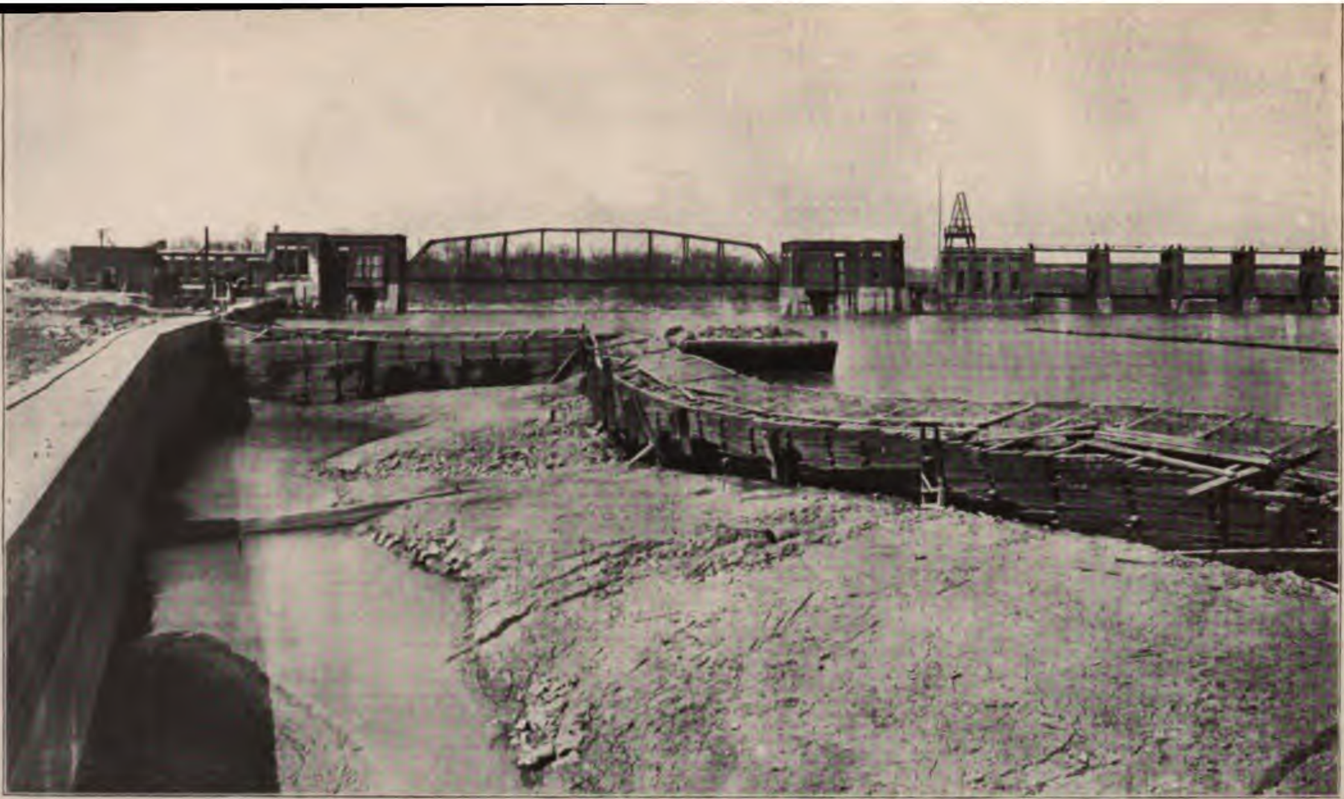
and he used army discipline. He regarded every worker as a thief who would not hesitate at petty larceny of time, even in the face of the foreman, and who delighted in grand larceny behind his back. His foremen were his spies; and he set himself to spy upon his foremen. But cost-analysis engineering is evolving a wholly different class of managers and foremen.

To most people, a cost-keeping system means nothing but a sort of bookkeeping; and they are unable to understand how a bookkeeper can develop into a successful manager. But the truth is that modern cost keeping involves cost analysis, and cost analysis involves a study and comparison of methods and machines, and such a study leads to improvements and to commercial success.

Cost keeping, in the sense that we use the term, has for its main object the determination of the efficiency of men. A proper system of cost keeping tells you daily what each workman or each gang of workmen has accomplished. It is better than a foreman, for it cannot "stand in" with the men. It is better than a foreman, for it costs you less and it tells you more. A cost-keeping system tells you who are your good men, and who are your lazy men. It shows you whom to discharge, and whom to promote. It tells you whose wages are too high, and whose are not high enough. And, finally, it leads to that ideal condition of industrial organization known as *profit-sharing*. How often have we read in novels of Utopia, where all men share in the profits of all business; and how often have we smiled with incredulity at the prospect? Yet Utopia is right here in America, in spots; and it is a Utopia far more rational than that of the dreamers. There are many firms that pay their men on a unit-price or bonus system. This is profit-sharing, and it is a profit-sharing begotten by the use of cost-keeping systems; for, when a manager has learned by cost keeping that certain men or groups of men produce more than others, he soon perceives the advantage of stimulating them to further use of brain and muscle by paying them either a bonus for each unit produced in excess of a prescribed minimum, or a unit-price for each piece of work performed. The men invariably respond to this stimulus, and often in a remarkable degree. It is nothing unusual for a man to increase his output 50 per cent upon the introduction of a bonus system of payment; and there are many instances of increase amounting to 200 per cent. Each man then becomes a contractor,



PART OF DAM CONSTRUCTION ON CHICAGO DRAINAGE CANAL AT LOCKPORT, ILL.

The temporary dam of stone, clay, and concrete here shown diverts water from the "Butterfly" dam and the new channel, to the "Bear Trap" dam and the Desplaines River.

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and works with the zeal of a contractor, for his earnings increase as his energy and ability increase. This is practical profit-sharing that any workman can understand. It is not something vague and intangible, like 5 per cent per annum. It is something very real and immediate, for a man can feel it in the pay envelope at the end of every week.

Cost keeping, then, leads to better management, although dispensing largely with submanagers. It substitutes the record card for the "big stick," yet the record card itself is the biggest stick ever devised.

The Science of Management. The managing of industrial enterprises is still more or less of an art; but the art is fast passing through the period of evolution that produces a science. There are, unquestionably, certain underlying principles of management which can be summarized into rules or laws. These rules or laws constitute the science of management, and it is our purpose to present certain of the more important laws of management.

Individual Incentive. When a group of men undertake to do a certain piece of work, such as shoveling earth into a wagon, the tendency is for each man to do as little as his neighbor. The inevitable result is that the shovels move with rhythmic precision, and the slowest man becomes the pacemaker for the rest. If any one of the men is ambitious to do a larger day's work, he is deterred by the knowledge that his employer will never know that it is he to whom the credit is due for a larger output. Then, too, the other men are apt to upbraid an ambitious man, and urge him not to set a "bad example" by working fast. To offset this tendency to fall to the lowest level of efficiency, employers have placed foremen over their employees, the duty of these foremen being to accelerate the motions of the men in any way possible. Each foreman has an *individual incentive* to get work done economically, for his employer studies the total amount of work done by the gang under the foreman, and rewards or punishes the foreman accordingly, the reward usually consisting of praise and an increase in salary. But the workmen under such a foreman have no *individual incentive*, and they will shirk their tasks as far as possible. Clearly, then, the first law of management is to create an *individual incentive* for every employee to do his best.

Creating Individual Incentive in a Gang. There are, and always will be, certain kinds of work that must be performed by a group of men working together, or, as we shall call it, a *gang* of men.

When this is the case, the first step to be taken is to devise a method of readily and accurately measuring the work performed *each day*—not each week or each month—by the gang. The next step is to notify the men that, for all work performed daily in excess of a specified number of units of work, a bonus or premium will be paid for each excess unit. Of this bonus, the foreman will get a specified percentage, and the men will divide the rest among themselves. Thus a powerful individual incentive is created. It is true that certain men in the gang will remain less efficient than certain others, but the general average output will be greatly increased. The foreman himself will have enough incentive to see to it that the lazy or inefficient workmen in the gang are discharged, for it will no longer pay him to play the part of indulgence for the sake of being "a good fellow."

Devising Ways of Dispensing with Gang Work. Simply because it has always been the custom to do certain classes of work by gangs, should not deter a manager from endeavoring to devise a way of splitting the gang up into individual units. Indeed, it should be self-evident that if the creating of individual incentive is the fundamental law of management, a great amount of study may profitably be devoted to increasing individual incentive by doing away with gang work entirely. To illustrate, let us assume that 12 men are engaged in shoveling earth into wagons, working in two gangs of 6 men, with one foreman supervising the 12. If a sufficient number of teams and wagons are used, there will always be 2 wagons in the pit being loaded, and 6 men shoveling into each wagon. As fast as a wagon is loaded, it pulls out, and an empty one takes its place. If a manager is told that he can do away with this system of gang work, he will usually reply that it is impossible. Nevertheless, it is possible to reduce this gang work to individual work in most instances, as follows:

Instead of having 2 wagons and teams in the pit all the time, have 6 wagons without teams—6 empty wagons. Assign two men to each wagon. Provide a dividing board between the sides of each wagon, running either longitudinally or cross-wise, so that each man has his definite half of the wagon to fill. Then pair the men off according to their respective abilities, putting the two best men on one wagon, the two next best on another wagon, and so on. When a team brings an empty wagon into the pit, let it be unhooked from the empty wagon and hooked to a loaded wagon, thus saving team time, which would otherwise be consumed in waiting for the wagon to be loaded.

It is possible to give many illustrations of this sort, but not desirable, for our object is to indicate the laws that should be applied, rather than to solve specific problems. The student of cost-analysis engineering will derive his greatest stimulus from applying the laws to specific cases that come under his own observations.

Prompt Reward. Most men believe in Heaven, and many believe in Hell; but few are greatly affected in their action by the hope of the one or the fear of the other. Any reward or punishment that is remote in the time of its application, has a relatively faint influence in determining the average man's conduct. To be most effective, the reward or punishment must follow swiftly upon the act. Hence a managerial policy that may be otherwise good is likely to fail if there is not a prompt reward for excellence. All profit-sharing systems have failed, principally because of failure to recognize the necessity of prompt reward, as well as because of failure to recognize the necessity of individual incentive. The lower the scale of intelligence, the more prompt should be the reward. A common laborer should receive at least a statement of what he has earned every day. If, in the morning, he receives a card stating that he earned $2.10 the previous day, he will go at his task with a vim, hoping to do better. But if he does not know what he has earned until the end of a week, his imagination is not apt to be vivid enough to spur him to do his best.

One contractor, known to the authors, has a large blackboard on which the hourly record of his brickmasons is chalked up. He has found that this constant record of where they stand in the day's race is a splendid stimulus.

Sufficient Reward. When a man produces more than has been his custom, he feels entitled to a very large percentage of his increased output. His sense of justice is keen on this matter, and rightly so. It is true that he is not entitled to all the increase, for his employer may have provided him with machines or tools of a better kind, for which payment must ultimately be made. Moreover, more rapid work with any machine means more rapid wearing-out of its parts, and a consequent expense to the employer. Finally, the employer who has used his brains to devise ways of increasing the output of the employees is entitled to a very substantial reward. No one begrudges Thomas

Edison his wealth. He has earned it by virtue of his inventions. In like manner, every man should be richly rewarded for every labor-saving machine or method which he creates or which he applies. However, employers are prone to try to take too large a part of the profit effected by an introduction of a system of unit-payment for work done.

Mr. Fred W. Taylor says that a workman should receive 30 to 100 per cent increase in wages upon the introduction of a piece-rate or bonus system of payment. Mr. Halsey says that the workman should receive one-third of increased value of his product resulting from an application of the bonus or premium system of payment. But the fact is that the employer should share liberally with his men; and, in the long run, the competition of other employers who are bidding for the services of workmen will force wages up to a point where the workman secures all but a very moderate percentage of the value of his daily product.

Educational Supervision. As previously stated, the old type of foreman mingles the functions of a spy with the functions of a mule-driver. The higher we go in the scale of human intelligence, however, the more noticeable is the fact that *the supervisors are teachers of the men they supervise.* These supervisors, foremen, managers—call them what you may—have learned that it pays better to spend time in training their men than to spend time in tongue-thrashing. Only of late years has it been discovered that systematic training of the least intelligent of workmen pays equally as well as the training of the most intelligent. The manager who recognizes the necessity of educational supervision, undertakes, first, a careful time study of each class of work. Then he analyzes the results, and deduces methods of securing greater economy. Having evolved a method of procedure, he reduces it to writing, and furnishes his foremen with detailed written or printed instructions to be followed, or, where the workmen are intelligent enough, the instructions are given to them directly; otherwise it is the function of the foreman to instruct the workmen.

Divorce of Planning from Performance. We have just spoken of the education of the workmen by the manager; but, before such education is possible, the manager must educate himself. In brief, he must study the problem and plan its most economic solution. According to the old-style method of management, each foreman was left largely to his own resources in planning methods, and, added to this duty, he had several other duties to perform, such as "pounding the men on the back" when lazy, seeing that materials were promptly supplied, employing and discharging men, looking after the condition of machines, etc. This multiplicity of duties can be properly performed, only by a foreman possessed of a multiplicity of talents. Since few foremen can comply with such a specification for brains, it follows that good foremen of the old style are rare indeed. The modern system of management consists in taking away from the foreman the function of planning the work, and in providing a department that does all the planning. This planning department should be under the supervision of the Cost-Analysis Engineer, for it is he and his assistants who, by unit-timing of work and by cost keeping, are best able to ascertain what methods should be applied to get the most economic results. Having planned a method, the Cost-Analysis Engineer delegates its performance to one or more foremen.

Subdivision of Duties. The previous rule of action comes under another, still more general in character—namely, the law of the *subdivision of duties.* Men are gifted with faculties and muscles that are extremely variable. One man will excel at running a rock drill, another at keeping time, another at surveying, and so on. It is clear, therefore, that the fewer the duties that any one man has to perform, the easier it is to find men who can perform the task well. But give a man many duties to perform, and he is almost certain to do poorly in at least one respect, if not in several. One foreman may have a great knack at "keeping an eye on" machinery, and in having few breakdowns and delays. Then it is the part of wisdom to burden him with no other duties, unless the magnitude of the work does not warrant dividing the duties among two or more men. Let him be the *machinery and tool foreman*, reporting directly to the Cost-Analysis Engineer, and subordinate to no other foreman.

Another foreman may have a special knack at teaching workmen how to use tools and machines. Let him have no other duty but to see that the men have the proper tools, get them promptly, and use them properly. Let him be the *gang foreman.*

According to the magnitude of the work, there may be different kinds of foremen, all coming in contact with the same men, perhaps, but each performing different functions.

Limitations of Military Organization. Most industrial organizations to-day resemble military organizations, with their generals and intermediate officers, down to sergeants, each man reporting to but one man higher in rank. There is little doubt that the present tendency in industrial organizations is to abandon the military system to a very large extent, and for the following reasons:

A soldier has certain duties to perform, few in number, and simple in kind. Hence the man directly in command can control the actions of his subordinates easily and effectively. Control, moreover, should come invariably from the same officer, to avoid any possibility of disastrous confusion, and *to insure the instant action of a body of men as one single mass.*

On the other hand, industrial operations do not possess the same simplicity, particularly where men are using machines; nor is there the necessity of action in mass. The military organization, therefore, should be modified to suit the conditions; and one of these modifications is the introduction of two or more foremen in charge of certain functions or duties of the same men or groups of men, as explained in the paragraph on *Subdivision of Duties.*

Opposition to Change. All men have a certain mental inertia which makes them resist any change of their methods and habits. Foremen are particularly resistant to change, because of their custom of giving orders more frequently than receiving orders. Hence the Cost-Analysis Engineer who is trying to introduce modern methods is sure to meet with violent opposition from foremen; and the older the foreman, the more violent the opposition. When the Cost-Analysis Engineer introduces a new method, he must personally attend to every detail, or it will surely "go wrong." The old foreman will see to it that it does "go wrong," just to show that the "new-fangled ideas" are worthless.

Opposition may also develop among labor unions, particularly if it is proposed to pay on the piece-rate plan—that is, to pay so and so much for each unit of work performed. The bonus plan and the premium plan (to be described later) are schemes to overcome this opposition to the piece-rate plan, but in essence they are all the same.

No manager of men can attain great success unless he has grit enough and tact enough to overcome the opposition to change which he will encounter from all quarters. If he realizes in advance that such opposition is as certain to manifest itself as it is certain that it takes power to change the direction or speed of motion of a heavy body, he will have possessed himself of one of the laws of successful management.

A man cannot impart motion to a very heavy rock by violent impact of his own body against it; but he can separate it into fragments, and move each fragment by itself. In like manner, no attempt should be made to change all the methods of an industrial organization at one stroke. Separate it into elements, and take one element at a time, beginning with the simplest. Apply your cost-keeping system to that element—it may be only the hauling of materials with teams—and effect the change desired. Then take another element of the organization, and apply the system to it. Continue thus, fragment by fragment, and you will overcome the opposition that would otherwise resist your greatest effort.

Respect Your Own Ability. One of the most common mistakes made by managers lies in assuming that a skilled workman necessarily knows better how to perform work than does the manager himself. A manager should first aim to familiarize himself with the methods used by the best workmen, and then, by an itemized time study, he should set his own wits to work to improve the methods. Workmen, for the most part, do their work just as robins build their nests—by the pattern of precedent. They put little or no brains into improving the process, because it usually means no money in their pockets to effect an improvement, and because they reason that an improvement that effects a saving in time may actually result in the discharge of some of their fellow workmen. It should be a cardinal law of management to give very little weight to the claims that workmen make as to their own skill or knowledge; and the same holds true as to foremen. Because a man has blasted rock for twenty years, should not make his opinion of such force as to prevent a manager from undertaking to show that man how to do rock-blasting more economically. We have frequently effected great economies in rock-blasting after a time study occupying fewer weeks than the blaster had occupied of years in the same sort of work. The trained mind of the Cost-Analysis Engineer enables him to analyze costs and methods, and to develop improvements which no amount of so-called "practical experience" can effect.

Weigh carefully every *reason* against any proposed change in method, and act accordingly; but pay no attention whatsoever to predictions of failure that are bare of reasons. Do not be influenced even by many positive statements that your proposed method has been tried and has failed; for its failure may have been purposely brought about, or some small condition essential to its success may have been absent.

Therefore, respect your own ability. The manager who cannot improve upon methods used by his men is not fit to manage.

Profit Does Not Mean Excellence. Many a manager points to the profits of his business as the profit of his ability. He forgets that to a plainsman a small hill looks like a mountain. The general level of mediocrity makes such managers fancy that they are quite extraordinary if their business shows a large profit.

The Cost-Analysis Engineer can frequently take a profitable business and convert it into a wealth-producer beyond all dreams of the ordinary self-satisfied manager. Nor should the Cost-Analysis Engineer himself grow satisfied. *There is positively no limit to the economies in production which may be effected by the human brain.*

The Human Engine. The human body is an engine, or rather a boiler and engine combined. Its fuel is about 3 pounds of solid food daily, containing about as much energy as one pound of carbon or coal. One pound of coal will develop energy enough to perform about 10,000,000 foot-pounds of work; that is, it will raise 10,000,000 pounds one foot high, if there is no loss of power. But in all boilers and engines there is a loss of power, due to heat lost by radiation, heat carried away in the escaping gases and solids, etc., and heat developed by friction. A steam boiler and engine suffers so much loss of heat energy from these sources, that it rarely develops an efficiency of more than 10 per cent of the theoretical energy of the coal consumed. Curiously enough, the human body is not much more efficient than a steam boiler and engine; so that, while the one pound of carbon fed into the human body has a theoretical energy of about 10,000,000 foot-pounds, the actual useful work performed by a man is seldom more than 1,500,000 foot-pounds a day, or about 15 per cent of the theoretical energy of the food consumed.

When a man is walking, his whole body rises and falls each step, the rise being about one-seventh of a foot. Hence, in walking 25 miles in a day, about 2,000 steps per mile, a man weighing 140 pounds does 1,000,000 foot-pounds of work in raising the weight of his own body, to say nothing of the energy consumed in swinging his legs. A man may walk the 25 miles in 10 hours, or he may walk it in 8 hours. In either case, he does substantially the same amount of work, and burns up substantially the same amount of food.

It should be clear, therefore, that when workmen are doing intermittent work, with periods of comparative rest, they are capable of working correspondingly harder during the periods of exertion. Thus, in running a rock drill, the physical labor is light, except when shifting the drill or when changing drill bits. At such times, the men should be required to work with great vigor in order to reduce the lost time.

It should also be clear that workmen should be taught to make no unnecessary movements of the body in doing work. Yet it is a fact that few workmen economize their energy by avoiding unnecessary motions.

It should also be clear that it pays to house workmen at no great distance from their work, so as to reduce the labor of going to and from the work; for every foot-pound of energy spent in going or coming reduces by that much the available energy of the man.

If it were practicable to measure the amount of resistance involved in doing each class of physical work, we could readily reduce to a science the setting of reasonable daily tasks. The authors are of the opinion that a careful study of *resistances* will eventually enable managers to fix certain tasks with great accuracy. To illustrate, let us assume that it is desired to know how much sand a workman should be able to shovel into a wagon box 5 feet from the ground in a day. It is not impracticable to measure the force required to push the shovel into the sand, and the distance pushed. The average weight of the earth on a shovel, and the weight of the shovel can be ascertained. The vertical height that this load is lifted, is easily measured. If the workman bends his body to fill the shovel, the weight of his body above the waist, multiplied by the height that the center of gravity of that weight travels will give the foot-pounds of work done in bending the body. And thus, by a calculation of each element of work done, an accurate forecast of the total possible work could be made.

Such a study as this will often disclose an unsuspected lack of economy in using certain tools. From such a study, for example, it is perfectly clear that the long-handled shovel, universally used in the far West for shoveling sand, gravel, etc., is a more economical tool than the short-handled shovel used in the East. Men have argued about this matter for years without coming to a definite conclusion, the reason being that workmen accustomed to the short-handled shovel prefer it, while workmen accustomed to the long-handled shovel show an equal preference for that type of tool.

COST GETTING

In taking of time and in the application of the cost of labor to the cost of work, there are probably as many systems as there are organizations doing work; and even within any one organization using a well-defined system throughout its entire operations, there will be no two men making the same interpretation of the rules laid down, or —more especially—whose methods of attack will be the same. But in spite of these many variations of method, there are several primary systems which are standard, and which can be found in one form or another on all properly conducted work.

The starting point of all cost getting is the taking of the time in the field, and it is here that the greatest variation in individual method is found. The most common way of taking this time record from which the pay-roll and the cost distribution is made, is for the time-keeper to go over the work with a notebook and put down therein with a pencil the number of each man and the particular part of the work that he is engaged on.

Two systems of record keeping, of which small cards form the basis, are also in vogue. One of these systems uses what is known as *punch-cards*—that is, cards in which the records of time, distribution, and performance are made by means of an ordinary conductor's punch; and the other has the record made in a way somewhat similar to the entries in a notebook—a written record being made on the cards with a pencil. Another system bases its records upon reports turned in by foremen.

Time-Keeper with Notebook. While the manner of taking time with a notebook varies according to the training and experience of the time-keeper, it may be said that there are in general two ways in which such notes are kept. In the first, the time-keeper has a list of the numbers of all men on the work, and, as he goes over the work, simply checks off the numbers, showing that each particular man is at work and indicating upon what branch of the work he is engaged.

A more common way, however, is for the time-keeper to make headings corresponding to the distribution used in making up the office records, and to write under each of these headings the numbers of the men working upon the part of the work so named. This method is often simplified by the time-keeper becoming so familiar with the foreman, and the numbers of the men under the particular foreman, that he is able to dispense with the headings entirely, and simply use the foreman's name or number in place of it. This, of course, makes the time-keeper's notes more or less unintelligible to anyone but himself, and makes it necessary for him to do office work as well as his field work. Moreover, not being a permanent or intelligible record, it is impossible for even the man who made the notes to return to them in case any dispute arises or a mistake is found to have been made, and get information after the notes have "grown cold." The time-keeper becomes so familiar with the appearance of the men who are on the work, that he learns to know their numbers, and often attempts to put them down without seeing their numbered checks. This is often a source of error, as the uneducated foreign laborer is very liable to make a mistake in stating his number; and if he does, there will exist no record of his having worked that day, and he will get no pay for it. The apportioning of the cost of his labor to any work that he may have been on, will also be the cause of trouble.

Of course, the time-keeper's memory serves him if any men are absent from the gang for any reason, and he is able to ask the foreman whether or not that particular man is working. A man may be away from the gang and be missed by the time-keeper altogether. In this case, no chance is given for correction of the record, unless the time-keeper goes over the work again soon after; and the consequence is that costs will be in error, and the men will be short of pay at the end of the month. This is especially liable to be true when night work is being done.

Men may be changed from gang to gang, or a whole gang may be changed from one job to another, and the time-keeper knows nothing of it unless he happens to be on the spot at just the right time. Such a change would not show in his time record; and while the men would get credit for their full time, the distribution of costs would be much in error. The difficulty in recording such changes can be seen from such items as the transportation and distribution of coal to various parts of the work, the transportation and handling of stores, and numerous other items which, while seemingly affecting the whole work, are directly chargeable to some particular operation.

In many instances of distribution, the item of *General Expenses*, which includes the expense of store-keeper, time-keeper, bookkeepers, clerks, and such office force as may be required, is rather difficult of disposition. Those items which are usually monthly, may be distributed daily at a rate per day found by dividing the monthly rate by the number of days in the month, or they may be lumped at the end of the month and apportioned to the various operations. If they are distributed from day to day, it is rather difficult to tell just what proportion of them should go to each operation, as the cost of any operation is liable to vary greatly from day to day. If they are left to the end of the month, it is impossible to tell from day to day the exact cost of the work.

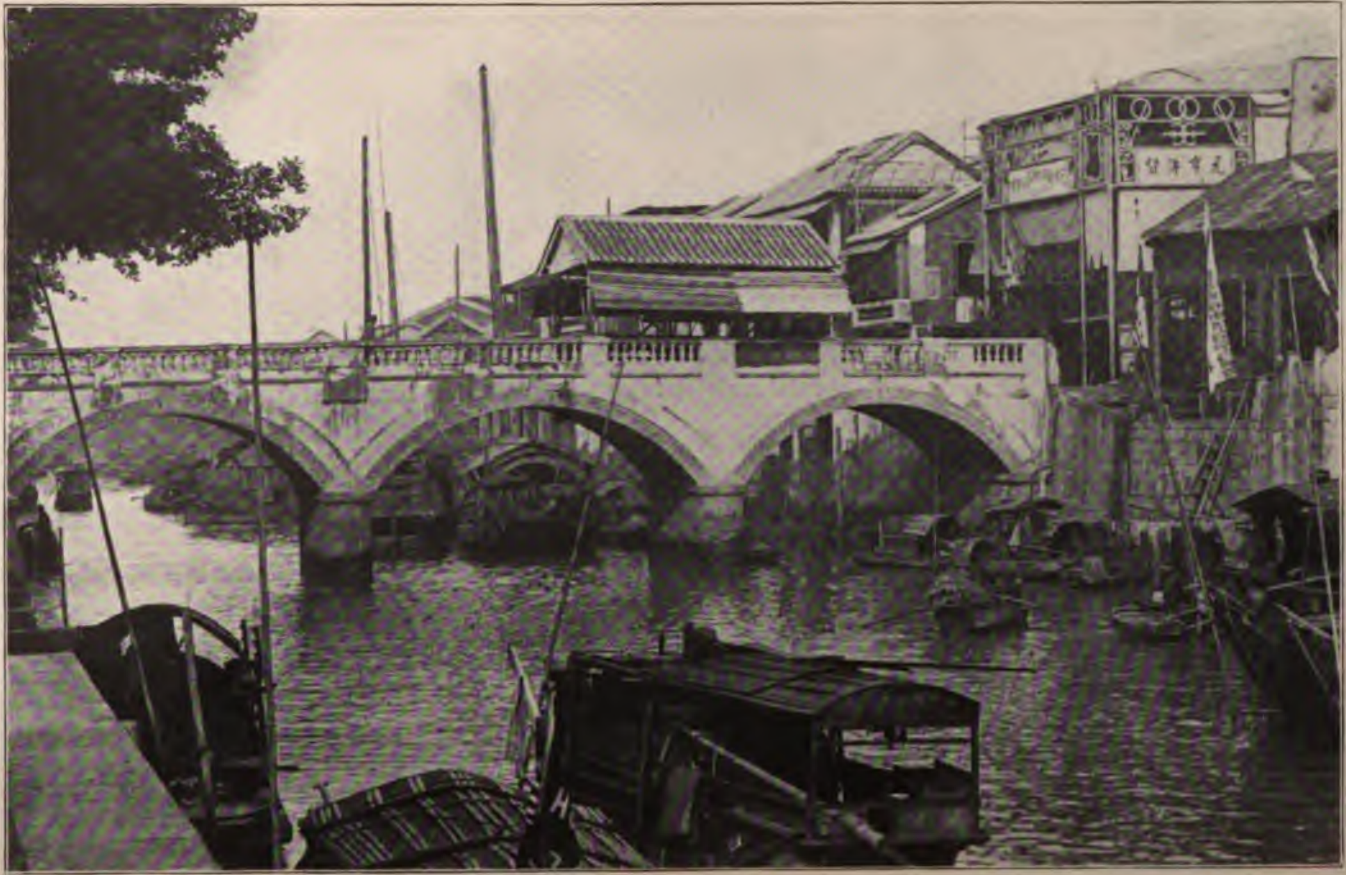
Overhead Expenses are another source of difficulty. Under this heading can be placed all salaries which do not ordinarily appear upon the pay-roll, such as the salary of the General Manager of the work, the Chief Engineer, and the officers of the company, and such expenses as office rent, telephone, office furniture, stationery, etc. Just where General Expenses leave off, and Overhead Expenses begin, is rather hard to determine, the line of demarcation varying in almost all cases.

One of the greatest troubles in distribution is caused by overtime of men who are on a daily and monthly basis. Under the same head might be placed *Lost Foreman's Time*—that is, the time which the monthly and daily men are paid for, and which produces no output.

TIME-KEEPER'S NOTEBOOK

A page from a time-keeper's notebook is reproduced in Fig. 3. The necessity for an explanation of such a record is apparent. The work on which this record was made, was a job of rock excavation on which two steam shovels were being used, and the time and output of each shovel were kept separately. The record was made on the 17th of the month.

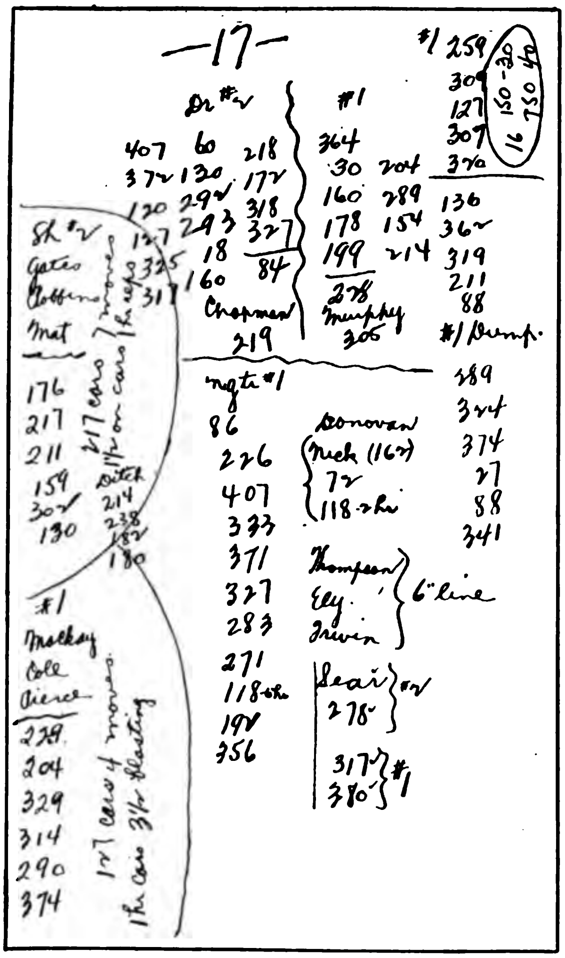
At the left of the page, within the curved line under the heading "*sh.* 2," we have the names of the shovel crew and the numbers of the pit men. At the side of these names and numbers, is the record of the



HARBOR SCENE IN CANTON, CHINA

English bridge over Pearl River, leading to foreign legations. No Chinaman is allowed to cross this bridge without a special permit. A large part of the more than two million people of the city live in boats on the river, some of which are here shown.

performance of the shovel of the day previous, the 16th, the shovel runner having given it to the time-keeper while the latter was on his first round on the 17th. Just below the record of the shovel, there are four numbers under a heading *Ditch*. In order to drain the shovel pit, it was necessary to use four men for cutting a ditch, and the time of these men is charged to the shovel. Underneath the record for the No. 2 shovel, is the record of shovel No. 1. It is recorded in exactly the same way as the other record.



—17—

Dr #2
407 60 218
372 130 172
120 292 318
127 293 327
18 84
160
Chapman
219

#1
364
30 204
160 289
178 154
199 214
278
Murphy
306

#1 259
309
127
307
370

150-320
750
16

136
362
319
211
88
#1 Dump.
389
324
374
27
88
341

Sh #2
Gates
Coffins
Mat
176
217
211
159
302
130
217 cars 7 moves
1½ on cars 1 hr repairs
325
317
Ditch
214
238
182
180

#1
Mackay
Cole
Pierce
228
204
329
314
290
374
127 cars 4 moves
1 hr cars 3½ hr blasting

ngt #1
86
226
407
333
371
327
283
271
118 2hr
192
356

Donovan
Nick (162)
72
118 2hr

Thompson
Ely
Irwin
6" line

Sear
278
#2

317
380
#1

Fig. 3. Page from a Time-Keeper's Notebook.

In the center of the page, at the top, is the record of the drill gang for each shovel. There were six drills working in gang No. 2; but the record does not show this clearly, as there are two columns of six numbers each, one column being the driller's numbers, and the other being those of the drill helpers. Then there is a column of four numbers representing the muckers. The time-keeper knows, of course, which is which; but if for any reason anyone else than himself had to use the notes in the office, they would be useless. The number under the line is that of the man who carries bits to and from the blacksmith. The name of the fireman of the boiler which furnishes steam to the drills is given, and the number of his helper.

The record of gang No. 1 is given in the same garbled manner, there being 5 drillers, 5 helpers, and 4 muckers in the gang, besides the man carrying bits, the fireman (whose name is given), and his helper. It will be noticed that in neither gang is the name or number of the foreman given, the time-keeper relying upon his memory to make the record complete in the office.

In the upper right-hand corner of the page, is the record of the blasting gang, in front of shovel No. 1. There are 5 men, including the foreman, whose number is given first. Within the ring is shown the number of pounds of powder used on the previous day as reported by the foreman. The record reads "on the 16th, 150 pounds of 30 % powder, and 750 pounds of 40 %."

Directly below this, midway down the page, is the record of the men working on dump No. 1. The foreman's number heads the list, the numbers of his men following.

Just to the right of the shovel records, and below the record for drill gang No. 2, is the heading "*ng. tr. # 1*;" and the men whose numbers are under this heading are engaged in laying and repairing the narrow-gauge track for the dump trains from shovel No. 1. Just below the middle of the page, is a list of names and numbers utterly unintelligible to anyone but the one who made it. The facts are these: *Donovan* is the man who looked after the storage of powder; within the bracket, *Nick* (the time-keeper, not knowing the last name, used his number, as well as the part of the name that he knows) is the blacksmith; and No. 72 is his helper; No. 118 helped the blacksmith for two hours, having been taken from the narrow-gauge track gang. The time-keeper had to depend entirely upon the blacksmith telling him this, or his record would have been incomplete. The next three men whose names appear in this column were engaged upon repairing a 6-inch pipe line; and the next two pairs within brackets, marked No. 2 and No. 1, are the pipe-fitters for the drill gangs and shovels No. 2 and No. 1 respectively.

DISTRIBUTION FROM TIME-KEEPER'S NOTEBOOK

The time-keeper, having taken his notes over the entire job, sends them to the office so that the time may be posted for each man, and the distribution made.

The time-keeper goes over his notes, and picks out the items that are chargeable to drilling. In gang No. 2, there are 6 drillers at 30 cents per hour; 6 helpers, 4 muckers, 1 man carrying bits, and one fireman's helper, all at 17 cents, and one fireman at 25 cents. From the note at the bottom of the page, he knows (although no one else would) that *Lear* at 20 cents and No. 278 at 17 cents, were also with this gang. This, with the foreman at $3.50 per day, figures to $29.08. These are the charges that go directly to drilling, being the cost of time of the men actually engaged upon that operation and nothing else. But besides this, there must be apportioned to this cost a certain part of the Superintendent's salary, a portion of the labor on the 6-inch water pipe and the whole water system, a portion of the time of the blacksmith, the watchman, the storekeeper, the time-keeper, clerks, the water boy, and numerous other items.

In exactly the same way, the cost of the operation of the steam shovels is figured. For instance, No. 2 has an engineer at $125, a cranesman at $100, and a fireman at $75, a month, and 6 pitmen at 20 cents per hour, making the total charge of crew $19.27. To this the time-keeper added $5.44 as the cost of digging the ditch that drains the shovel pit. To charge this whole amount against the shovel for that day, is manifestly unjust, as the work of draining through this ditch will continue for many days, always facilitating the work of the shovel. The cost of subsequent days' work is lessened, while the cost of this particular day, as given with the $5.44 charge against it, is entirely too high. The spreading of an item of this kind is an extremely difficult matter, but it must be done. The steam-shovel cost must also have its proportional share of the charge for Superintendent, water system, blacksmith, etc.

The charge for narrow-gauge track is $14.86, being the time of one foreman at 20 cents, and 9 men for 8 hours and one man for 6 hours at 17 cents. The charge against No. 1 dump is $8.40, being the time for one foreman at 20 cents, and 5 men for 8 hours at 17 cents. The cost of blasting is figured exactly the same way, and the 900 pounds of powder used entered in the material account charged against the work in front of shovel No. 1.

The headings for the distribution of steam-shovel work, aside from *Drilling* and *Blasting*, would be *Shovel crew*, *Pit crew*, *Dump crew*, *Laying shovel track*, *Train crew*, and *Laying narrow-gauge track*, all of which in the end can be summarized under *Loading and Transporting*, and the unit-cost of moving a yard of material figured from this summary.

PUNCH-CARDS

The manner in which the time-keeper takes his notes in a notebook has been shown, and the impracticability of many of its phases pointed out. Two punch-cards for use on such work as that mentioned —namely, rock excavation with steam shovels and dump trains—are shown in Figs. 4 and 5. They are the *Steam-Shovel Card* and the *Train Record*. The shovel card is kept by the shovel runner or the fireman, and the train record is kept by the dinkey runner. Each keeps his own record separately; and, at the end of the day's work, the records must check each other.

The steam-shovel card shows the date, the number of cars loaded per hour, and the total number loaded per day. It also shows the time of starting and stopping the shovel for any reason, the stops for moving up being indicated in a different way from other stops; and thus a record of moves is kept automatically. The time of the shovel crew and the exact number of hours worked by the pit crew, are also shown, together with the cubic feet of coal consumed by the shovel. The causes of delays and the condition of the shovel are written in the blank spaces under their proper heading at the bottom of the card; but with this exception, the entire record is made with the use of an ordinary conductor's punch.

The train card shows the number of trips made by a train each day, the time of leaving the shovel on any trip being shown to the nearest 5 minutes. The number of cars hauled by all the trains during any hour must check with the number of cars loaded, as shown on the shovel card. The train card, besides showing the date, shows the total number of cars hauled (the total of all cards must check the total cars as shown by the steam-shovel record), the cubic feet of coal consumed, the average yardage per car, the haul in stations of 100 feet, the number of the dinkey engine, and a report of its condition, whether it be good, fair, or bad. This card is signed with the dinkey runner's name.

It will be seen that the record is very much more complete than that taken by the time-keeper, and is more reliable as to methods, being made while the work is going on; and the greater part of it is checked by having two records made separately, instead of taking a verbal

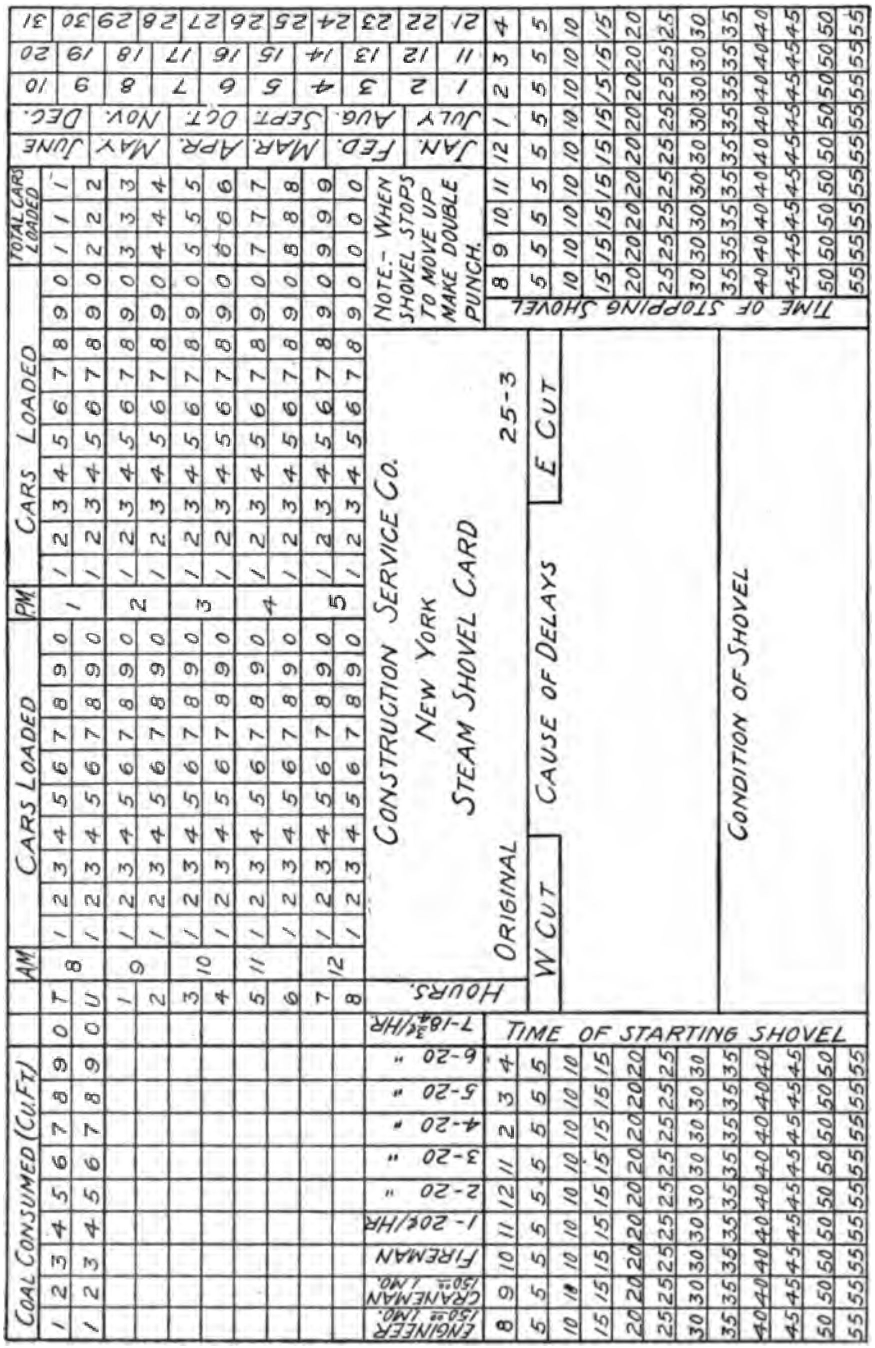


Fig. 4. Punch-Card for Recording Work of Excavation with Steam Shovel.

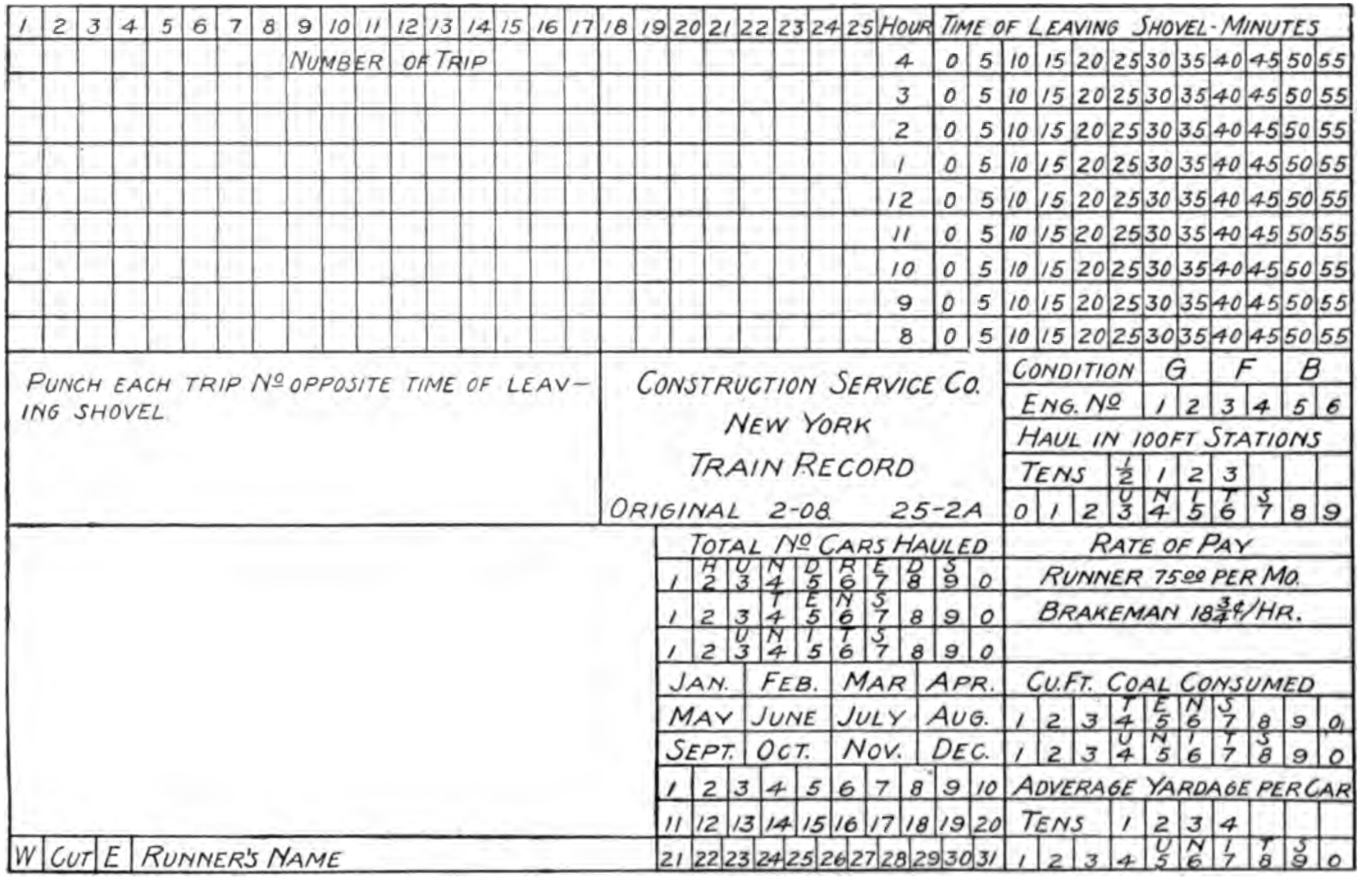


Fig. 5. Punch-Card for Recording Work of Dump Trains in Excavation Work.

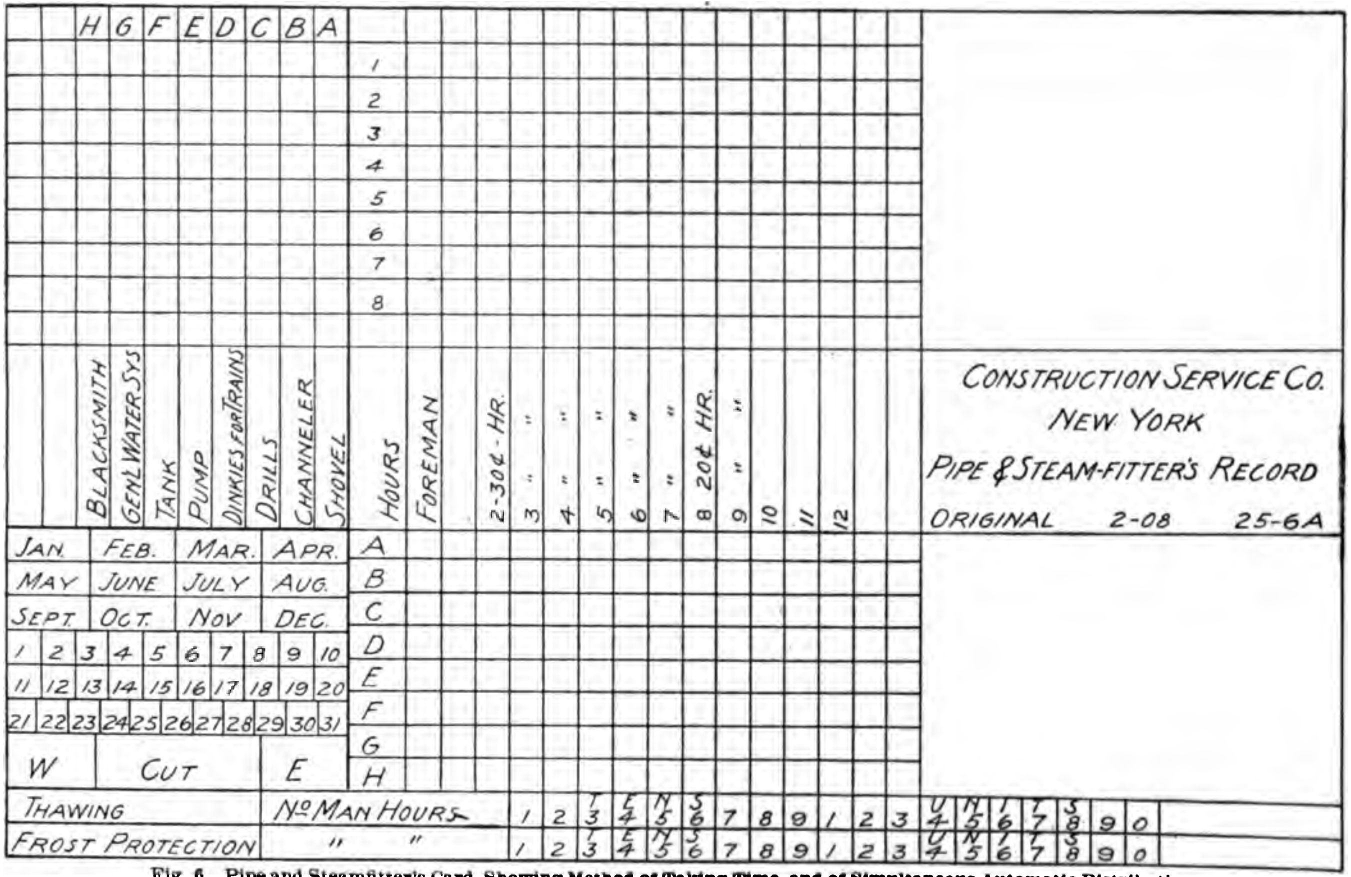
report from the shovel runner the following day as in the example previously shown.

Neither the train record nor shovel card, however, show any distribution of time, but are really performance records. The pipe and steamfitter's card reproduced in Fig. 6 gives an excellent example of how the time is taken and the distribution automatically made all at one time.

The classifications of labor are: Shovel, Channeler, Drills, Dinkey, and Trains, Pump, Tank, General Water System, and Blacksmith, being lettered, it will be noticed, from *A* to *H* at the head of the column. Each card provides space for the record of the foreman and 14 men. These eight classifications will probably cover all the work that the pipe and steamfitters are called upon to do; but if not, there are two extra lines on which can be written any classifications out of the ordinary.

There will be certain men assigned to certain regular work, as in the case previously quoted under the head of the Time-Keeper's Notebook, where there were two pipemen for each drill outfit. If these men spend their entire day of eight hours doing nothing but looking after the water supply for the drills, a punch mark would be made above the number of each of them on the card and opposite the figure 8, which represents the hours worked. To the left of the eight, and in the same line, and also in the vertical column opposite the word *Drills*, another punch mark will be made. Again, opposite the letter *C*, which is the key for the classification of drills, and in the column assigned to each man, and below his name, another punch mark will be made. This gives the workman full time, showing that he worked eight hours on drill water supply and nothing else. Suppose the foreman worked three hours on the general water system, three hours on the pumping station, and two hours directing the repair of the water tank. There would then be on the record a punch mark in his column opposite 3, 6, and 8; at the left of 3 in the column headed *G*, another punch will appear; at the left of 6 in the column *E*, another punch will be found, and still another at the left of 8 in the column *F*. In the column under the foreman's name, punch marks would be made opposite *E*, *F*, and *G*, showing that he worked on these three classifications.

In the same way, the time and occupation of each man under this foreman can be indicated, no matter how many changes he may make



CONSTRUCTION SERVICE CO.
NEW YORK

PIPE & STEAM-FITTER'S RECORD

ORIGINAL 2-08 25-6A

H G F E D C B A

1 2 3 4 5 6 7 8

BLACKSMITH
GEN'L WATER SYS
TANK
PUMP
DINKIES FOR TRAINS
DRILLS
CHANNELER
SHOVEL
HOURS
FOREMAN
2-30¢-HR.
3 " "
4 " "
5 " "
6 " "
7 " "
8 20¢ HR.
9 " "
10
11
12

A B C D E F G H

JAN.	FEB.	MAR.	APR.
MAY	JUNE	JULY	AUG.
SEPT.	OCT.	NOV.	DEC.

1	2	3	4	5	6	7	8	9	10	
11	12	13	14	15	16	17	18	19	20	
21	22	23	24	25	26	27	28	29	30	31

W CUT E

THAWING	NO. MAN HOURS	1	2	T 3	E 4	N 5	S 6	7	8	9	1	2	3	U 4	N 5	I 6	T 7	S 8	9	0	
FROST PROTECTION	" "	1	2	T 3	E 4	N 5	S 6	7	8	9	1	2	3	U 4	N 5	I 6	T 7	S 8	9	0	

Fig. 6. Pipe and Steamfitter's Card, Showing Method of Taking Time, and of Simultaneous Automatic Distribution.

in his work during the day. The time, however, is recorded only to the nearest hour.

Provision is made in the lower left-hand corner, for the punching of the date; and along the lower edge is the place for the recording of the number of hours used in thawing the pipes, etc., and in providing protection for them. This latter record was found necessary, because the work on which these cards were used was done in an extremely cold locality and continued throughout the entire year.

When the records are made in the field and are sent in to the office to be transferred to permanent records, it is not necessary for the man who made the record to be at hand to interpret his notes, as there is absolutely no opportunity given him to allow his note taking to vary in the least from day to day, the record being absolutely automatic.

PROCESS COST SUBDIVISION

While the object of the regular distribution of cost is the obtaining of unit-costs, there is another cost analysis which may be called a refinement of the cost-keeping system, and which, if properly used, can bring about a marked reduction in all costs. While this will be discussed more fully in the chapter upon *Reduction of Cost,* it is a form of time-keeping, and so will be touched upon here. On more or less rough construction work, it seems rather absurd to attempt to reduce the various processes of any operation to such a fineness that they may be timed to minutes and even to seconds. Conditions vary so greatly, the character of the work being done changes so much from time to time, and the personnel of the organization is sometimes shifted so much, that it seems impossible to reduce performance to any satisfactory basis which may be used as a standard. Nevertheless, without attempting to reach such a basis, careful watching and timing of the different parts of the work will result in much better performance and increased profits, as can be clearly shown.

Take, for instance, a driller working with a steam drill in fairly even rock, with no marked obstacles in his way and with very little mucking to do. Notice the exact time at which his tripod is in place and the drill ready to work. The driller places his bit in the drill, turns on the steam, and the drill starts. Note the time of starting the drill; note the time when the drill stops, the bit having gone down its full length; and do the same with each subsequent bit, noting carefully the exact time consumed in changing. When the last bit is down its full length and the hole is finished, note the time required to take out the bit, move the weights, loosen the tripod, and make everything ready for the moving. Then note just how many men are required to move the drill, and just how long it takes them to do it; and finally, how long it takes the driller to get his drill again in working order and started.

It will be found that a large majority of drillers take entirely too much time in the changing of bits, and that almost invariably there are too many men helping to move a drill, and that they take too long for it. Another source of delay is preparing the drill for work after it has been moved. It is perhaps just as well to take plenty of time for this, in order to get the drill properly set and adjusted before starting it; but the loss of time between the adjustment and the starting may be said to be about the same as that lost in changing bits, if not a little more.

When the driller takes too long in changing bits, it is largely his own fault, and he should be watched more carefully by the foreman, and, if necessary, instructed. If time is wasted in the moving of the drill, it is the fault of the foreman alone. By a careful timing and balancing of the various processes in drilling, the most competent men can easily be picked out.

In the case of concreting, the minutes lost in the handling of a batch of material from the stock pile to its final position as concrete, often amount to a great deal. Suppose on a small job a half-yard mixer is being used, and it averages for 8 hours 30 batches per hour, or 120 yards per day If it is possible to reduce the time of each batch 15 seconds, the output of the plant will be increased over 14 per cent; or, figured on a basis of 120 yards, there will be an increase of 17 yards, which—at, say, $5.00 per yard—would mean a handsome increase in the daily profits. And still, 15 seconds seems to be almost too trivial a matter for which to spend time and perhaps a little extra money in the way of time-keeping.

Starting with the unmixed material in the stock pile, notice how long it takes the men to load their wheelbarrows with sand and stone; then the time that the material remains in the wheelbarrow, both at the beginning and stopping end of the trip to the mixer; and also the time in transit. If the material is dumped into measuring boxes, note the time that it remains in the boxes. If it is dumped directly from the wheelbarrows into the mixer, it is necessary to take the time of mixing from when the first wheelbarrow was dumped until the batch is dumped. The mixer may be said to be the governor of the whole operation; for the men handling unmixed material can handle it no faster than the mixer takes it, and the men handling the mixed concrete can get it no faster than the mixer furnishes it to them. For this reason the observation of the operation of the mixer should be made with special care. It is not our intention to tell how, or to give advice concerning the mixing of concrete; but it is desired to show how, if any time is to be saved, it will be through the saving of seconds in each operation.

If the mixed concrete is to be dumped as a batch into the hopper or hoist, the question of time saving is much simpler than if portions of the batch have to be dumped into wheelbarrows. If, however, it is necessary to dump into wheelbarrows, a basis for the time necessary to empty the mixer can be found only by careful timing and noting the action of the men during the timing.

The time between the filling and the emptying of the wheelbarrow of concrete, will of course vary greatly according to the haul; but here again, careful timing and observation will soon establish a basis from which the most economical manner of distributing the concrete can be made; and exactly the same thing is true of the return of the empty barrels.

All of this may seem to be a digression from the subject of cost getting; but in fact it is merely a discussion of a very refined form of cost getting, and a branch of the subject which has perhaps been given too little attention. When the daily output of a job is up to or above the average, everything looks bright, and no one who is responsible feels overburdened with care. If, however the output falls too low, some glaring cause is at once sought, and the fall of output blamed to some unforeseen circumstance or accident. This is all very well, as accidents affecting output cannot be entirely avoided, and unforeseen conditions will make great differences in performance; but the careful analysis of process cost subdivision will bring about results that will astonish those "practical men" who think that they have got their unit-costs down to the lowest point simply because their output is generally large and everyone on the work seems to be working to his top notch.

OUTPUT

The reason for compiling the data for which the time-keeper is responsible, is that, from the analysis of the distribution made, the contractor is able to tell what work is being done with profit; and, if any particular operation shows loss, the analysis will help more than anything else to discover the reason for the loss. In figuring his profit on any work, the contractor must figure on a unit-cost basis, exactly the same as he figures when he prepares his bid. In order to do this, he must have an exact measurement of output. In many classes of work, this measurement is extremely simple; but in others no little ingenuity is required to devise a scheme which will give the information wanted exactly and without requiring much work.

The payment for work is based upon the engineer's estimate. The *monthly estimate* is usually more or less a guess, made simply for the purpose of paying the contractor approximately according to what he has done. The monthly estimate is generally a pretty fair approximation of the exact amount of work done; and the *final estimate* covers everything included in the contract that has not already been taken care of.

The contractor's measurements of work done each day should agree quite closely with the engineer's estimate; but, if the work is difficult to measure, the contractor has many times more opportunity of making errors in his measurement by going over it daily than the engineer has who only goes over it once. A careful consideration of the differences in the amount of estimates will sometimes show the contractor how his estimates can be made to balance with those of any particular corps of engineers, and he can govern his daily measurement accordingly.

There are few measurements in the field which can be reduced to a unit, or rather which can be counted directly. Linear measurements are easy enough to get; the measurement of area is a little more difficult; while the measurements of volume, especially in rough work, are often extremely difficult to make in a satisfactory manner. Measurement by weight is often found to be of great advantage, if proper facilities can be arranged for weighing.

The measurement of drill output is extremely simple. The holes for any one day's work can be marked as they are finished, and, at the end of the day, all measured; or they can be measured as finished, and their depth taken, and hence the entire day's work is easily determined. This, of course, is a linear measurement; and in the same class would fall such work as laying track, ballasting, grading with a road machine, and the measurement of the work of track and wheel scrapers.

The measurement of quantities whose units are areas is only a little more difficult. Paving, for instance, is very easily measured, the distance from curb to curb generally being constant, and so really reducing the measuring to a linear measurement—that is, the length of the section of pavement laid. Brick laying, while really a cubical measurement, is taken in the same way, the area of the face of the wall laid being taken, and multiplied by the standard number of bricks to any given thickness of wall. This really reduces the measurement for brick laying to a unit-basis, the unit being one brick. Painting and plastering are measured in the same way; and so also is roofing. On road work, plowing and sprinkling are estimated per unit-area; and in quarry work, channeling is so estimated.

The determination of volume on construction work is liable to be very difficult. Take, for instance, the output of a steam shovel cutting through rock. The walls of the cut will be very irregular both in line and in slope, no matter how skilfully the shovel is operated; and the face of the cut is liable to be even more irregular. No absolutely exact measurement can be made; and for this reason it is common practice to estimate the contents of the cars rather than attempt to estimate the size of the pit excavation during any one day. Generally the size of the pit is roughly measured, and the yardage figured from this measurement. It is also figured from the number of cars loaded, and, if carefully done and the estimate of the volume of the cars loaded is correct, both figures should balance at the end of the month with the monthly estimate, which, on account of the large volume measured, can practically ignore such irregularity as would affect the other two measurements. In earth excavation, the measurement is much simpler, because the pit is more regular and the cars can be fully loaded.

There are natural working units that lend great simplicity to calculations of cost—such, for example, as a floor panel in a building, a column, a bridge panel, a pier of masonry, etc.

Another unit of measurement is often obtained through the percentage of a total or of another unit, such as the amount of sand in a yard of concrete. Knowing the mix, a percentage of the total yardage of concrete will be the amount of sand that has been moved.

Care should be taken properly to subdivide the units of measurement. The ordinary unit of concrete work is the cubic yard or the cubic foot. The mistake is frequently made, of estimating the cost of forms and of reinforcement only in terms of the cubic yards of concrete. The cost of forms should be estimated also by the number of feet, board measure. Reinforcing steel should be estimated by the pound.

One difficult kind of work to obtain costs on by the regular method, is the laying of cut stone. A very simple way to obtain this is to paint on each stone a number, and let the time-keeper get the dimensions of the stone after it has been cut, before it has been placed in the wall. Then the stone layer simply records the number of each stone as it is laid.

A check on the measurement of the quantity of the work done is frequently obtained by the measurement of the quantity of the work left undone or of the material remaining in the stock piles.

COST SHOWING

The object of cost keeping is to furnish accurate and early information to those in authority, both as to where they stand financially on the work, and what necessities or opportunities there are for improvement in economy.

In order to accomplish the object of cost keeping, it is necessary that there be some efficient method of cost showing; and it is essential that the system of cost showing, in combination with the system of cost keeping, shall meet the following specifications:

1. It shall be accurate.
2. It shall be simple.
3. It shall be easy to study.
4. It shall be easy to compile.
5. It shall be capable of being compiled in a very short time after the receipt of the original figures.

It needs no argument to prove that the cost-showing system should be accurate. . If it be full of errors, its usefulness is entirely obviated; and 1 per cent of error in it will do a great deal more than 1 per cent of damage to its efficiency, in assisting the manager to increase the efficiency of the work. There is, however, a limit to the desirable precision of such an affair. The cost of putting in ties on a certain railroad for a certain month, for instance, may have been 7.2143 cents. If the last two figures are interesting from the statistician's point of view, they are utterly useless to a practical manager. If the previous month's performance has been, we shall say, 6.94 cents per tie, this month's figures will have shown an increase in cost of 0.27 cent, which is approximately 3.9 per cent of the previous month's figure. In other words, the tie-placing efficiency has decreased 3.9 per cent. It is very questionable whether the figure 4 per cent, although not quite so precise, would not be rather more useful to the manager than the figure 3.9 per cent; and, personally, the authors would favor the briefer work. The degree of refinement to which these records should be carried, is, in the last analysis, a matter for the individual judgment of the manager himself. The student should bear in mind the folly of unnecessarily elaborate figures.

The second specification, that the cost-showing system shall be simple, is almost as important as the first. If it be not simple, the chances for inaccuracy will be tremendously multiplied. It will take more work to carry it on; and the straightening-out of errors and discrepancies will be so difficult, and will require so much of the time of persons in authority, as to leave them no opportunity to do their other work. Plainly, it should not be necessary for a manager to do a lot of detailed work on cost-keeping or cost-showing systems himself.

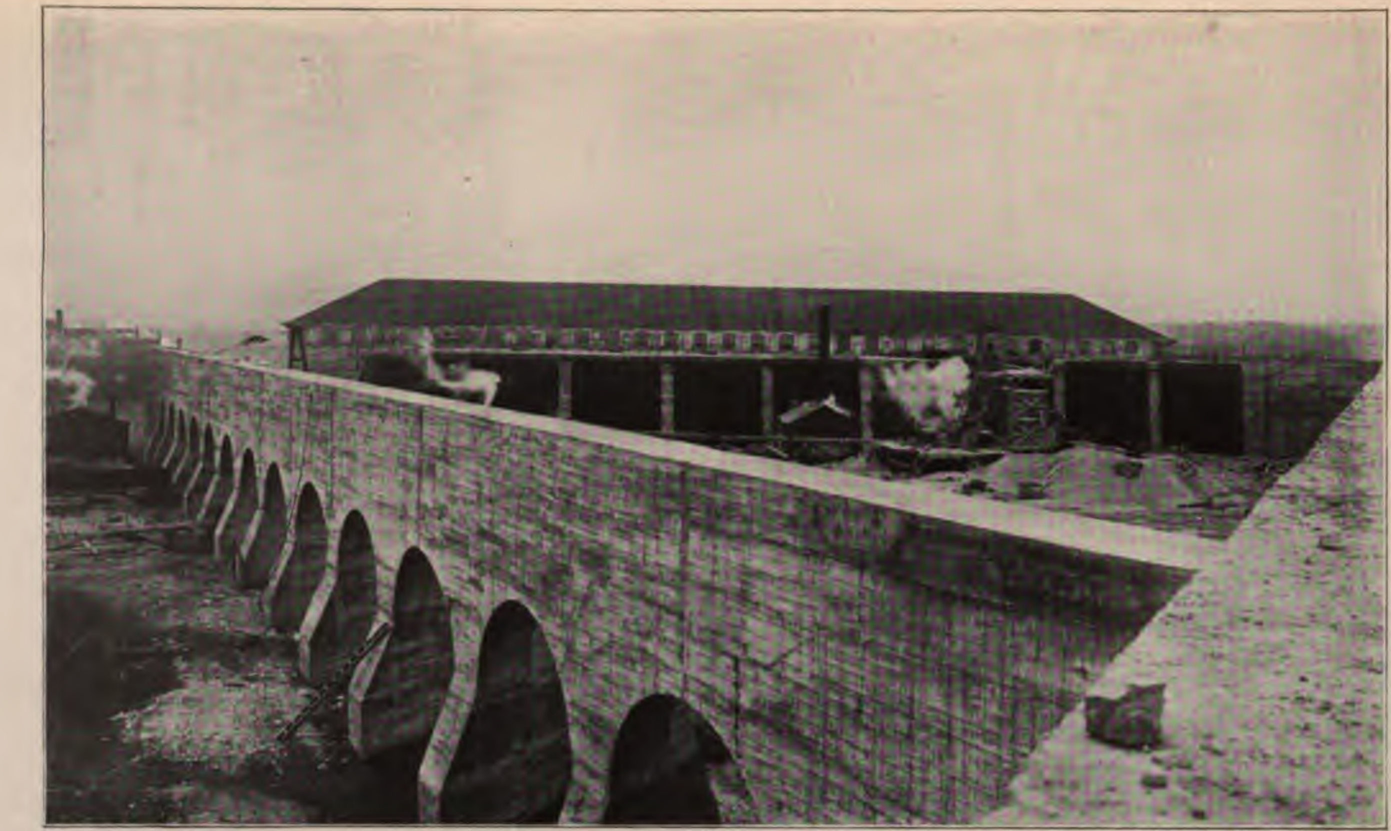
Specifications Nos. 3 and 4 are more or less included in specification No. 2. Specification No. 5, however, is also of great importance. Information that is stale is about as useless as no information at all. If you tell a foreman on Monday that the work of his gang for the week ending ten days before was not up to the mark, he will not have much respect for your cost-keeping system; he will certainly not remember sufficiently well the causes that produced his bad work, to remedy them; or he will be able to pick out of the haze of history enough excuses to let himself out of the responsibility of his bad work, and to put his manager at sea as to where this foreman and his gang really stand. It is therefore of prime importance that the arrangement for showing the manager what his costs are, with the salient conditions affecting such costs, shall be so rapid as to be "red-hot" all the time.

The commonest arrangement of cost showing—and the only one ordinarily found at the present day on most contract work—is an abstract prepared on a piece of yellow paper by the time-keeper for the inspection of the manager each morning; and this has so few disadvantages that it would be very satisfactory, were it not that it is impossible from it to compare at a glance the work done, let us say yesterday, with that done previously. It is, however, far better than any other system which lacks any of the essentials indicated above.

USE OF CHARTS

The best method that has so far been devised is by the use of charts showing to scale the different unit-costs for the various days in the month. Such charts are illustrated in Figs. 7 and 8. They are from the records of the Construction Service Company. One of these (Fig. 7) indicates the cost of channeling rock. It will be seen from the line *A*, that during this month the number of square feet channeled varied from 75 to 375, and that the labor cost varied from a maximum of 62 cents to a minimum of 8 cents. On the 8th, the morning crew did not work, because, as it happened, of severe weather, which accounted for the low output on that day. On the 24th, the night shift allowed the pipes to freeze up. It may be mentioned that the foreman of the night shift, whose name appears in brackets on the chart, has since turned his attention to other fields of industry than channeling. On the 28th, the channeler had reached a point where the cut was frequently filled up by earth that slid in from the side and caused such a large amount of sludge as to cushion the blows of the blade; and the chart shows on that day a high cost, due to the cleaning away of this earth.

The chart illustrated in Fig. 8 is that for steam-shovel work on the same contract of the Construction Service Company. Line *A* indicates the approximate number of yards moved per day. Line *B* shows the pay-roll, ranging from $165 to $310 per day, and including a percentage for incidentals; while line *C* represents the values of the *B* quantity divided by the *A* quantity, and gives the unit-cost in labor per yard for excavating and moving rock. It will be noted that the Sundays are skipped. These came on the 5th, 12th, 19th, and 26th of the month. There were some men employed on each of these Sundays; but their time was so distributed over the rest of the month as not to show for the Sundays, as the steam shovels did not work on that day.



POWER HOUSE ON CHICAGO DRAINAGE CANAL AT LOCKPORT, ILLINOIS

Power House in background; in front of it is the forebay, with screens protecting the conduits leading to the turbine chambers: in foreground is the arched concrete fender wall.

Courtesy of R. Isham Randolph.

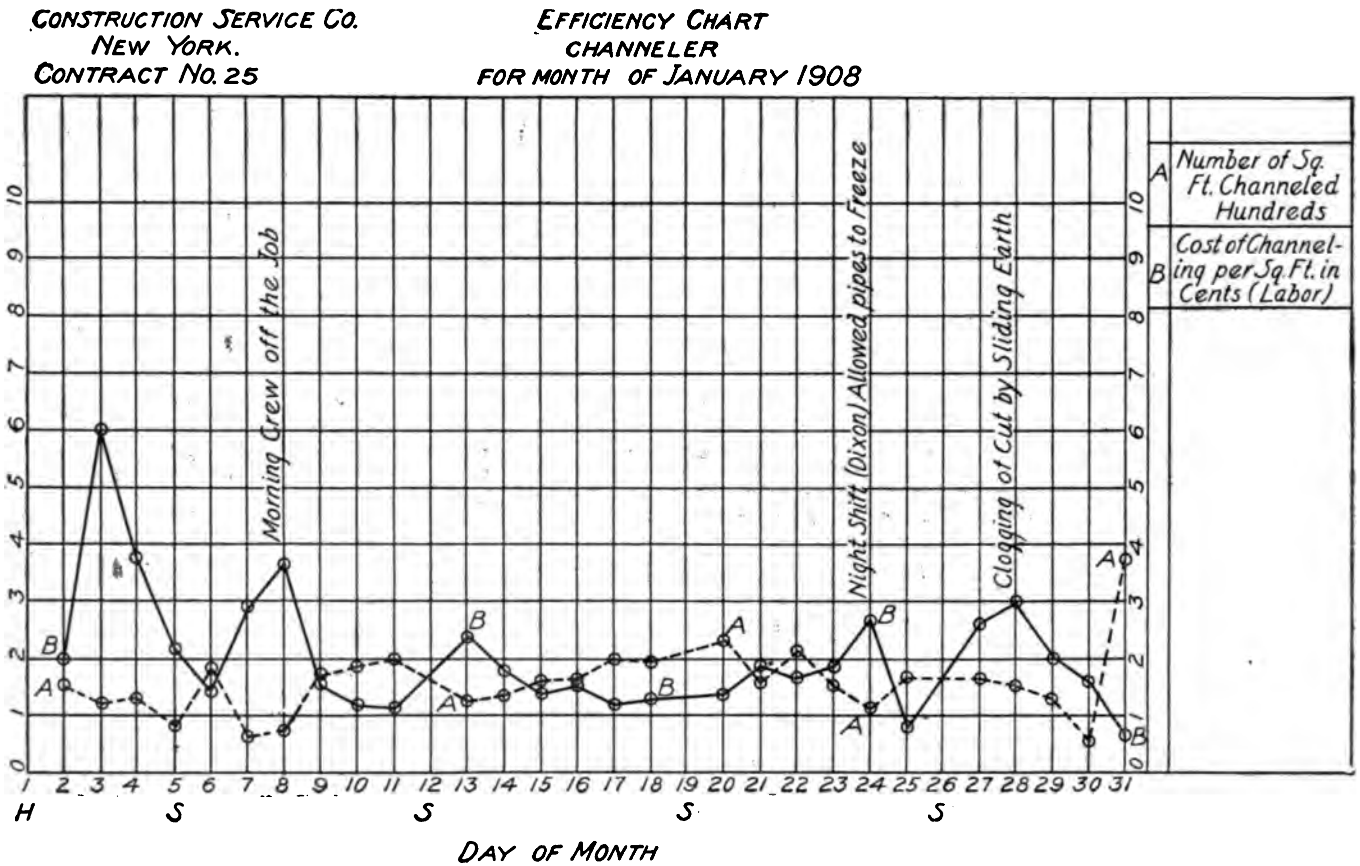


Fig. 7. Efficiency Chart Indicating Cost of Channeling Rock.

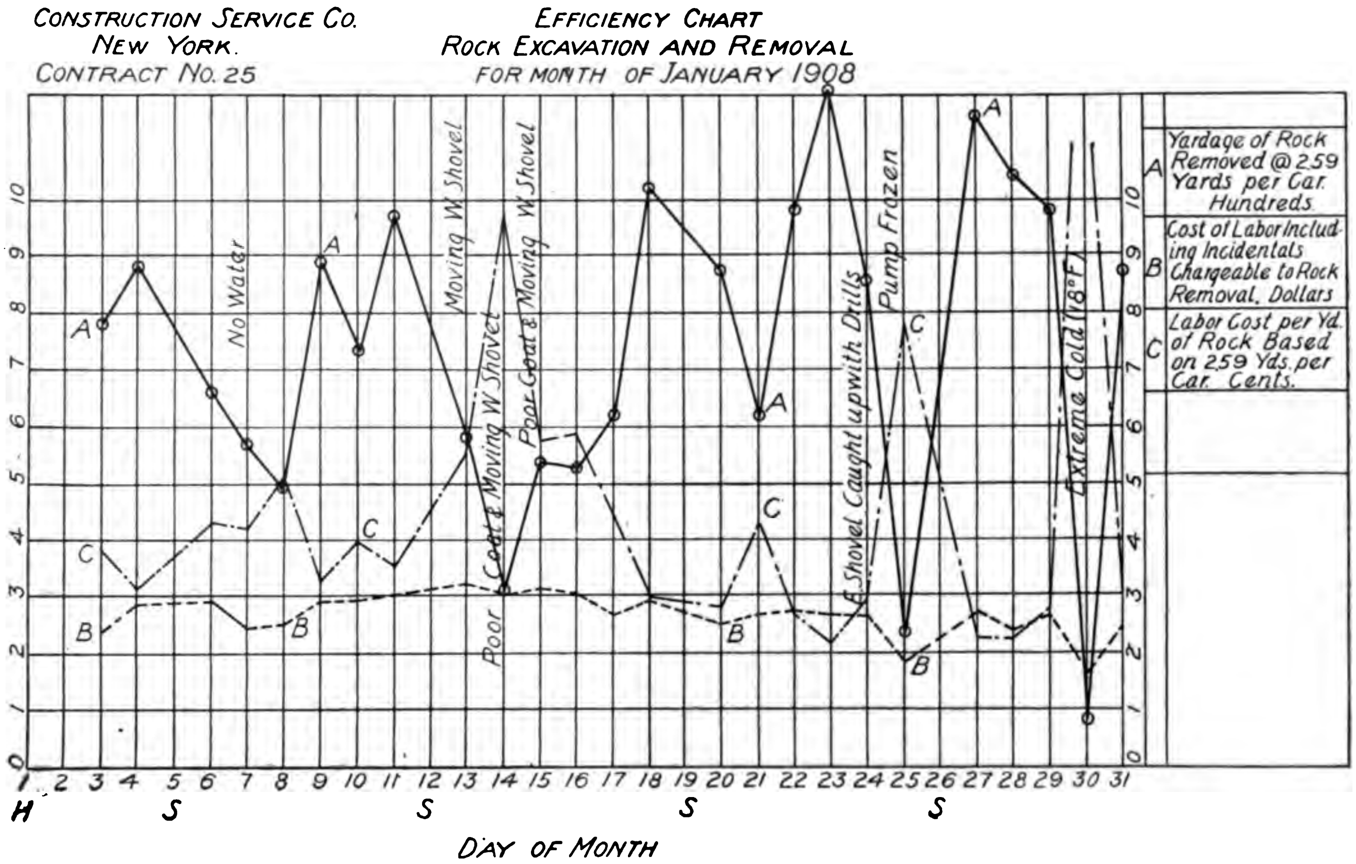


Fig. 8. Efficiency Chart Showing Costs in Connection with Steam-Shovel Work in Rock Excavation and Removal.

These charts are of a size to be filed in one of the standard loose-leaf books, and their range is from zero to about 12; thus it is possible to show any quantity to scale for any day in the month. This company has not found it of advantage to plot more than 4 lines on any one chart.

Charts such as these may be marked upon a tracing prepared for this purpose by the time-keeper each morning; and at the end of the month the lines connecting the points may be inked in, and the chart blue-printed, and the blue-print filed in a convenient place for immediate reference.

There are several ways of working out the unit-cost from the figures, such for example as:

1. Performance per time unit
2. Performance per dollar;
3. Cost per unit of performance.

The first of these is not, properly speaking, a cost statement, although it is a function of a cost statement and for certain purposes is more convenient. The number of feet of rock drilled per drill hour, is a very convenient form for record.

When drilling under conditions of snow and ice, more muckers have to be employed than at other times. If the cost of mucking is included in the cost of drilling, as it frequently is, the true index of how well the drills are getting on is the number of feet per drill hour, rather than the cost of the operation to the contractor.

The second method is the reciprocal of the third. Other systems will suggest themselves by virtue of the peculiar requirements of each case in practice.

Checking by Charts. A great advantage of the chart system of cost showing, is that it acts as an automatic check upon the cost-keeping system in general. As indicated earlier in this volume, it occasionally happens that a punch-card is not turned in, or the time-keeper fails to get certain data. This is immediately discoverable by the gap on the chart, and thus the chart acts as a check on the cost-getting department. This will not entirely obviate the necessity for inspection to ascertain whether the time-cards are properly kept and the work is properly done.

The showing of costs should be made *daily* for the men immediately identified with the field work; they should be made *weekly* for the general manager, and *monthly* for the home office. These monthly office reports are sometimes valuable in the planning of the financial arrangements for the work. On a job involving, say, a pay-roll of $5,000 a week, with monthly estimates, early information as to performance over the month is of very great value.

Showing the men certain charts and records will serve to increase their interest in their work, but this should not be overdone. It is as well that the men should not know the actual cost of their work to the contractor in dollars and cents. If, on the contract price, the contractor is making a handsome profit, the men want more money. If the contractor is not making a handsome profit, the men are apt to think that they are on a losing job, and become discouraged accordingly. The economy of the contractor's work should be private information, since it might do him considerable damage by becoming known to competing contractors. The charts showing the performance per unit of time, however, are not subject to the restrictions above mentioned.

COST KEEPING

In the foregoing there has been nothing that is a part of the regular bookkeeping, with the exception that part of the time-keeper's records are necessary to the bookkeeper. It should be appreciated at the start, that the bookkeeper's work is of great importance, that it cannot be superseded by a cost-keeping system, and that it should not be divided up with the cost-keeping system. The scoffers at cost analysis are inclined to take the ground that a bookkeeper, a cost-keeper, a cost-analysis engineer, are more or less clumsy substitutes for managerial intelligence; and they point to the proposition that in the last analysis it should be easy to let the office boy run the job with a textbook at one elbow and a calculating machine at the other.

It is insisted upon at the start, that cost keeping is as important as bookkeeping, but that it has an entirely different function; and in applying cost keeping to construction work, it is very important that a distinct line of demarcation be drawn between the two branches.

Another error that is frequently made by antiquarian students, men who are studying old methods of engineering and construction rather than those of to-day, is that a cost-keeping system is assumed to be complete by the man who runs it, when he knows how many feet of hole his drillers are able to average per hour, per day, or per week. The cost analyst will point to the fact that in the literature of the subject many false statements are made as to the costs of certain items of work, and will show that no allowance has been made for depreciation, repairs, etc., not to say profit, interest on the contractor's money, and a host of other things. The student is warned that a proper cost-keeping system must of necessity take into consideration *all* the items of cost on the job; and, further, it should take them into account with such detail that it will be a real, living help to a man in estimating future costs on similar work.

Now, as a general thing, the essential similarity of items has been lost sight of when these items are parts of work which is not generally dissimilar. For example, the item of earthwork in the construction of a large dam may be very similar in its essential cost to, and may be of the greatest use in assisting a contractor or engineer to figure the cost of, earthwork under similar climatic conditions on a railroad embankment; yet those who are most interested in the subject are inclined to classify dams as an entirely different sort of structure from railroads. The designing of a dam is a different matter from the design of a railroad; but to build one will often involve the same kind of tools, the same kind of machinery, the same kind of men, the same kind of "horse sense," and the same general principles of construction, as to build the other. Therefore, if his costs are properly subdivided and intelligently kept on one kind of construction, the contractor or engineer will be materially aided, not only in estimating the cost of the work upon the other, but in being in close touch with his work after he has started.

Every construction organization ought to have a schedule of standard items which may be called *ledger accounts*; and its books ought to be kept in such a manner that the records of the total and of the unit-amounts for these items on past work and on current work may be immediately available for the benefit of its officers. No two contractors will have the same arrangement for distributing cost; no two will have the same items for the accounts; but there are certain fundamental items that will come into use on almost every large piece of work, and some of them have a peculiar significance, and should be treated with special care.

Estimates on Ledger Items. In making estimates it is important to have this list in sight, in order that important items may not be omitted. Such a list, which will cover a large portion of the ordinary charges, is here given:

1. GANG LABOR:
    (a) Hourly rate;
    (b) Monthly rate.

The men who work by the month are apt to have to spend a good deal of non-productive time, on account of weather conditions, etc.; while the men who are on an hourly basis, as a general thing, do some profitable work whenever they are paid. It is feasible to figure a good deal more closely on the cost of work for those men who have practically no lost time to be taken care of, as *emergencies* or *incidentals*. It will be noted, also, that the cost and the time of hourly and daily men can be figured and charted day by day; whereas it is impossible to know exactly what the charges will be for labor that is paid by the month, until the end of the month. In order to make report charts showing cost as completely as possible, it is a frequent practice to add a certain percentage to the cost of the known items, to cover the so-called *lost foremen's time;* and to make at the end of the month a correction of a greater or less size, in order to make the cost-keeping end tally with the bookkeeping end of the work.

2. GENERAL LABOR, ETC. This item will comprise the labor of the men who have something to do with more than a few parts of the work. A watchman's time is not spent in drilling, or on a steam shovel while it is running. Nevertheless a proportionate part of his salary should be divided among the different branches of the work. Sometimes this will be a very small item, sometimes a large item. For example, if a steam shovel is excavating 30,000 yards of material per month, the watchman's unit-charge to excavation may be very small; but if the shovel is tied up for nearly the whole month, the charge per unit for watchman's time may be alarmingly high. This is one reason why unit-cost and total cost should always go together. A blacksmith's time, part of which is spent in sharpening drills, need not be all chargeable to drilling, because he may spend a good deal of time in repairing the steam shovel or fixing hand tools, etc.

3. OVERHEAD LABOR. Clerks, bookkeepers, messengers, office force, and General Manager are ordinarily included among the items of *overhead charge*, as well as salaries of general officers.

4. OVERHEAD MATERIALS. In this classification, there are

included stationery, office furniture, supplies, etc. When the office furniture is disposed of upon the completion of the work, its value should be credited upon this item.

5. OVERHEAD INCIDENTALS. These may include various items, such as telephone, office rent, telegraph messages, express charges on incidentals not directly connected with plant, etc.

6. PREPARATORY COSTS. These include the cost of getting ready to do the work, and, depending upon the nature of the job, may include any or all of the following items:

(*a*) Temporary roads;
(*b*) Temporary trestles;
(*c*) Clearing and grubbing;
(*d*) Snow removal and drainage;
(*e*) Traveling expenses to job;
(*f*) Preliminary estimates, calculations, and surveys;
(*g*) Freight and handling of materials to and from job;
(*h*) Freight on preliminary supplies;
(*i*) Handling of preliminary supplies;
(*j*) Licenses and premiums on bonds, etc.
(*k*) Legal expenses;
(*l*) Loss on initial operations;
(*m*) Right of way and cost of site;
(*n*) Sheds, storehouses, and other temporary buildings;
(*o*) Tools, less final value.

7. SUPPLIES. These are chargeable F. O. B. the job, or at the railroad station nearest to the work. They include all supplies for carrying on the work, as distinct from *material*, including explosives, coal, oil, waste, etc., and may include a *charge for water*.

8. INTEREST AND DEPRECIATION ON PLANT. This item is variously estimated by different people, and may vary greatly. It is impossible to establish an absolute rule; but on the average contractor's plant, it may be stated that $\frac{1}{10}$ of 1 per cent per working day is a very fair general average figure. The average steam shovel, for example, will work perhaps 200 days, under favorable weather conditions; and on this basis the interest and depreciation charge will be 20 per cent per year, and is not far from a fair figure. Some contractors allow 33 per cent per year on such material as road machinery, including crushers, steam rollers, etc. This is a little high, provided that a reasonable charge is made for repairs.

9. REPAIRS TO PLANT. How much money it takes to keep the equipment in proper condition for performing efficient work, is a

question on which the limits of space prevent a detailed discussion. On such a machine as a standard-gauge steam locomotive in constant operation to the limit of its capacity, repairs may run as high as 20 per cent per year; and on a rock drill the repairs may be 50 per cent or more per year.

10. Rent, Stumpage, etc. The item of *rent* includes the rental of ground and the storage buildings, if any, outside of the office expenses. *Stumpage* is the cost of standing timber, the purchaser being privileged to leave the stump after cutting down the tree.

11. Materials of Construction. These are chargeable F. O. B. the job, or at the railroad station nearest to the work.

12. Handling of Supplies.

13. Freight, when not included in item No. 11 or No. 7.

14. Unloading, Hauling and Storing Materials and Supplies.

15. Re-handling Materials and Supplies.

16. Interest on Cash Capital Exclusive of Plant.

17. Taxes and Insurance on Property (including boilers).

18. Accidental Insurance, to protect workmen and the public.

19. Advertising, Medical Expense, and Charity.

20. Discounts on Bonds, Warrants, or Notes.

21. Contingency Labor.

22. Contingency Materials.

23. Contingency Supplies.

24. Cost of Finding and Recovering Lost Freight and Supplies.

25. Profit.

COST REDUCTION

The ultimate aim of cost analysis is economic efficiency; and any system or method of cost analysis which does not result in the lessening of the total cost per unit of work performed, must necessarily be a failure.

After the costs on work have been partially analyzed, it becomes the province of the engineer to introduce methods and devices whereby the expense of obtaining the various data may be more than offset in the general economy of the work. It was long ago realized that shop practice could be economized by methods of systematization; and we have an early instance of the appreciation of this fact in the story of the struggles and methods resorted to by James Watt in the construction of the early steam engines. The troubles arising from incompetent workmen, drunkenness, and the necessity of doing work in different parts of the country far removed from headquarters, were as real then as they are now, with this disadvantage, that in the eighteenth century the press, the telephone, and the professional schools had not reached a development admitting of intelligent co-operation in the attack upon this problem.

Within the last score of years it has been found that cost-analysis problems applied to shop work give most amazing results. When the piece-work system was introduced, it was believed that the final solution of the problem had been attained. The men were then placed upon the footing of contractors. A man got so much pay for accomplishing so much work; and it was most clearly to his interest to accomplish the maximum of work in order to get the maximum of pay. It was immediately evident that the good men would soon show such a contrast to the poor men on the work as to inspire a constant rivalry, thereby resulting in a very much higher output. In order that the desire to accomplish more work should not interfere with the quality of the work, all materials were systematically and rigidly inspected.

For a good many years the piece-work system flourished, and it is still flourishing as compared with its predecessors. It is vigorously fought by trades unions and by the less able among the men. It is tolerated by those of mediocre ability, and it is heartily endorsed by the most skilful. In order to remove the resistance of labor unions, a modification of the piece-rate system, known as the *bonus system*, has been devised. It will be described in a subsequent paragraph.

Remarkable development has been achieved in the shop by the most brilliant work on the part of the men who have applied cost analysis, favored by the fact that in the shop one has conditions of work which are practically invariable from day to day. It is possible, then, to compare the work done per unit of time in the morning, with the work done per unit of time in the afternoon, or for each hour in the day, and thus to determine the effect of fatigue of the operators upon their efficiency and the effect of such specific influences as the character of artificial light, the grade of steel in tools, and even the economic value of providing reading rooms, white-enameled lavatories, and recreation for the operatives. To cost analysis has been largely due the development of the special high-speed steels and an amazing number of improvements in machinery, entirely aside from the stimulus and education of the workmen.

In field work, however, comparatively little has been accomplished in the world at large along these lines, not because the opportunity is lacking, but because certain of the difficulties appeal more glaringly to the pioneer in the field, and offer some peculiar discouragements. The conditions are not uniform from day to day. The locus of the work is changing, the weather is variable, and a very large number of external agencies will be continually interfering with the scheduled regularity of the work. The method or process whereby a piece of work can be done more economically, may be instituted at just the time when some apparently trivial variation of the weather, or breakdown in a water system, or interruption in train service, may produce an entirely opposite effect which will more than nullify the advantages obtained from the improved process or method, and will sometimes cover a period of a good many days and possibly weeks, making it appear that the improved method is not only a failure but a dragon in disguise.

Field work is constantly presenting obstacles and difficulties which have to be met and fought, calling for emergency judgment on the part of the men in charge and on the part of all the men on the work to a greater or less extent; and here alone is one of the chief reasons why a contractor comes to depend almost exclusively upon the personality of his superintendent or foreman, to the exclusion of systematic analysis.

It is abundantly demonstrable that when results are properly charted, and when a careful record is kept of the causes of interruption and the extent of the accidental obstacles, the problem becomes much simplified. It is astonishing how intimately a manager may come into touch in a short time with the obstacles to his work, and with the most efficient methods for their removal, by means of proper reports and cost analysis, and especially by the intelligent use of charts.

Effects of Weather. The principal obstruction to economical construction work in the temperate zone, is due to *rain*. If it is raining at about the time when the men come upon the work, they will rarely fail to retire to their homes or to some point so far from the scene of operations as to make it difficult if not impossible to get them back upon the job if the weather should clear.

If a rain comes on in the noon hour, they generally leave and do not return to work. If it should start to drizzle in the middle of the afternoon, and the men are under fairly good discipline, they stand a very good chance to stay the day out rather than miss getting a full day's pay.

Next to rain as an obstacle is *freezing weather.* On concrete work, if the concrete is laid at temperatures below 22 degrees, and particularly when a slow-setting brand of cement is used, special precautions have to be taken, and even then the work is liable to be rejected. For work in which steam engines are employed, as dinkeys, steam shovels, steam drills, etc., continued cold weather is likely to result in the freezing of supply pipes, the freezing of valves, and the breaking of pipe connections, necessitating a good deal of frost protection and a well-disciplined and well-handled gang of pipe-fitters under a responsible foreman charged with the express duty of keeping the water lines clear. In very cold weather—say below zero—the freezing of water will seriously interfere with the economy of the work on the water end alone. Where steam drills are used and the weather is exceedingly cold, the steam from the drills blown upon the men condenses and freezes upon their clothes, involving great inconvenience and suffering. Under such conditions the men will not work so many hours per day as otherwise, and frequently they will not work at all at critical times in the progress of the work, to the great detriment of economy.

It has been observed that a combination of stress of weather a few hours before the coming of the paymaster seems to be more discouraging than at any other time, and it is seized upon as an excuse to quit work.

In very windy weather, more coal is burned than at other times; and sometimes a boiler which is capable of running seven or eight drills in ordinary weather will not be able to furnish steam for more than 70 or 80 per cent of this amount in the high, cold winds.

For the above reasons it is essential to keep a record of the temperature and weather.

Accidental Conditions. Besides the weather, there are a host of accidental conditions that may arise to influence the economy of the work and complicate a precise study of the performance. Some of these, named at random, are:

1. The blowing-out of the gasket in a main water supply pump.
2. A shipment of poor coal.
3. The wrecking of a trestle.
4. A derailment of cars.
5. The breaking of machinery on a shovel.
6. The burning-out of a boiler, due to carelessness on the part of a watchman over night.
7. The non-arrival of necessary and important material.
8. Irregular blasting, due to irregular spacing of drill holes, or to bad loading, or to poor detonators or a poor exploding machine, or to irregularity in the character of the rock itself.
9. Erratic action on the part of one or more of the men, due to drunkenness, ill-temper, or general contrariness.
10. Errors on the part of foremen in co-operation, some of which are not detected in time to be eliminated; and a host of others.

In working out any special problem, care should be taken that such accidental causes affecting performance—whether they decrease the performance or, as may happen, increase it—should be carefully noted and be a part of the regular report on the work. These features of a report ordinarily are ignored as being unimportant, but they are of the utmost value to the success of the work.

STIMULATING THE MEN

The art of persuading a man who is turning out 500,000 foot-pounds of work in ten hours, to turn out 800,000 foot-pounds of work in ten hours with a trivial increase in pay, is on its face difficult; but is by no means impossible, and a list of some of the ordinary means of doing this should not be out of place here.

1. **Watching the Work.** If, on the average work under the observation of a foreman whom they know, the men are made to realize that their individual performance is being watched and recorded by someone who is above and beyond their own foreman, there will usually result an increase in performance of from 10 to 20 per cent per man; and particularly if a tab is kept upon the performance of the gang as a whole, the foreman will add his own stimulus to that applied by the men themselves, resulting in highly inceased efficiency.

On such work as teaming, where teams are hauling earth along a road for a considerable distance, a punch-card is very valuable.

The driver knows that the time of his trip is being recorded and compared with the time for the same work done by other drivers; and it has the effect of concentrating his mind upon his performance, which in itself causes him to use more care in cutting down delays and keeping his team up to their work.

In the operation of drilling, most valuable results have been achieved by giving each drill runner a card on which, at the completion of each hole, the time of the finishing of the hole is punched; and also the time of starting the new hole, in the same way. This card will then show the length of time that it took to drill the hole, and the length of time required to move his drill. He will be stimulated to move quickly, which in soft rock is an exceedingly important element of the drilling work, and he will be stimulated in the effort to get his holes down rapidly.

Where earth or rock is being loaded by steam shovels and hauled by dinkey trains, great economy can be arrived at by providing each dinkey runner with a punch-card or a report card on which he indicates the time when his train left the shovel and when it returned again to the shovel. This card then indicates the time for a round trip, and his mind is constantly being stimulated to look out for causes of delay; and, if he is at all conscientious, as most men are, he will instinctively attempt to make the best time. Some remarkable results have been achieved by this means alone in recent work.

When concrete is being mixed by hand, if a record is made of the time when each batch is finished, there will inevitably be an increase of activity of all the men in the mixing gang.

2. **Discharges.** The principle of natural selection of the men can be very advantageously applied. Where the supply of labor is adequate, it is advisable to make a rule of discharging a few of the poorest men every few days, taking on new men to fill their places. This necessarily results in an increase of the ability of the average men on the work, and it gives a healthy spur to the men who are not discharged. In carrying this method out, it should be done judiciously and with care to avoid discharging good men, lest the discipline of the work be interfered with. Any man who is not willing to do his best, or who is caught loafing deliberately, is an economic disadvantage to the work, and should be allowed to go. Likewise, any man who with good intentions is so dull as to hinder the progress

make it more than well worth while to secure the very best of the labor attainable.

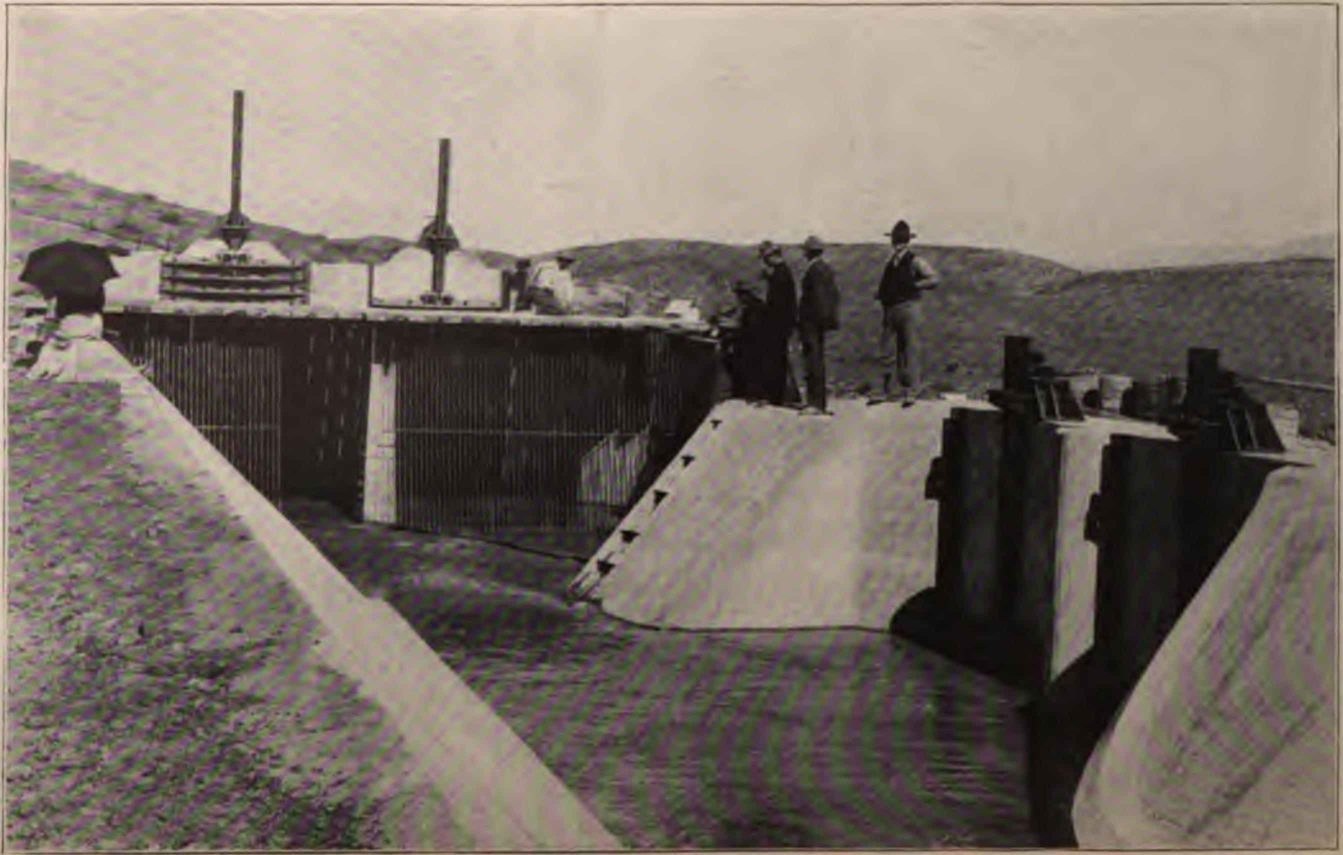
7. **Prompt Pay.** Men will work very much more contentedly when they can count upon their pay with promptness and regularity. There is nothing that demoralizes a piece of construction work more than the postponement of a pay-day. Special care should be taken that each man's pay is accurate. A man will seldom be overpaid unless there is "graft" on the job, but it sometimes happens that through errors on the part of the time-keeper or bookkeeper a man's pay is short, much to the agony of the man himself.

8. **Early Hours.** A good deal of money is lost by the men not starting to work promptly at the commencing hour, and by quitting before the final hour. On a recent piece of work that had to be drastically reorganized, an entire blasting gang rested on their shovels for over one hour, because their foreman had decided to quit without notice, and the man who was supposed to be in charge of the work arrived late himself, and was detained at the other end of the job. On this particular piece of work, it was not the habit to blow a whistle at the commencing or the quitting hour, and the men started work in the morning and quit work in the evening according to their own timepieces. It was noted that nobody on the whole job quit a minute after he should have quit, or started a moment earlier than he was paid to start.

In factory work it is feasible to have all the men go through a gate which is closed one minute after the hour and not opened again for perhaps 25 minutes, so that, if a man is two minutes late, he loses a half-hour's pay. This has the merit of not working injustice to anyone, and, after being instituted, seems to be accepted by the men with a reasonable degree of contentment. It is not easy to start a strike because some men lose their jobs from being late.

Such a system as this, however, is practically impossible on outside contract work; and while it may be feasible to institute a modification of the time and clock method, it is not known that this has yet been successfully done. Probably the most satisfactory way of insuring prompt arrival of the men, is to measure the output of each gang and make each foreman responsible for it, thus giving him a personal incentive to get his men on the job promptly.

9. **Enough Foremen.** It is necessary, in any organization,



SLUICE GATES AND ENTRANCE END OF COTTONWOOD PRESSURE PIPE

Salt River Project, Arizona.

to have the chain of responsibility lead through a sufficient number of foremen; otherwise a superintendent or supervisor will find himself "spreading out too thin," and will be attempting to perform a lot of work that should be done by a foreman. One superintendent can supervise the work of 20 or 30 foremen with a favorable layout, and each foreman can supervise the work of from 10 to 25 men. If, however, there be more gangs than there are foremen, the superintendent will find himself trying to play the part of foreman in instructing the men, and not able to do his own work, which is to instruct and supervise the foremen. In the matter of drilling, a number of able managers are not in favor of having a foreman over the drills. It is calculated that by substituting a boy to keep the records of the drilling, and putting bonuses on the drills, the difference between the pay of a foreman and the pay of a boy is saved, with no appreciable loss in performance.

There are strong grounds for the opinion that there should be no process, such as drilling, without a foreman, where the work is on a large scale. When 10 drills are working, they will employ altogether 20 men on the drills, a number of muckers clearing the ground, and a pipe man. The work of these men cannot fail to be improved by their being at all times under the watchful eye of a man to whom they are responsible for the quantity and quality of their work. Aside from this, if the foreman is an expert driller, the instruction that he can give to the less able of the drill runners will be worth ten times its cost.

The same argument applies to all processes in the field.

10. **Education on the Work.** As a general thing, men who take money for their labor are more than willing to deliver a square deal to their employer; and it will almost invariably be found that the more familiar a man is with the difficulties and possibilities of his fellow-workmen, the more efficient he will be himself. For this reason it has been found highly satisfactory, in some lines of work, to change the men around on the job. In a certain concrete building 12 stories high, the upper stories were built in a small fraction of the time required for the corresponding lower stories. The greater part of the extraordinary increase in efficiency was attributed to the fact that the men were so educated that a man at the top of the building knew how the men at the mixer and in other parts of the job were doing their work, and knew that the superintendent in charge was measuring the speed of

the deliveries from the hoist. The disadvantage of this method is that it takes a long time to work up the efficiency. It is, however, an admirable method for disciplining an organization.

Discipline. To the practical man, or to the intelligent student, there seems to be no necessity for arguing in favor of *discipline* as an essential to economical field work. So large a percentage of contract work in the field is badly disciplined, and the general principles seem to be unknown to so many field organizations, that a brief statement of them appears to be called for in this volume.

By *discipline* is meant the cultivation of a spirit of:

1. Co-operation;
2. Obedience;
3. Responsibility;
4. Personal loyalty.

The subject will be discussed on the basis of *reorganization of work*, because here the chief difficulties are met.

The three types of organization that are most clearly defined in the way of discipline, are those of a *military nature*, *railroad work*, and *factory work*.

On construction work, it is not feasible to introduce a military form of discipline. In the first place, the penalties of the military service are not permissible; and in the second place it is not usually practicable to have so thorough a system of distribution of responsibility; while in the third place the same men are not here together for a long enough period to make military discipline practicable.

In railroad work a man is usually employed for a long term of years, but rather small pay, with a large chance of promotion. From the day of his initiation on the work, he is impressed profoundly with the necessity of protecting lives and of keeping trains moving all the time; and, in a short time, he comes to the frame of mind in which his pay, his personal convenience, and his personal prejudice are subordinate interests. The initials of the superintendent on a little slip of paper are sufficient to make him do almost anything within the limit of endurance; and, as a general thing, he does it ungrudgingly (but not uncomplainingly), and with a cheerfulness that is in many respects astonishing.

Such a degree of discipline is entirely feasible on any contract work of long duration, and it should be obtained if economy is desired.

It is not possible to institute successfully radical reforms and methods, without first securing good discipline on the work; and when work is badly disorganized, the discipline should be the first point of attack.

It is necessary to have, first, a system of locating responsibility. If a dinkey becomes derailed, if the spacing of drill holes be erroneously made, if a steam shovel be out of line, if the wrong methods of loading be pursued, if a pump be out of order, if necessary material or supplies be wanting, if the pipes freeze up, if, in short, one hundred and one little things happen that cause confusion on the work, it should be possible to find someone who, by some crime of omission or commission, is responsible for the trouble, and who can be made, in some degree at least, to bear the brunt of it. The only man who seems destined to be entirely free from the consequence of his mistakes, is the clerk of the Weather Bureau.

The organization should be laid out from the bottom upward, rather than from the top downward. The laborer is responsible practically for carrying out the instructions of his foreman to the satisfaction of his foreman and of no one else, and for this reason he should not work under the impression that anyone except his own foreman is likely to discharge him, to criticise him, or to praise him. If his foreman be the right sort of man, the laborer, with his dozen associates, will have, besides the interests of the work at heart, a strong feeling of personal loyalty to the foreman; and this feeling will be reciprocated by the foreman. If a foreman be noticed vigorously complaining that the men he has to deal with are inefficient, incompetent, and a disgrace to civilization compared to the men he had to work with some years before, he may as a general thing be put down as a "blow-hard" and of little value to the organization. The most successful are usually the ones who are ordinarily quiet, cool under emergency, and yet of sufficient determination to inspire among the men a wholesome respect for them. A man who loses his temper on the work for any reason, does not, as a general thing, make a good foreman or superintendent.

The relations obtaining between the men and their foreman, should obtain to a more marked degree between the foreman and their superintendent. Briefly stated, every man on the job should have to look for orders from one man and only one man; and he should be responsible to that man for the satisfactory performance of those orders.

Conflicting orders can be avoided only by systematic compliance with the rule just above outlined.

It sometimes happens after periods of financial depression, or as a result of special conditions, that it is feasible to reduce the pay of a good many men on the work. This should always be done with great care and after an intimate knowledge has been obtained of the personalities of the men affected. As a general thing, if you cut down a man's pay 10 per cent, you will cut down the work 20 per cent, at least for a time; and it frequently happens that after such a pay reduction, small, petty depredations on the work are committed. Articles get stolen; machinery is damaged by "sore heads." It is usually unwise to reduce the pay of a few men. As a general policy, where a small percentage of the working force is to be affected, it is better to discharge a few men outright, and endeavor by economic methods to increase the output of the others.

Differential pay is a prolific source of trouble, and it is very common. By this is meant the payment to different men of different rates for the performance of the same work. The men who obtain the less pay think that their pay ought to be raised; and the man who gets the most pay can in no probability appreciate the fact that he may be over-paid. Rather than cut his pay down, it is well, if possible, to put him at some other class of work.

It is often necessary for economical reasons to place men who have been paid by the month, upon an hourly basis; and even when by this they average rather more than they formerly received, it usually causes discontentment. A man likes to know how he is coming out at the end of the month regardless of the weather, and it is an additional source of anxiety to him not to know what his pay envelope will contain. When he keeps his own record of the hours worked, he is likely to disagree with the time-keeper, and this can lead to a good deal of disaffection and dissatisfaction.

A frequent cause of disaffection on work is due to the habit too often indulged in by time-keepers, of gossiping with the men. The time-keeper and the storekeeper necessarily come into contact with a very large percentage of the men every day; and if the time-keeper particularly be disposed to gossip, he has abundant opportunity to gratify his desire, and can produce a great deal of trouble. For this reason the general character of the time-keeper should be carefully scrutinized before employing him; and he should be cautious, when making his rounds, to confine himself strictly to business. If the men on the job know as much about the work as the General Manager, if they know all the ins and outs and ups and downs of the contract, they necessarily discuss it among themselves, and a great deal of restlessness is produced, which is very difficult to stamp out, because, by the time it has reached a pronounced stage, the men have learned so much about the politics, as it were, of the job, as to interfere with the discipline.

If there be dissensions at headquarters, if the parties that control the work are at war, and if the methods and performance of the manager or superintendent be not absolutely satisfactory to every one of the officials, nothing can be worse than to let a suspicion of this matter get around among the men. How a general manager or superintendent can prevent this, if an officer be disposed to talk, is a problem that no attempt will here be made to solve. In reorganization, such a condition is one for which the manager should be continually on the alert, and he is advised to be suspicious of the time-keeper and storekeeper.

Labor-Saving Devices Involving Plant. If the men are in a reasonably good state of discipline, it is feasible to make changes in the layout involving special apparatus or plant; and in deciding upon such measures, a question arises as to how much money it is justifiable to spend for a new plant. A piece of work under reorganization is ordinarily a piece of work that is more or less in financial difficulties, and the purchase of plant for the economizing of the work is usually looked upon by the officers as a dangerous move. Particularly is this the case when any changes of this kind turn out to be unsuccessful. A small amount of money wasted on special apparatus is always in sight—at the scrap heap, if nowhere else—whereas a good deal of money wasted in fruitless labor can be easily lost.

If the amount of saving on a certain operation by the installation of special material be sufficient to pay for this material in a few weeks, the purchase of the material can be immediately justified, and the cost of the apparatus can be charged as *current expenses* to be shortly recovered in the economy of the work. Where expensive and heavy machinery is to be installed, however, the matter should be gone into with the greatest care and detail.

A few of the articles which come within the class chargeable to *current expenses*, are:

1. The use of water jets for increasing the speed of drilling in soft rock.
2. The use of hickory wands for stirring up sludge in drill holes, and increasing the speed of drilling.
3. The use of special explosives and good exploding machines, and of loading tubes for blasting.
4. Small grading machines for spreading earth and macadam.
5. Special wheelbarrows or carts for moving material.
6. Special small tools for the blacksmith, including a trough in which he can set his bits to be hardened, with the points in the water.
7. A sufficient supply of picks and shovels.

Some of the items of plant that may be classed in the other category, are:

1. Special wagons and scrapers for hauling earth.
2. Concrete mixers especially adapted to the work in hand.
3. Derricks.
4. Locomotive cranes.
5. Cableways.
6. Bit-sharpening machines.

Labor-Saving Devices Involving No Plant. Where the labor preparatory to introducing the improved methods is considered, it should be taken as equivalent to a plant charge as affecting the interest of the contractor. If, for example, it has been the practice to drill and blast immediately in front of a steam shovel on rock excavation, and it is desired to have the drilling and blasting so far ahead of the shovel as to avoid the occasional necessity of holding up the shovel, the money involved in the work done ahead should be considered in the nature of a temporary investment and charged to money expended on plant which will not come back for a period of perhaps one month.

A steam-shovel crew has a good deal more pride in its work, and will continue working under more disagreeable weather conditions, than a drilling gang; and when drilling in front of a shovel, severe weather conditions may cause the drilling work to stop without interrupting the operation of the shovel. If, then, the drills are working too close to the shovel, the shovel may catch them.

On the other hand, it is unwise to blast far ahead of the shovel, for a number of reasons. In the first place, there is no advantage in investing money in drill holes, except to avoid such a contingency as outlined above. In the second place, it is impossible to tell how effective the blasting has been until the shovel has attacked the broken rock; and if the blasting is done far ahead of the shovel, poor blasting may go undetected until an immense amount of financial damage has been done.

To cite a specific instance—On a recent piece of important work, a cut several hundred feet long was drilled to a depth of supposedly fourteen feet, and blasted with more or less unsatisfactory results. The steam shovel was then put in, and excavated to a depth of five or six feet. The subsequent cut of from eight to nine feet deep had to be entirely re-drilled and re-blasted. The drilling in the already partly broken rock was immensely difficult, the drills sticking a great deal and a good many of the holes having to be abandoned; while the blasting was unsatisfactory because of the fissures.

The rapid reorganization of work can be furthered by the issuance of *special instructions to foremen in the field.* This practice has been admirably followed by Frank B. Gilbreth, and is described in his "Field System." As illustrations of such orders, are the following to drill and blasting foremen, issued on some recent work:

INSTRUCTIONS TO FOREMEN

RULES FOR DRILLING—

Drill foremen are requested to do their utmost to enforce the following rules for drilling:

1. Each drill at the beginning of a hole is to be supplied with a complete set of sharp bits and a pump, which will be laid alongside of the drill tripod by the drill tender, under the directions of the foreman.

2. As soon as a hole is finished, one of the muckers at the direction of the foreman will assist the two drill runners to move the tripod; and the mucker under the direction of the foreman will then pump out the hole that has just been completed.

3. The foreman will then personally, with a wooden rod, measure the depth of the hole, and punch said depth on the drillers' card, the mucker placing a well-made round plug in the hole and hammering it home.

4. Foremen will see that the drills are so distributed as to keep them as near as possible to the manifold, from which the steam supply is taken.

5. Pipe connections are to be made by a pipe-fitter who will be assigned to each drill gang, and who may be assisted by the foreman and by a mucker when necessary. No pipe-fitting is to be done by drill runners or helpers unless absolutely necessary.

6. The time for drilling is from 8 A. M. until noon, and from 12.30 until 4.30 P. M.; and the moving of the dinkey supplying the drills with steam is never to be done within those hours, unless absolutely necessary, and

when it is necessary, a note to that effect must be made on the quarry card.

7. Whenever for any reason the drills are out of steam, the drill foreman will indicate the time when the steam pressure failed and the time when pressure was again turned on, with a reason why the pressure gave out. This to be written on the quarry card.

8. Foremen will see that each drill is in proper working order, and supplied with an exhaust pipe of the proper length and with a throttle. Whenever for any reason a drill is not in perfect condition, the foreman will immediately report it and make requisition through the storekeeper for the necessary parts and repairs.

The attention of all concerned is particularly called to these rules, the enforcement of which is essential to the economic performance of the work; and all concerned are particularly urged to make a most earnest effort to see that, as far as it is possible, every drill on the work shall be in actual operation for 8 hours on every working day.

By order of,

RICHARD T. DANA, General Manager.

INSTRUCTIONS TO FOREMEN

RULES FOR THE GUIDANCE OF BLASTING FOREMEN—

Blasting foremen are requested to do their utmost to enforce the following rules for the conduct of blasting operations:

1. Attention is called to the fact that dynamite will freeze in about 45° temperature, and that at this temperature, a little above or a little below, dynamite is exceedingly sensitive to shocks, and should be handled accordingly.

2. Whenever a part of the charge in a hole has been exploded, leaving any unexploded dynamite in the hole, do not under any circumstances blow out the unexploded dynamite with a steam jet. You may, however, put a stick of dynamite down into the hole on top of the unexploded powder and endeavor to fire the entire hole in this manner.

3. The blasting foreman will provide himself with a measured wooden rod for tamping, and personally see that every hole that is loaded is down to grade. If he should find any hole that is not down to grade, he will immediately report the fact to the superintendent or assistant superintendent on the work, and make a note of the same on his time-card.

4. The attention of the foreman is called to the fact that dynamite is not as efficient in a hole full of water as it is in a dry hole, and every effort should be made to load the holes as dry as possible.

5. The tamping of the holes should be of the heaviest and stickiest clay that can be obtained, and this tamping should extend the entire length of the hole above the powder.

6. Never thaw dynamite in front of the fire, or on a hot stone removed from a fire, or by piling sticks in a boiler or in an oven.

By order of,

RICHARD T. DANA, General Manager.

In hauling earth, the principal elements of team expense are the time to haul and the standing still to load. This last item can be very materially reduced by the simple expedient of having on the work an extra wagon or two. A team can be changed from one wagon to another in about one and a-half minutes, and the same number of teams on a short haul will do easily 15 per cent more work by this trick.

The same principle applies to the mixing of concrete involving extra wheelbarrows; and here it may be mentioned that the arrangement of the concrete platform is seldom economical. The men, if left to themselves, will usually not have sufficient runways, so that a man with a loaded wheelbarrow will be painfully struggling up a plank, while a man with an empty wheelbarrow is waiting for him to get out of the way. Much can be accomplished by having the men move in procession so that no man with a wheelbarrow will ever have to stand and wait for another man to get out of his way. Of course the ideal method of handling concrete into a mixer is to do it from bins with chutes; but the great majority of this class of work is not done in this manner.

On contract work, the emergency charges for the moving of plant are usually considerably higher than they ought to be, owing to the fact that the work is done by men who are not especially skilful in this kind of work. The direction of these processes should be given to a man who is especially good at it; and the work should be provided with a good supply of gin poles, snatch blocks, tackle, etc.

If a piece of work has been under personal observation for considerable time, a great many sources of improvement in the performance can be detected that are entirely inadvisable upon casual inspection; and the student of economics is urged to devote a large amount of time to the most careful and complete study of minor and apparently trivial operations. Too much respect is usually given to established methods, just because they are established methods; and the analysis of a process that is apparently simple and of minor importance, but which is repeated scores of times in a day, is nearly always given too little importance as compared with the process that is elaborate and complicated, and which may in itself be of great importance, but which, on the particular work at hand, is dependent upon apparently minor processes. To illustrate—A shovel, loading eight or nine thousand yards of rock per month, was inspected; and the first impression obtained was that the reason the shovel output was so small was because of the inefficient layout of the shovel work itself. It was found, however, that the shovel was actually able to work a good deal faster than the drills and the blasting could provide broken rock for it; and the ultimate solution of the problem was found in the reorganizing of the drilling, in order to do more work with the same number of drills, and in the use of improved methods in blasting. The handling of the shovel took care of itself as soon as the other problems were solved.

The cost of spreading broken macadam on a road, to the average contractor, is not far from 12 cents per cubic yard; and the work is done with shovels and forks. This method is one that has been pursued for a great many years; and there are very few contractors who realize that it is exceedingly expensive. Some contractors, however, are doing work of this kind with the aid of a road machine that requires for its operation two or three men and four horses. A small grader machine that can be operated by one man and two horses for rough spreading, assisted by one man on the ground with a potato hook, has been known to do this work for about 2 cents or less per cubic yard.

In bridge-erecting work, a great deal of money can be saved over ordinary methods by the designing of special tools, such as dolly bars; and a good system of keeping detailed cost on such work will be sure to result advantageously. Much labor is lost in the erecting of roof trusses and in the erection of trusses in general, by crude and old-fashioned methods. The pneumatic riveter which strikes a great many light blows per minute has revolutionized field riveting; but the use of such a machine for cutting rivets has been unsuccessful in competition with hand labor on at least one large piece of work in New York City.

In painting, considerable time is ordinarily lost by the painters in preparing their own staging. Whenever possible, these preparations should be done for them under the direction of a skilled man; and the use of small wenches on the staging whereby the painters can quickly raise and lower themselves, has been found of great value.

On contract work, the blacksmith is in a position peculiar to himself. He is classed as an expert, paid by the month, and is supposed somehow to get all the work done that comes to him. He has general charge of his department, and he gets very few orders and practically no instruction from the superintendent or manager. He is nearly always an interesting personality, and, outside of a very limited field, extraordinarily ignorant. The excuse on a great deal of uneconomical work is that it is impossible to get a competent blacksmith who knows how to do the work that he is called upon to perform. Tools will not hold their edge, or they break. Upon the matter being referred to the blacksmith, he will usually come back with a complaint about his coal, or the grade of steel with which he is supplied, or his tempering solution, or the condition of his forge. He should be provided with a thoroughly good set of tools, and the superintendent should know that his tools are of the best. He should next be carefully and thoroughly instructed as to how to harden and temper steel. A convenient shop for the blacksmith, and proper methods of forging and tempering, will add incalculable value to the organization.

Introduction of New Methods. It should be adopted as a cardinal principle, that there are no methods in the field which are not capable of improvement along the line of economy; and it should be remembered that a very small improvement in any one method is invariably worth a great deal of thought and time in arriving at it. The systematic perusal of the proceedings of the engineering societies and the engineering press, will result in the suggestion of new and improved methods and of a good many bad and unimproved methods; and the trained expert should be able to sift the wheat from the chaff, and apply only such as will fit his special needs.

The literature of shop development and shop economics is rich in illustrations and suggestions that can be adapted to field work, and should be gone over very carefully for this purpose. In this connection it should be urged that it is a duty of a professional man to publish new methods. There is no room for argument on the proposition that the principle of free trade showing the other fellow two blades of grass growing where one grew before is an advantage to all concerned.

Design of New Methods. When there is crying need for improved methods in the field on account of special necessity, it behooves the man in charge to invent improved methods and design improved apparatus. The cardinal elements of such design include the following:

1. Simplicity.
2. Low first cost, so that if the experiment is not successful, nothing will be lost.
3. The use of standard sizes of material.
4. Generality of application.

Whenever possible, a new method or a new machine should be so constructed as to apply to as large a proportion of the whole work as possible, and every effort should be made toward the standardization of materials and apparatus.

In attempting work in blasting, it should be remembered that the use of new and untried explosives is attended with peculiar dangers. The men are familiar with the use of the standard grades of powder; and while they are ignorant of how dangerous it is to take liberties with dynamite, they are at a great disadvantage when a new explosive is given to them for trial. If it looks like dynamite and is exploded with the ordinary detonating cap, its peculiarities do not receive much attention.

Men in the field are instinctively opposed to new ideas, and it will invariably be found that new methods meet with stubborn opposition. A foreman to whom a new method is suggested will not expect it to be successful, particularly if he has ever heard it condemned; and it always seems as if the thought were father to the wish, for, when ordered to try it in the field, if he can make it fail, he will do so with unerring accuracy. As a general thing, however, when it is successfully demonstrated, he will become a loyal supporter of it. In presenting a new method to a foreman or superintendent, it is well not to encourage the raising of objections. It is better to let the objections raise themselves in the application of the process; and a man who has not gone on record as saying that in his opinion a new scheme is no good, is a much more loyal supporter of the new scheme than when he has committed himself against it.

One of the difficulties in improving the efficiency of work, is the extraordinarily ingenious line of excuses that the men will present for not getting their work done properly. Of these, perhaps the most hard-worked is that of improper and insufficient material. When a man is berated for poor work, and presents the argument that he was unable to do so and so because he ordered material for it several weeks previously and the material has not yet arrived, the situation is embarrassing. The best preventive of this is to have small requisition blanks measuring about 2½ by 4½ inches, made up into pads of about fifty each, and to give each foreman a pad. Each blank should have a space for the date, the articles ordered, the time when the article is needed, the particular part of the work where the article is needed, the class of work for which the article is needed, and the foreman's name. The foreman should then be instructed that material will be purchased through the storekeeper, and that non-delivery of material will not be accepted as an excuse. The storekeeper should then go around a job at least once a day, and get from the foremen their requisition slips; and an intelligent storekeeper will see to it that useless and unnecessary material or superfluous material is not ordered. Material that is ordered on requisition, and is not in the storehouse, should be purchased if necessary on a rush order, because, contrary to the ordinary apparent belief, it is economical to spend a dollar for material in order to save two dollars in labor.

The Field Layout. In laying out the plan of campaign on starting a new piece of work, it is important to consider the proposition from the capitalization end, as well as from that of pure construction. It is usually not appreciated by the engineer or the owner, that the contractor is doing a piece of delicate financiering, for the performance of which his own available money is usually inadequate, and that he is therefore obliged to borrow money on the work as it goes along, and to depend upon his monthly estimates. It is sometimes specified in the contract, that the contractor shall own all of his plant in fee, but it may be said that this arrangement is seldom lived up to. He can in addition nearly always borrow the amount of his pay-roll a month in advance, from his bank. He can also sometimes borrow money, giving as security his interest in the money retained on the contract, which is ordinarily something like 10 per cent. Therefore, provided that all goes well, if he gets his estimates when they are due, if his pay-roll is not more than the amount of his monthly estimate, and if no very large and disastrous contingencies interfere with the progress of the work, the contractor can swing a large piece of work with a comparatively small capital. If, however, things do not go well; if, through the failure of the owner's engineer, or through the insolvency of the owner, or through liens and attachments upon the work brought by dissatisfied creditors, the contractor does not receive his monthly estimates on time; if, in order successfully to prosecute the work, it is necessary for him to buy a large amount of additional machinery at a time when payments on old machinery are due; or if the portion of the work that he is doing is bringing him in less than the amount of his pay-roll and immediate materials and supplies, unless he has a large capital back of him, which capital is at once available, he is liable to be placed in an exceedingly embarrassing position. At such a time, if there should come a period of financial stringency, bankruptcy may stare him in the face, even though he has at the same time a contract on which he can be reasonably sure of making a large profit.

It is therefore of great importance that the work be prosecuted in such a manner as to have a continuous running profit, if possible. A contractor may turn in what is known as an *unbalanced bid*. In that event it will be very easy for him to start a certain portion of the work upon which he will lose money before he reaches the portion on which he expects to make money. Unless, as above indicated, the contractor is provided with a large fund for contingencies, great care should be taken to avoid this. The nature of unbalanced bids will be explained below.

As a case in point, on a certain contract involving over a million dollars, the company that was organized to conduct the work was provided with a small working capital, bought its plant on a time basis, and proceeded with a small working capital, uuder the impression that it would not be necessary to borrow any money, that the work immediately commenced would be sufficient to pay all the expenses and leave a profit, which profit would gradually accumulate and enable a running fund to be maintained which would take care of future contingencies. The idea was admirable. It happened that the work was in earth and rock excavation also known as *unclassified*, and was taken at a price which would admit of a large profit in any event. The rock work, if taken economically, would cost more than the contract price; the earth work, if taken economically, would cost considerably less than the contract price. The original plan contemplated starting the earth excavation at a point to which another contractor was to excavate, and it was not deemed feasible to commence the earth excavation until the other contractor had cut up to the line between the two contracts. Dependence was placed upon the other contractor doing his work on time, which he did not do; and it was then decided that it would be impracticable to commence in the earth, and work was accordingly commenced in rock, which work was conducted at a considerable loss. The strong financial position of the contracting company was the only thing that prevented it from going to the wall with a most excellent contract partly completed and a lot of good money tied up.

We shall assume, for purposes of illustration, that a certain contractor desires to bid on some public work involving the removal of 100,000 cubic yards of earth work and 50,000 cubic yards of rock work. He estimates that he can do the earth work for 30 cents per cubic yard, or $30,000, and rock work for 80 cents per cubic yard, or $40,000, making a total of $70,000 for the entire 150,000 yards, or 46.66 cents per yard for an average of the earth and rock; and he puts in his bid at this figure.

If the contract has been obtained as one of the Erie Barge Canal contracts, the work will be let *unclassified*, as it is called. By this is meant that no discrimination in monthly estimates will be made between rock and earth removed; that the earth and rock removed will be measured in excavation, and the contractor will be paid for these two materials indiscriminately. Now, we shall assume that he can make a profit of 4 cents per yard on the earth, and 10 cents per yard on the rock, so that his total profit on the contract will be $9,000. According to the terms of his contract, he will be paid on the monthly estimates 46.66 cents per yard removed, less 10 per cent—or 42 cents, the 10 per cent being retained until the completion of the contract.

Suppose, now, that he starts in on the rock, and he excavates the 50,000 yards at a cost to him of $35,000.00 for which he will receive 42 cents per yard, or $21,000.00. He will then be out of pocket $14,000.00; but there will be coming to him as held by the State $2,333.33.

Before he can begin to "see daylight" on his contract, he must proceed to excavate earth until he has made up the $14,000.00. He gets 42 cents in cash, and it costs him 26 cents, so that he must excavate 87,500 yards of earth, for which he will get the $14,000.00, and he will have held up $4,083.33 additional. There will then be remaining 12,500 yards to be excavated on which he will get $5,250.00, with $583.33 held back. He will have been obliged to do 91⅔ per cent of his contract before he stops putting money into it; and the money that he has put into it he will not be able to draw interest on, because he will not be drawing interest on the 10 per cent retained. The amount of money that he had to put up to cover shortage on his contract will have been $14,000.00, on which he will have to pay interest to his bank. If, on the other hand, he commences the earthwork first, he does 100,000 yards of earthwork, costing him 26 cents, on which he gets back immediately 42 cents, and he has $16,000 for working capital, in addition to $4,666.66 held up. He then does the rock work, and the rock work never exhausts his capital, and he has no interest to pay except on his plant, which he can easily do out of his $16,000.

This is not only a practical problem in how to handle a contract without being wiped out financially, but it is an exceedingly important one as defining where the ultimate success in the operation lies. It can readily be seen that when a contract is taken on close figures, the entire success of the financial operation will depend upon the proper layout, as indicated above.

Unbalanced Bids. We shall assume again, for purposes of illustration, that a certain contractor desires to bid on some public work involving the removal of 100,000 cubic yards of earthwork and 50,000 cubic yards of rock work. He estimates that he can do the earthwork at a profit for 30 cents per cubic yard, or $30,000; and rock work for 80 cents per cubic yard, or $40,000. If the work in the above example were *classified*, and the contractor were paid so much money for each yard of rock and so much money for each yard of earth excavated, and his bid read 80 cents for rock and 30 cents for earth, it would be said to be a *balanced bid*. Other contractors, seeing his bid, would know that he considered that he could do the rock work at a profit at 80 cents, and earthwork at a profit at 30 cents. In order to prevent them from obtaining this information, the contractor can *unbalance* his bid, as it is termed; and in this event he would bid perhaps as follows--namely, 100,000 yards of earth at 40 cents, or $40,000; and 50,000 yards of rock at 60 cents, or $30,000. The total amount of this contract would be the same, and he would make the same profit; but his competitors would be deceived as to his basis of doing work.

The disadvantage of this from the contractor's point of view is that, in the event of an error having been made in an estimate of quantity, he might find himself doing less than 100,000 yards of earth and more than 50,000 yards of rock, in which event he would stand to lose money.

Material Supply. In concrete work particularly, it is all-important that material—cement, sand, and stone—be promptly shipped, and at the same time not too promptly shipped. If the shipments are not promptly made, there will be a failure of material to arrive, which will throw the men out of work, with all that this implies in high costs. If the material is shipped too rapidly, it will be necessary either to unload it into a stock pile, which will involve the re-handling of the material; or to pay demurrage charges to the railroad company, if the shipments are made by rail.

In such work, at a time when there is likely to be any freight congestion in the country, stock-pile facilities should be provided to care for a supply of material to carry the work for one to two weeks.

On a piece of work involving, say, two large concrete mixers capable of mixing 300 yards of material each per day, there will be used 900 yards of stone and sand per day, which, on a ten-day basis, will mean a very respectable stock pile. This 9,000 yards of material, costing perhaps one dollar per yard, means an investment of $9,000 in stock pile, on which interest must be paid at the rate of, say, 6 per cent, or $2.00 per working day, which means a trivial item compared with the advantages derived from having a constant supply of material. The total cost of this stock pile as against no stock pile at all, is the cost of one re-handling of material out of the stock pile, which at 5 cents per yard would be $450. This amount is very much less than the damage that would accrue from not having any stock pile at all. On most concrete jobs, there is usually provided a large storehouse for cement; and when the work has to go over from one working season to another, it is frequently the custom to leave the cement in storage. This is frequently a cause of loss of money, because the cement, being hydroscopic, absorbs moisture from the atmosphere, and is liable to spoil in consequence. This can be avoided by keeping the storehouse dry and warm through the winter, but this again is an expensive matter.

Old versus New Machinery. In planning construction work, the question always comes up as to whether to use old or new machinery. No hard and fast rule can be prescribed. A case occurred upon an important contract where there were needed some new boiler tubes for the boiler that ran the main supply pump. The purchasing agent of the contracting company, who happened also to be the President and Chief Engineer of the company, bought some second-hand boiler tubes, which were forthwith put into this boiler. The saving on the boiler tubes was probably $8 or $10. The loss caused by a breakdown of the same boiler was nearly $50. In purchasing second-hand material, if the material can be thoroughly and rigidly inspected, it is perhaps wise to purchase it, and sometimes money can be saved; but as a general proposition, no second-hand material should be purchased for a contract, unless it is done with the determination of putting this material in first-class condition before it is used. The best inspection, as a general thing, will not disclose the exact condition of old material. By this it is not meant to intimate that new material should be purchased for every new contract.

Use of Maps. A precaution on construction work that is very seldom taken by contractors generally, and one that is a most certain saver of money, is to have a complete map of the work to a large scale carefully prepared, on which should be indicated day by day the progress of the work. This map, if kept up to date, will enable the manager of a company, or the president and directors, to know in detail the progress of the work, without necessarily going out on the work; and from it can be found the quantities of needed materials, such as rail, pipe, etc.

Standard Instructions. Every organization doing field work would do well to follow the custom admirably illustrated by Frank B. Gilbreth, of issuing regular standard instructions to foremen and to employees generally. These instructions have been published in book form by the Myron C. Clark Publishing Company, and are an admirable example of the type. The idea follows that of the old Railroad Company's "Book of Rules" that will tend toward evading similar accidents in the future. In this manner eventually a contractor can obtain a control of his organization, and a freedom from accidents, that will be extremely valuable.

Chronological Charts. These are intended to show the proposed time of completion in certain parts of the work. A valuable aid to a manager on work requiring a large amount of material, and where there is a small amount of available space, is a chart showing the time and quantity of expected materials and supplies. This will enable him to see at a glance where he may expect to be in the matter of his materials, and will tend to relieve his mind of one of its most annoying problems. These same charts can also show him the estimated times of completion of certain parts of the work.

REVIEW QUESTIONS.

PRACTICAL TEST QUESTIONS.

In the foregoing sections of this Cyclopedia numerous illustrative examples are worked out in detail in order to show the application of the various methods and principles. Accompanying these are examples for practice which will aid the reader in fixing the principles in mind.

In the following pages are given a large number of test questions and problems which afford a valuable means of testing the reader's knowledge of the subjects treated. They will be found excellent practice for those preparing for Civil Service Examinations. In some cases numerical answers are given as a further aid in this work.

REVIEW QUESTIONS

ON THE SUBJECT OF

STRENGTH OF MATERIALS.

PART I.

1. When a $\frac{3}{4}$-inch round rod sustains a pull of 10,000 pounds, what is the value of the unit-tensile stress in the rod?

2. What do you understand by Hooke's Law?

3. What are the dimensions of a square white pine post, needed to support a steady load of 6,500 pounds with a factor of safety of 8?

4. How large a force is required to punch a 1-inch hole through a $\frac{3}{4}$-inch plate of wrought iron, if the ultimate shearing strength of the material is 40,000 pounds per square inch?

5. Compare the ultimate strengths of wood along and across the grain; also the ultimate tensile and compressive strengths of cast iron.

6. Make a sketch of a beam 20 feet long resting on end supports, and represent loads of 6,000, 3,000, 1,000, and 4,000 pounds at points 2, 5, 11, and 16 feet from the left end, respectively. What is the value and sign of the moment of each of these loads about the middle of the beam? Also about the left end?

7. A beam 15 feet long is supported at two points, 2 feet from the right end, and 3 feet from the left end. If the beam sustains a uniform load of 400 pounds per foot, what are the values of the reactions?

8. Compute the values of the external shear and bending moment for the loaded beam described in question 6, at sections 1 4, 10, and 15 feet from the left end.

9. Draw shear and moment diagrams to scale for the beam described in question 7.

10. Suppose a T-bar 2 inches deep, has a flange 3 inches wide, and is $\frac{1}{4}$ inch thick throughout. Locate the center of gravity by computation.

REVIEW QUESTIONS

ON THE SUBJECT OF

STRENGTH OF MATERIALS.

PART II.

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1. A cantilever beam 6 feet in length projects from a wall and sustains an end load of 300 pounds. The cross-section being as in Fig. 38, find the greatest tensile and compressive unit-stresses, and state where they occur.

2. An I-beam weighing 30 pounds per foot, rests on end supports 25 feet apart. Its section modulus is 20.4 inches^3, and its working strength 16,000 pounds per square inch. Calculate weight of the beam.

3. A wooden beam 15 feet long, 4 × 14 inches in cross-section sustains a load of 4,000 pounds 5 feet from one end, and 2,000 pounds at the middle. Compute the greatest unit shearing stress.

4. What do you know about radius of gyration? Give an example.

5. Find the factor of safety of a 24-inch 80-pound steel I-beam 17 feet long, used as a flat-ended column to sustain a load of 150,000 pounds. Note.—Use "Rankine's Formula."

6. A steel Z-bar is 20 feet long and has square ends; the least radius of gyration of its cross-section is 3.1 inches, and its area of cross-section is 24.5 square inches. Calculate the safe load with a factor of safety of 6. Note. Use "Rankine's Formula."

7. Make sketches of the following:

Lap joint single-riveted;
" " double-riveted;
Butt " single-riveted;
" " double-riveted.

REVIEW QUESTIONS

ON THE SUBJECT OF

GRAPHICAL STATICS.

1. Define concurrent and non-concurrent forces, equilibrant, and resultant.

2. What do you understand by the "Triangle law?"

3. Determine the magnitude and direction of the resultant of the 400- and 800-pound forces of Fig. 47.

4. Compute the magnitude and direction of the resultant of the 600- and 700-pound forces of Fig. 47.

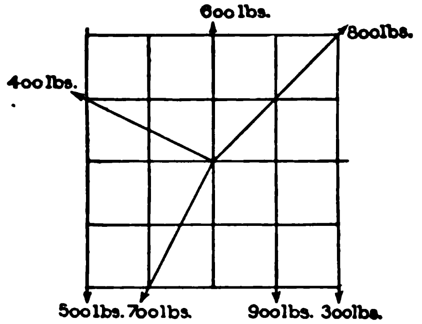


Fig. 47.

5. Determine the values of the horizontal and vertical components of the 700-pound force of Fig. 47.

6. Determine the magnitude and direction of the resultant of the four forces acting through the center of Fig. 47; also of their equilibrant.

7. Compute the resultant of the four parallel forces represented in Fig. 47.

8. Find the resultant of the 300-, 400-, 500-, and 800-pound forces of Fig. 47.

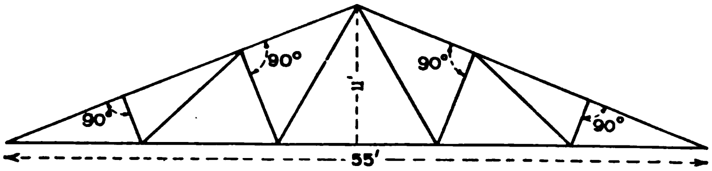
9. Suppose that the truss of Fig. 48 is one of several used to support a roof, the trusses being 16 feet apart. What is the

probable weight of each? What is the total roof load borne by each truss if the roofing weighs 18 pounds per sq. foot?

10. What is the total snow load for the truss if the snow weighs 20 pounds per square foot (horizontal)?

11. Compute the total wind load for the truss of Question 9, when the wind blows 75 miles per hour.

12. Supposing that the right end of the truss of Fig. 48 rests on rollers, and that the left end is fastened to its support, compute the values of the reactions (*a*) when the wind blows on the left; (*b*) when it blows on the right, 90 miles per hour.



Upper joints equally distant apart.

Fig. 48.

13. Construct stress diagram for the truss of Fig. 48 for the following cases:

(*a*) When sustaining dead load only as computed in answer to Question 9.

(*b*) When sustaining wind pressure on the left, the truss being supported as described in Question 12.

(*c*) When sustaining wind pressure on the right, the truss being supported as described in Question 12.

14. Make a complete record of the stresses as determined in answer to the preceding question for cases *a*, *b* and *c*, and for snow load as computed in answer to Question 10. Compute from the record the value of the greatest stress which can come upon each member due to combinations of loads, assuming that the wind and snow loads will not act at the same time.

15. Suppose that the truss of Fig. 49 is one of several used to support a roof, the trusses being 12 feet apart. What is the probable weight of a truss and that of the roofing supported by one truss, if the roofing weighs 15 pounds per square foot?

16. Compute the apex loads for the truss of Fig. 49 for snow if it weighs 20 pounds per square foot (horizontal).

REVIEW QUESTIONS

ON THE SUBJECT OF

ROOF TRUSSES

---

1. Name and describe the three classes of roof trusses, and give a sketch of one truss of each class.

2. Give a sketch of the Fink truss and the Modified Fink truss.

3. Given $W = \frac{3}{4} al \left(1 + \frac{l}{10}\right)$, compute the dead panel load of an eight-panel Fink truss of 80-foot span, if the distances between trusses is 20 feet and the roof covering is composed of corrugated steel.

4. Tell how a felt and asphalt roof is made and laid.

5. What is a non-condensation roofing?

6. Design the purlins if they are to be spaced 6 feet apart and the trusses are to be spaced 16 feet apart. They are to carry 40 pounds per square foot of roof surface.

7. Write one page on the economical pitch and spacing of roof trusses.

8. If the trusses are of 80-foot span and are spaced 20 feet center to center, and are eight-panel Fink, compute the stresses in the knee-braces if they join the columns 8 feet from the top. The columns are 25 feet long; the normal wind pressure on the roof is 12 pounds; the pitch of the roof is $\frac{1}{4}$; the normal wind pressure on the side of the building is 25 pounds; and the columns are considered free.

9. In the trusses of question 8, above, compute the bending moment in the posts.

10. If in question 8, above, the girts are placed 4 feet apart, design them.

REVIEW QUESTIONS

ON THE SUBJECT OF

COST-ANALYSIS ENGINEERING

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1. What is the object of cost keeping and cost analysis? Of cost distribution?

2. What is "cost?"

3. What five essentials must a cost-keeping system possess?

4. What are *overhead expenses?*

5. What are the four most common ways of time-keeping? Give the basis of each method.

6. What are some of the difficulties that confront the timekeeper when taking time in the field?

7. How is brickwork measured? plastering? pavement? earthwork?

8. What is a *bonus?* How may a bonus system be applied to construction work, and what are some of its advantages?

9. Devise a method for measuring the work done by a steam shovel in each shift. Method to be operated by an engineer or inspector, and must be quick and easy. No instrument to be used.

10. Upon what is the payment of the contractor from time to time based? Does this always give the contractor all that is due him?

11. What is an *unbalanced bid?* Why are they sometimes used? What is the objection to them?

12. What advantage can you see in process cost analysis? Make such analysis for erecting centering for concrete factory.

13. Devise method for measuring brickwork so that work of each bricklayer can be determined and credited to the right man. Do same for dimension stone work.

14. What advantages are claimed for the piece-work system?

15. What is an *unclassified* contract in excavation?

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